8.1 For a 5 point running average filter

$$h[n] = \tfrac{1}{5}\delta[n] + \tfrac{1}{5}\delta[n-1] + \tfrac{1}{5}\delta[n-2] + \tfrac{1}{5}\delta[n-3] + \tfrac{1}{5}\delta[n-4]$$

$$H(z) = \tfrac{1}{5} + \tfrac{1}{5}z^{-1} + \tfrac{1}{5}z^{-2} + \tfrac{1}{5}z^{-3} + \tfrac{1}{5}z^{-4}$$

$$H(\hat{\omega}) = \tfrac{1}{5} + \tfrac{1}{5}e^{-j\hat{\omega}} + \tfrac{1}{5}e^{-j2\hat{\omega}} + \tfrac{1}{5}e^{-j3\hat{\omega}} + \tfrac{1}{5}e^{-j4\hat{\omega}}$$

(a) $x[n] = 10\,\delta[n-50]$

Step 1: Since $x[n]$ is so simple just compute
$y[n] = x[n] * h[n]$

(b) $x[n]$ is a sampled audio signal of 20,000 numbers

Step 1: Compute the difference equation

$$y[n] = \sum_{k=0}^{4} \tfrac{1}{5}\, x[n-k]$$

Step 2: Use the difference equation to determine
$y[n]$ (probably using MATLAB)

(c) $x[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$

Step 1: Use convolution, that is, the
difference equation

8.1 (d) $x[n] = 3\cos(0.1\pi n - \pi/3) + 2\cos(0.4\pi n - \pi)$

Step 1: Find $H(\hat{\omega})$ for the 5 point averager.

Step 2: Evaluate $H(\hat{\omega} = 0.1\pi)$ and $H(\hat{\omega} = 0.4\pi)$

Step 3: Obtain $y[n]$ by applying the magnitude + phase of $H(\hat{\omega})$ to $x[n]$.

8.1 (e) Step 1: find $h[n]$ and convolve with $10\delta[n-50]$

Step 2: Use $H(\hat{\omega})$ as in part (d) and apply to the sinusoidal parts of $x[n]$.

Step 3: add the two solutions.

8.2 (a) $H(z) = \dfrac{1 - z^{-4}}{4}$

(b) $H(\hat{\omega}) = \dfrac{1}{4} - \dfrac{e^{-j4\hat{\omega}}}{4}$

(c) $H(\hat{\omega}=0) = \frac{1}{4} - \frac{1}{4} = 0$

$H(\hat{\omega} = 0.75\pi) = \frac{1}{4} - \frac{1}{4} e^{-j4\left(\frac{3}{4}\right)\pi}$

$= \frac{1}{4} - \frac{1}{4} e^{-j3\pi} = \frac{1}{4} - \frac{1}{4}(-1) = \frac{1}{2}$

8.2(c) $H(\hat{\omega} = 1.5\pi) = \frac{1}{4} - \frac{1}{4}e^{-j4\left(\frac{3\pi}{2}\right)} = \frac{1}{4} - \frac{1}{4}e^{-j6\pi}$

$= \frac{1}{4} - \frac{1}{4}(1) = 0$

There is aliasing present, so the third term in $x[n]$ must be corrected to place it in the range $-\pi \le \hat{\omega} \le \pi$. $x[n]$ becomes:

$$x[n] = 3 + 2\cos(0.75\pi n - \pi/4) + 11\cos(0.5\pi n + \pi/3)$$

Applying the filter the output, $y[n]$, becomes:

$$y[n] = 3(0) + 2\left(\tfrac{1}{2}\right)\cos(0.75\pi n - \pi/4) + 11(0)\cos(0.5\pi n + \pi/3)$$

$$= \cos(0.75\pi n - \pi/4)$$

Finally, $\hat{\omega} = \frac{\omega}{f_s}$ $\quad \omega = \hat{\omega} \cdot 8000$, so

$$y(t) = \cos(6000\pi t - \pi/4)$$

8.3(a) $H(z) = 1 + 2z^{-2} - z^{-4} + 2z^{-5}$

$$y[n] = x[n] + 2x[n-2] - x[n-4] + 2x[n-5]$$

$$h[n] = \delta[n] + 2\delta[n-2] - \delta[n-4] + 2\delta[n-5]$$

$$H(\hat{\omega}) = 1 + 2e^{-j2\hat{\omega}} - e^{-j4\hat{\omega}} + 2e^{-j5\hat{\omega}}$$

