ECE 2025 Fall 2001 Problem Set #9

9.1

(a) $$\delta(t+5) * \left[\delta(t-10) + 3e^{-t}\cos(5\pi t)u(t) + \sin(50\pi t)\right]$$

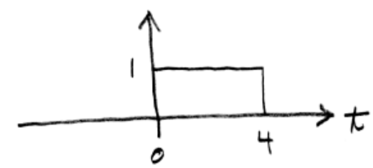
$$= \delta(t-10+5) + 3e^{-(t+5)}\cos(5\pi(t+5))\,u(t+5) + \sin(50\pi(t+5))$$

$$= \delta(t-5) - 3e^{-(t+5)}\cos(5\pi t)\,u(t+5) + \sin(50\pi t)$$

(b) $$u(-t+4)u(t)\left[\delta(t+1) + \delta(t-1) + \delta(t-5)\right]$$

$$= \delta(t-1)$$

$u(t)u(-t+4)$



(c) $$\frac{d}{dt}\left[e^{-(t+1)}\sin(5\pi t)u(t+1)\right]$$

$$= e^{-(t+1)}\sin(5\pi t)\delta(t+1) + \frac{d}{dt}\left[e^{-(t+1)}\sin(5\pi t)\right]u(t+1)$$

$$= \sin(-5\pi)\delta(t+1) + \left[e^{-(t+1)}5\pi\cos(5\pi t) - e^{-(t+1)}\sin(5\pi t)\right]u(t+1)$$

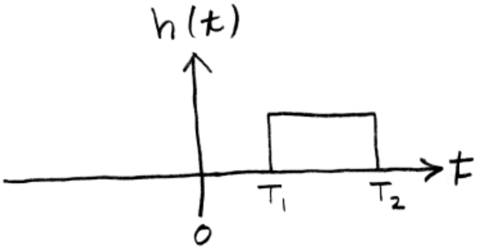
$$= e^{-(t+1)}\left[5\pi\cos(5\pi t) - \sin(5\pi t)\right]u(t+1)$$

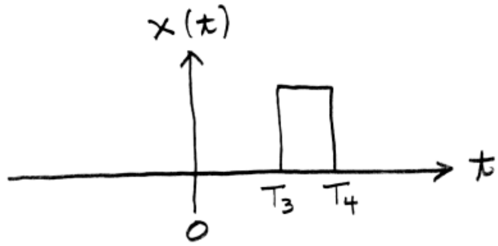
(d) $$\int_{-\infty}^{t} e^{-(\tau-1)}\left[\delta(\tau) + \delta(\tau-1)\right]d\tau$$

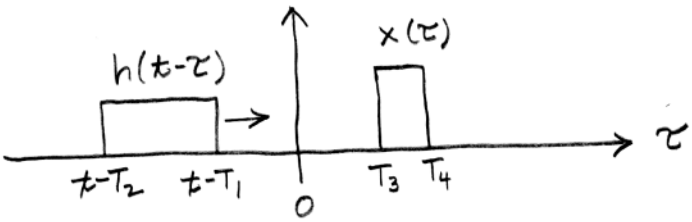
$$= \begin{cases} 0 & , \quad t < 0 \\ e^{-(0-1)} & , \quad 0 \le t < 1 \\ e^{-(0-1)} + e^{-(1-1)} & , \quad t \ge 1 \end{cases}$$

$$= \begin{cases} 0 & , \quad t < 0 \\ e & , \quad 0 \le t < 1 \\ e + 1 & , \quad t \ge 1 \end{cases}$$

9.2







$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau$$

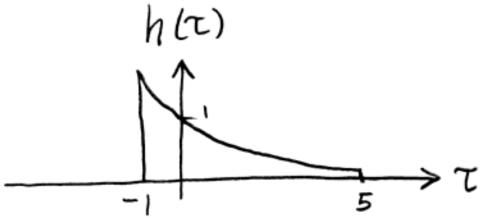
$$\therefore\ y(t) = 0 \quad \text{if} \quad t - T_1 \le T_3 \quad \text{or} \quad t - T_2 \ge T_4$$

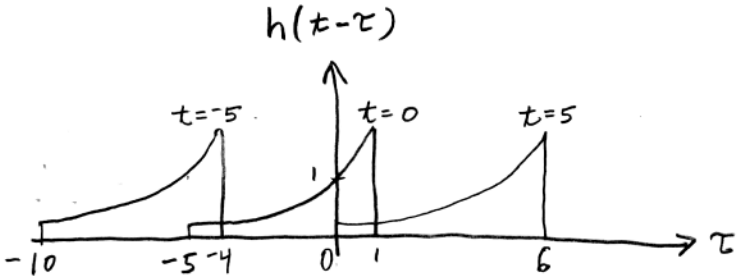
$$t \le T_1 + T_3 = T_5$$

$$t \ge T_2 + T_4 = T_6$$

9.3 $$h(t) = \begin{cases} e^{-t}, & -1 \le t < 5 \\ 0, & \text{otherwise} \end{cases}$$