8.3(b) $y[n] = x[n] + 2x[n-2] - 3x[n-3]$

$h[n] = \delta[n] + 2\delta[n-2] - 3\delta[n-3]$

$H(\hat{\omega}) = 1 + 2e^{-j2\hat{\omega}} - 3e^{-j3\hat{\omega}}$

$H(z) = 1 + 2z^{-2} - 3z^{-3}$

8.3(c) $h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] + \delta[n-4]$

$y[n] = x[n] - x[n-1] + x[n-2] - x[n-3] + x[n-4]$

$H(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$

$H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}$

8.3(d) $H(\hat{\omega}) = [1 - \cos(3\hat{\omega})]e^{-j4\hat{\omega}}$

$H(\hat{\omega}) = \left[1 - \frac{1}{2}e^{j3\hat{\omega}} - \frac{1}{2}e^{-j3\hat{\omega}}\right]e^{-j4\hat{\omega}}$

$= e^{-j4\hat{\omega}} - \frac{1}{2}e^{-j\hat{\omega}} - \frac{1}{2}e^{-j7\hat{\omega}}$

$H(z) = -\frac{1}{2}z^{-1} + z^{-4} - \frac{1}{2}z^{-7}$

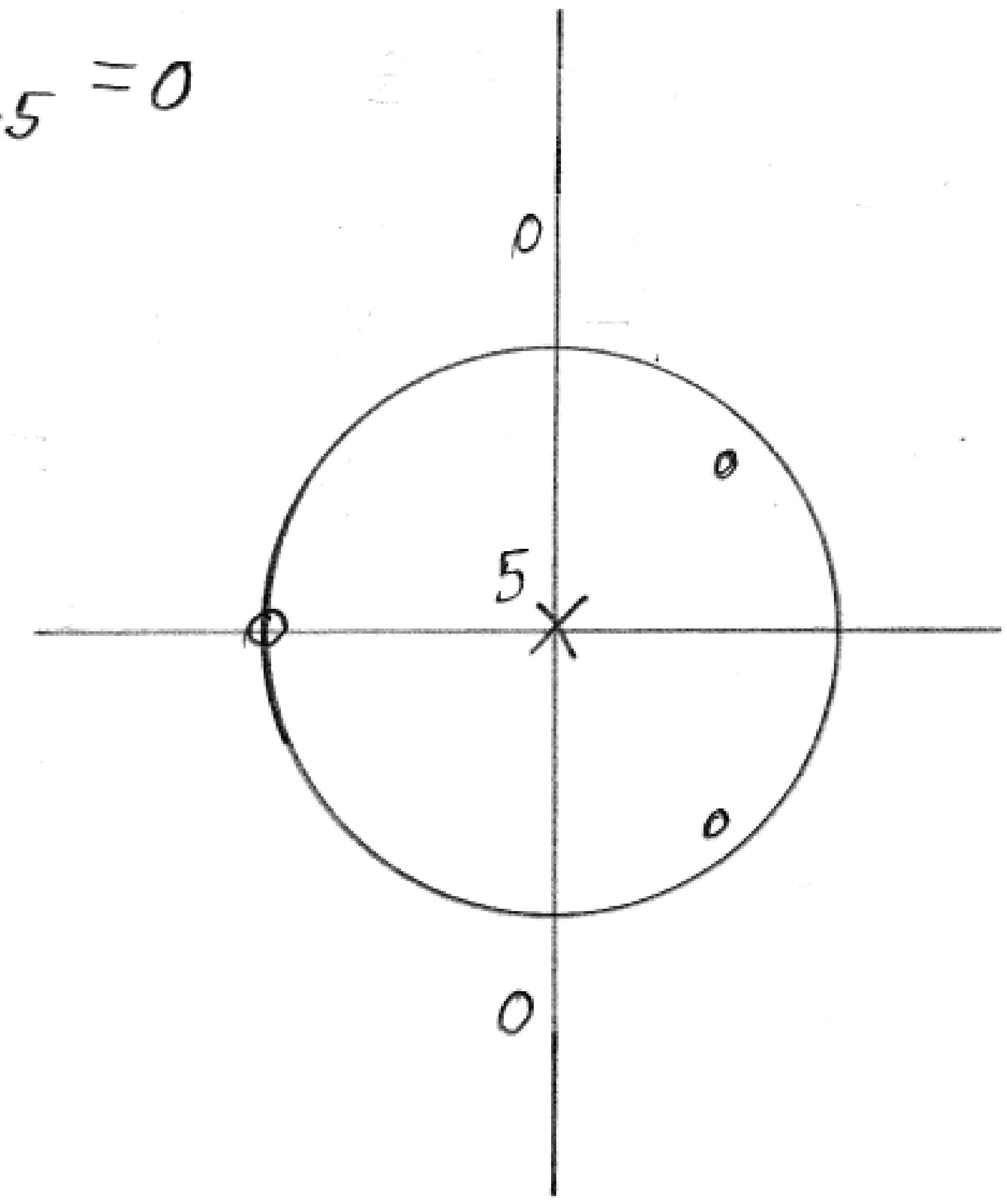
$h[n] = -\frac{1}{2}\delta[n-1] + \delta[n-4] - \frac{1}{2}\delta[n-7]$

$y[n] = -\frac{1}{2}x[n-1] + x[n-4] - \frac{1}{2}x[n-7]$

8.3(e) $H(z) = 1 + 2z^{-2} - z^{-4} + 2z^{-5}$

$= \dfrac{z^5 + 2z^3 - z + 2}{z^5}$ $\quad p_{1-5} = 0$

$p_{1-5} = 0$
$z_1 = -0.1304 + j1.5891$
$z_2 = -0.1304 - j1.5891$
$z_3 = -1$
$z_4 = 0.6304 + j0.6240$
$z_5 = 0.6304 - j0.6240$

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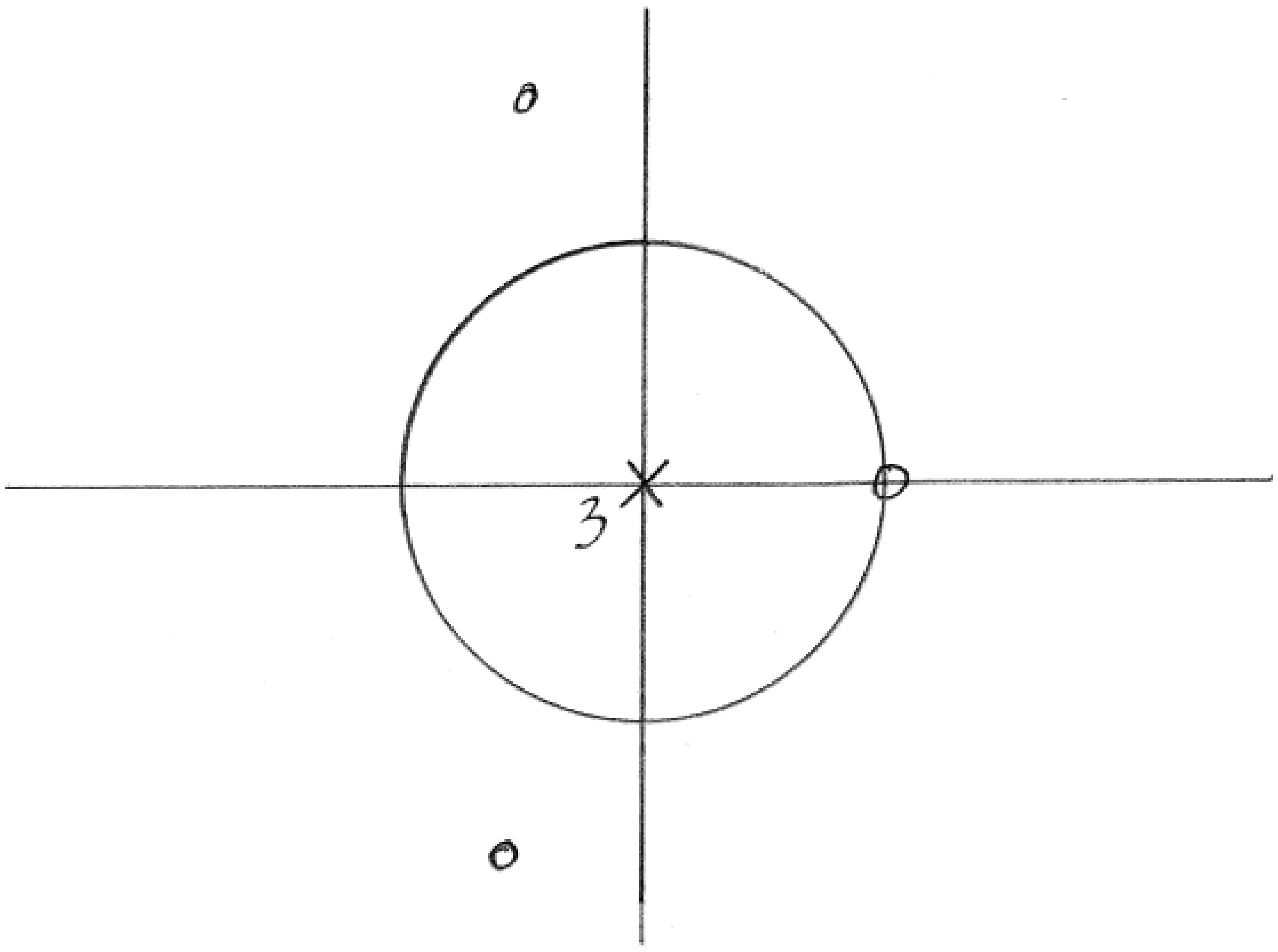
$H(z) = 1 + 2z^{-2} - 3z^{-3}$
$= \dfrac{z^3 + 2z - 3}{z^3}$

$p_{1-3} = 0$
$z_1 = -0.5 + j1.6583$
$z_2 = -0.5 - j1.6583$
$z_3 = 1$

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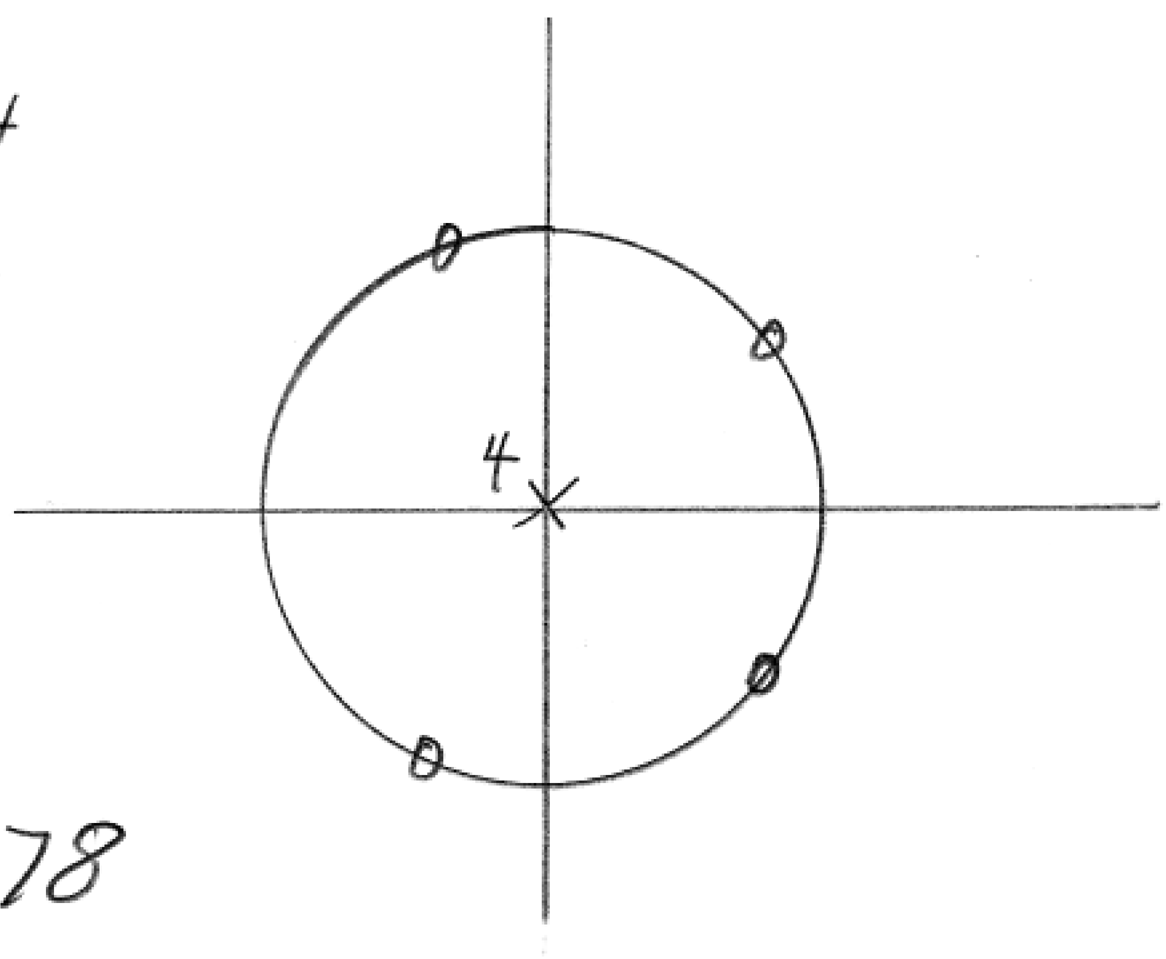
$H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}$
$= \dfrac{z^4 - z^3 + z^2 - z + 1}{z^4}$

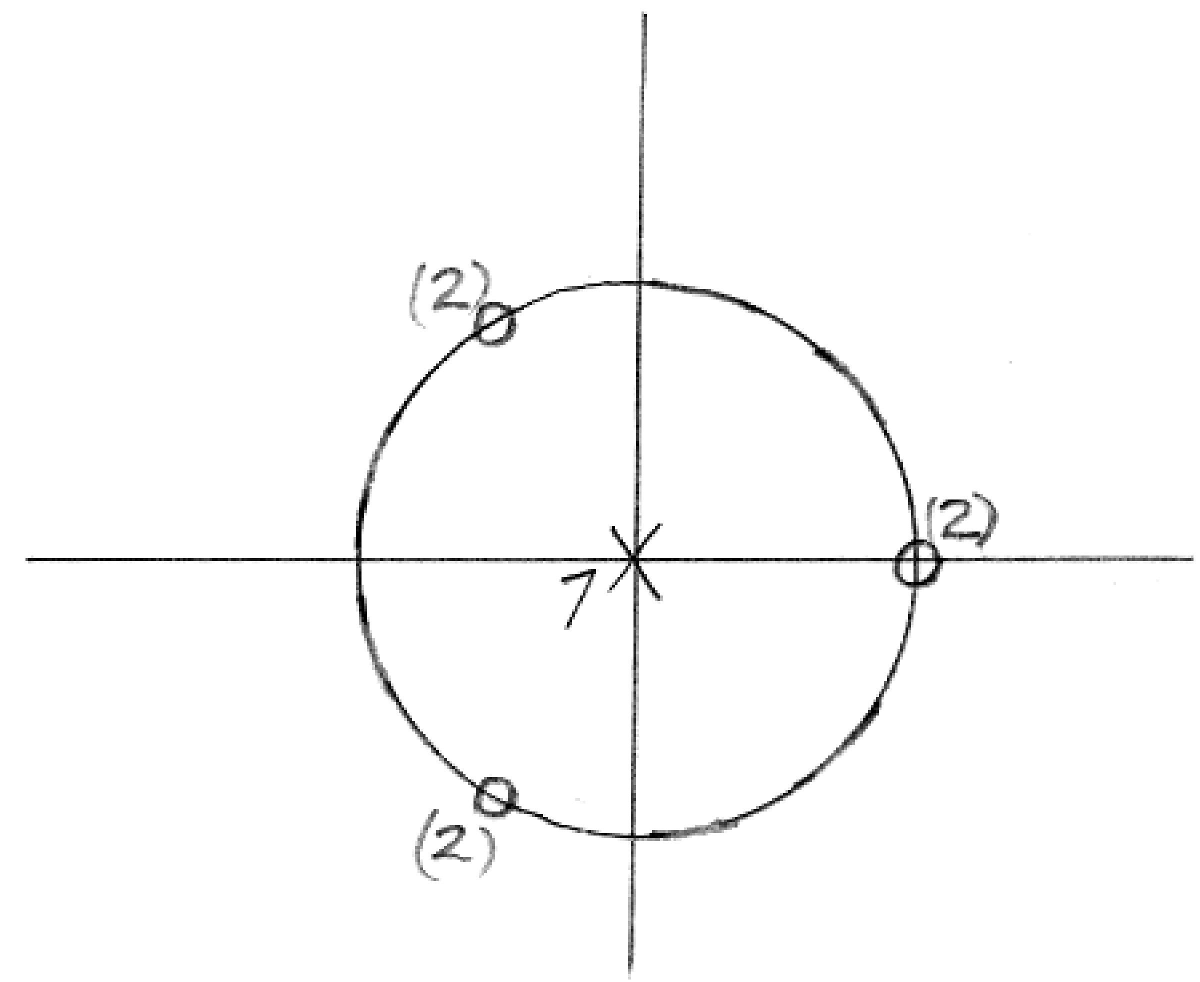
$p_{1-4} = 0 \quad z_{1\&2} = -0.3090 \pm j0.9511$

$z_{3\&4} = 0.8090 \pm j0.5878$



$$H(z) = -\tfrac{1}{2}z^{-1} + z^{-4} - \tfrac{1}{2}z^{-7}$$

$$= \frac{-\tfrac{1}{2}z^{6} + z^{3} - \tfrac{1}{2}}{z^{7}}$$




$p_{1-7} = 0$

$z_{1+2} = 1$

$z_{3+4} = e^{j2\pi/3}$

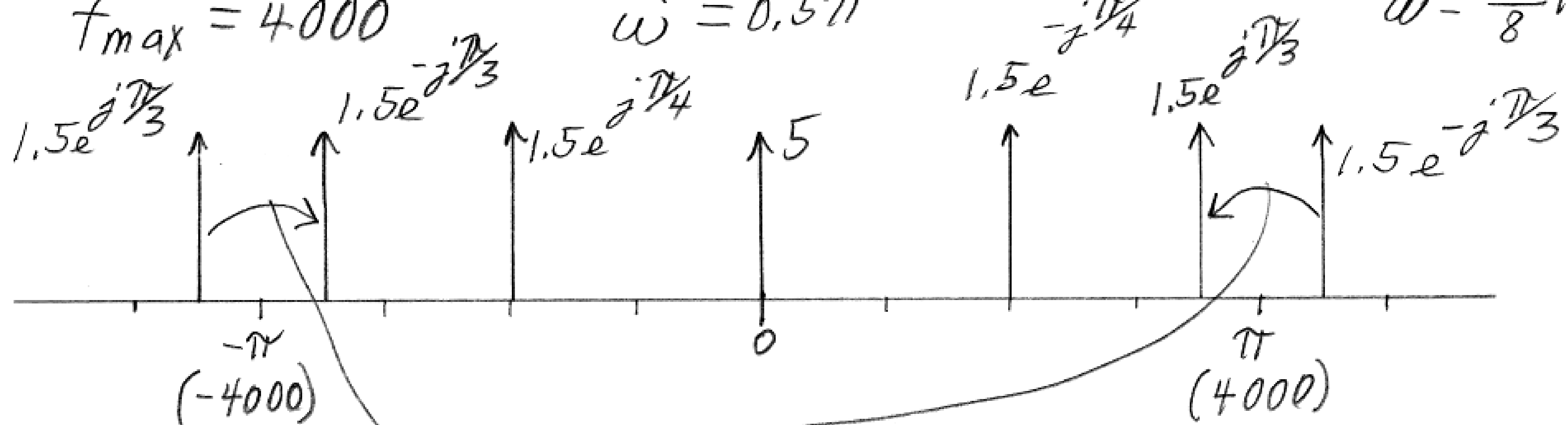
$z_{5+6} = e^{-j2\pi/3}$

$H(z) \to 0$ when $z \to \infty$ so there is a zero at infinity

8.4 $x(t) = 5 + 3\cos(4000\pi t - \frac{\pi}{4}) + 3\cos(9000\pi t - \frac{\pi}{3})$

$f_s = 8000$ $\quad f = 2000$ $\quad f = 4500$

$f_{max} = 4000$ $\quad \hat{\omega} = 0.5\pi$ $\quad \hat{\omega} = \frac{9}{8}\pi$




Frequency above $f_{max}$ aliases to below $f_{max}$, so that $x[n]$ becomes

$$x[n] = 5 + 3\cos(0.5\pi n - \tfrac{\pi}{4}) + 3\cos(\tfrac{7}{8}\pi + \tfrac{\pi}{3})$$

$H(z) = 1 + z^{-2}$ $\quad H(\hat{\omega}) = 1 + e^{-j2\hat{\omega}}$ $\quad H(0) = 1 + 1 = 2$

$H(\frac{\pi}{2}) = 1 + e^{-j\pi} = 0$ $\quad H(\frac{7\pi}{8}) = 1 + e^{-j\frac{7\pi}{4}} = 1 + \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$= 1.85e^{+j0.39}$

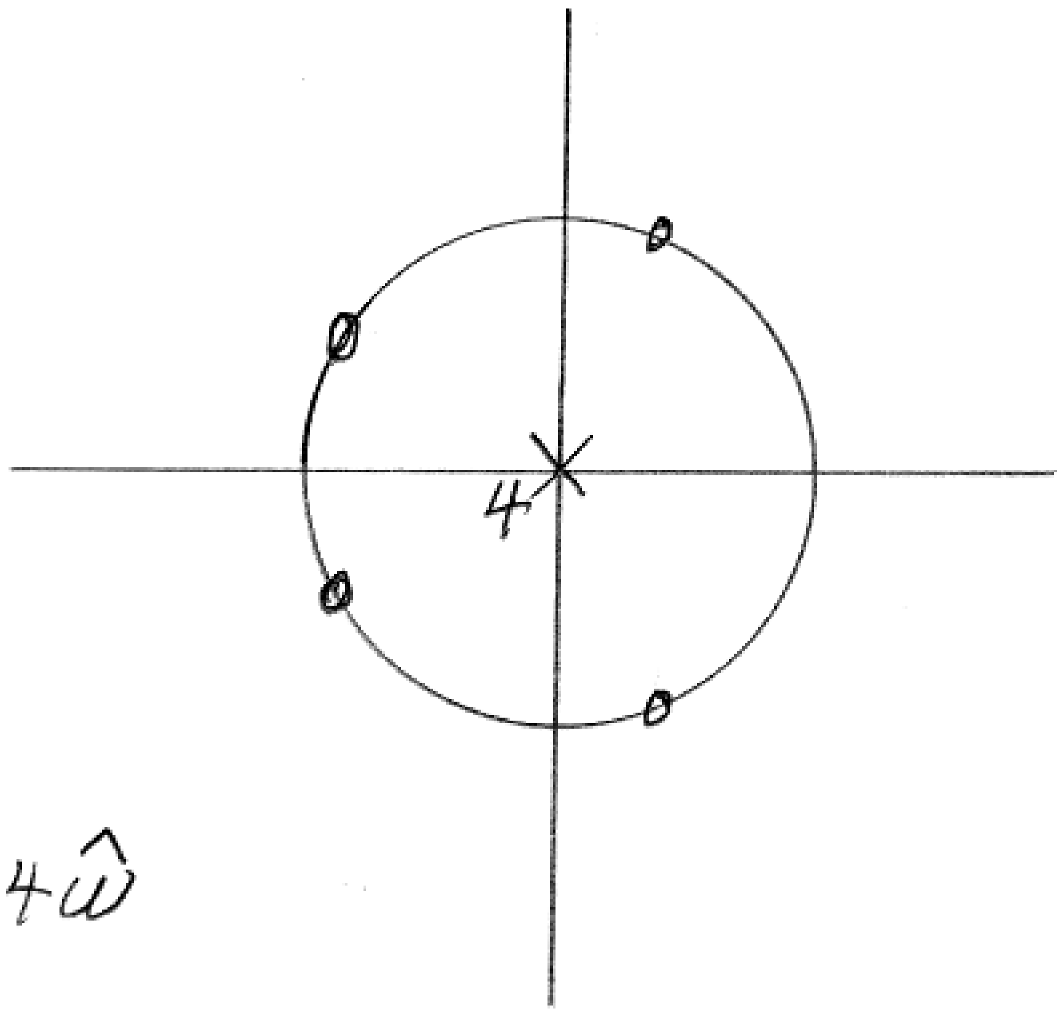
$$y[n] = 5(2) + 3(0)\cos(0.5\pi n - \pi/4) + 3(1.85)\cos\left(\tfrac{9\pi}{8}n + \pi/3 + 0.39\right)$$

$$y(t) = 10 + 5.55\cos(7000\pi t + 1.44)$$

<u>8.5</u> $y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$

$$H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$$

$$= \frac{z^4 + z^3 + z^2 + z + 1}{z^4}$$




$p_{1-4} = 0 \quad z_{1\&2} = 0.3090 \pm j0.9511$

$z_{3\&4} = -0.8090 \pm j0.5878$

$$H(\hat{\omega}) = 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$= \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}}$$

<u>8.5(d)</u> $x[n] = 1 + 2\cos(n\hat{\omega}_0)$

From $H(\hat{\omega}) = \dfrac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}}$, there are zeros everywhere $\sin(5\hat{\omega}/2) = 0$

$$5\hat{\omega}/2 = \pi k \quad \boxed{\hat{\omega}_0 = \frac{2\pi}{5}, \frac{4\pi}{5}}$$

8.5(d) cont. $\left|H(0)\right| = \lim_{\hat{\omega}\to 0} \frac{\sin 5\hat{\omega}/2}{\sin \hat{\omega}/2} \to \frac{5\hat{\omega}/2}{\hat{\omega}/2} \to 5$

$$\boxed{y[n] = 5}$$

8.6 $H(z) = (1-z^{-1})(1-e^{j\pi/3}z^{-1})(1-e^{-j\pi/3}z^{-1})$

(a) $H(z) = (1-z^{-1})(1-e^{-j\pi/3}z^{-1} - e^{j\pi/3}z^{-1} + e^{j0}z^{-2})$

$= (1-z^{-1})(1-z^{-1}(e^{j\pi/3}+e^{-j\pi/3}) + z^{-2})$

$= (1-z^{-1})(1-2\cos(\pi/3)z^{-1} + z^{-2})$

$= (1-z^{-1})(1-z^{-1}+z^{-2}) = 1-z^{-1}+z^{-2}-z^{-1}+z^{-2}-z^{-3}$

$= 1-2z^{-1}+2z^{-2}-z^{-3}$

$\therefore y[n] = x[n]-2x[n-1]+2x[n-2]-x[n-3]$

8.6(b) for $x[n]=\delta[n]$

$y[n] = \delta[n]-2\delta[n-1]+2\delta[n-2]-\delta[n-3]$

8.6(c) $x[n] = \delta[n-1]-2\delta[n-3]+\delta[n-4]$

$X(z) = z^{-1}-2z^{-3}+z^{-4}$ $\quad Y(z) = X(z)\cdot H(z)$

8.6(c) cont. $X(z) = z^{-1} - 2z^{-3} + z^{-4}$

$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$

$$Y(z) = z^{-1} - 2z^{-3} + z^{-4}$$
$$-2z^{-2} + 4z^{-4} - 2z^{-5}$$
$$+2z^{-3} - 4z^{-5} + 2z^{-6}$$
$$-z^{-4} + 2z^{-6} - z^{-7}$$

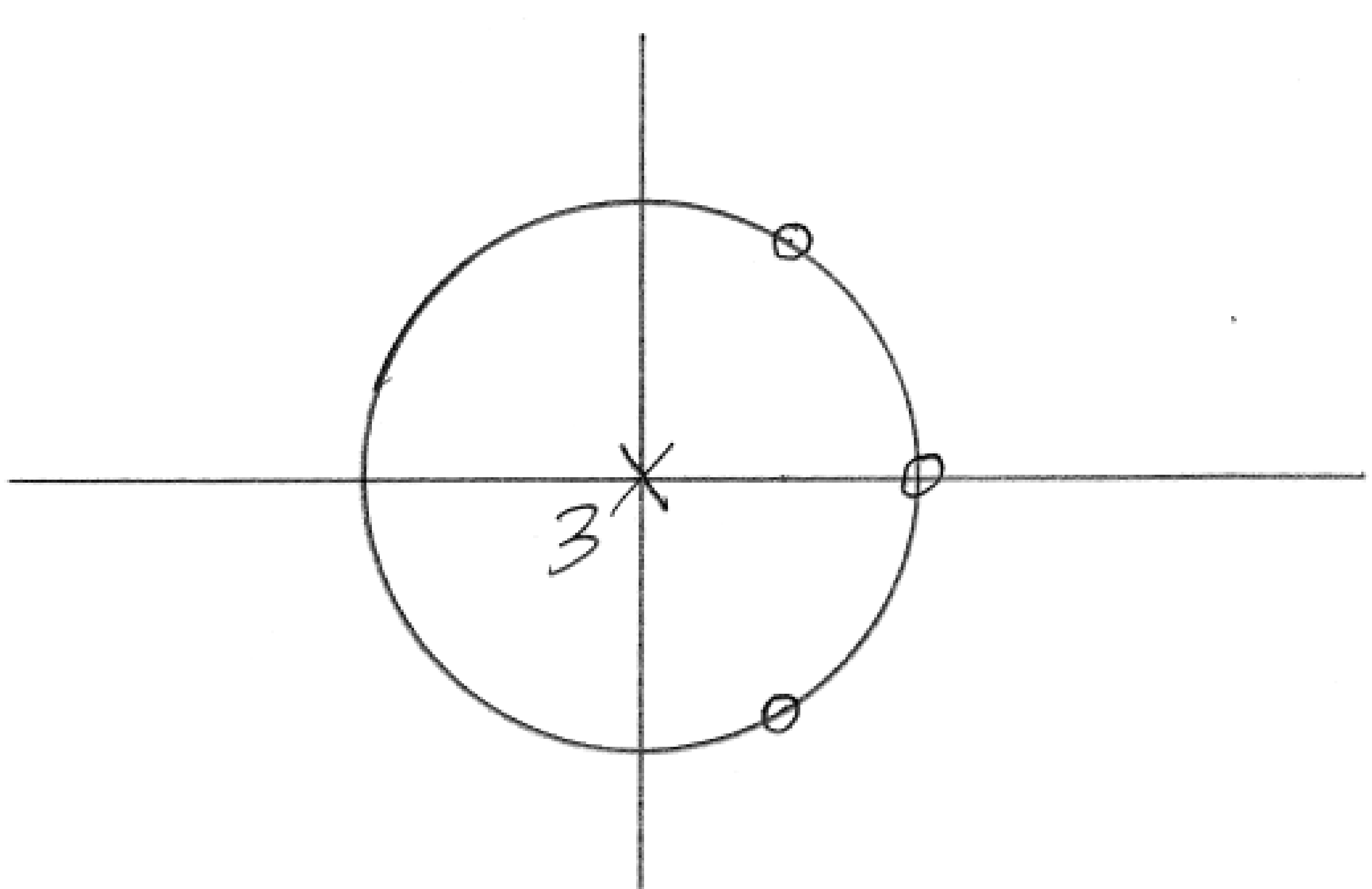
$$y[n] = \delta[n-1] - 2\delta[n-2] + 4\delta[n-4] - 6\delta[n-5] + 4\delta[n-6] - \delta[n-7]$$

8.6(d) $H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$

$$= \frac{z^3 - 2z^2 + 2z - 1}{z^3}$$

$p_{1-3} = 0$ $\quad z_1 = 1$

$z_{2+3} = e^{\pm j\pi/3}$




8.6(e) $y[n] = 0$ for $\hat{\omega} = 0$

$\hat{\omega} = +\pi/3$