(a)





(b) $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-1}^{5} e^{-t}\,dt = -e^{-t}\Big|_{-1}^{5} = e^{1} - e^{-5} < \infty$

$\Rightarrow$ system is stable

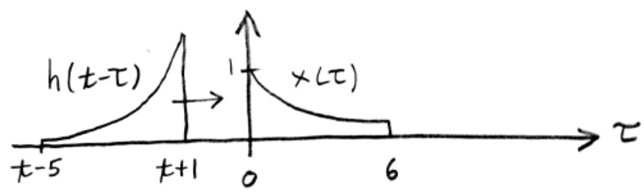
(c) $h(t) \neq 0$ for all $t<0$ $\Rightarrow$ system is not causal

(d) $x(t) = \delta(t-1) \Rightarrow y(t) = h(t) * \delta(t-1)$

$$= h(t-1)$$

$$= \begin{cases} e^{-(t-1)}, & -1 \le t-1 < 5 \\ 0, & \text{otherwise} \end{cases} \quad (0 \le t < 6)$$

(e) $$x(t) = \begin{cases} e^{-t/4}, & 0 \le t < 6 \\ 0, & \text{otherwise} \end{cases}$$



$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)\,d\tau$$

$$= \begin{cases} 0, & t+1 < 0 \\ \int_{0}^{t+1} e^{-(t-\tau)} e^{-\tau/4}\,d\tau, & 0 \le t+1 < 6 \\ \int_{t-5}^{6} e^{-(t-\tau)} e^{-\tau/4}\,d\tau, & 0 \le t-5 < 6 \\ 0, & t-5 \ge 6 \end{cases}$$

$$= \begin{cases} 0, & t < -1 \\ e^{-t}\frac{4}{3} e^{3\tau/4}\Big|_{0}^{t+1}, & -1 \le t < 5 \\ e^{-t}\frac{4}{3} e^{3\tau/4}\Big|_{t-5}^{6}, & 5 \le t < 11 \\ 0, & t \ge 11 \end{cases}$$

$$
= \begin{cases} 0 & , \quad t < -1 \\ \frac{4}{3}\left(e^{-\frac{1}{4}(t-3)} - e^{-t}\right) & , \quad -1 \le t < 5 \\ \frac{4}{3}\left(e^{-(t-9/2)} - e^{-\frac{1}{4}(t+15)}\right) & , \quad 5 \le t < 11 \\ 0 & , \quad t \ge 11 \end{cases}
$$

9.4

$$
y(t) = \int_{t-3}^{t} x(\tau)\,d\tau
$$

(a) $h(t) = y(t)\big|_{x(t)=\delta(t)}$

$$
= \int_{t-3}^{t} \delta(\tau)\,d\tau
$$

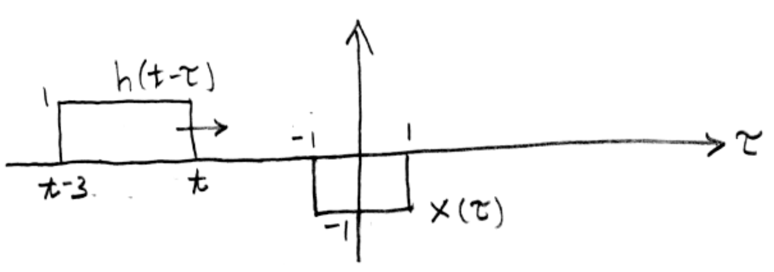
$$
= \begin{cases} 1 , & t-3<0 \text{ and } t \ge 0 \\ 0 , & \text{otherwise} \end{cases} \qquad (0 \le t < 3)
$$

(b) $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_0^3 1\,d\tau = \tau\Big|_0^3 = 3 < \infty$

$\Rightarrow$ system is stable

(c) $h(t) = 0$ for all $t<0$ $\Rightarrow$ system is causal

(d) $x(t) = u(t-1) - u(t+1)$



$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\,x(\tau)\,d\tau$$

$$= \begin{cases} 0 & , \; t<-1 \\ \int_{-1}^{t} -1\,d\tau & , \; -1 \le t < 1 \\ \int_{-1}^{1} -1\,d\tau & , \; t \ge 1 \text{ and } t-3<-1 \\ \int_{t-3}^{1} -1\,d\tau & , \; -1 \le t-3 < 1 \\ 0 & , \; t-3 \ge 1 \end{cases}$$

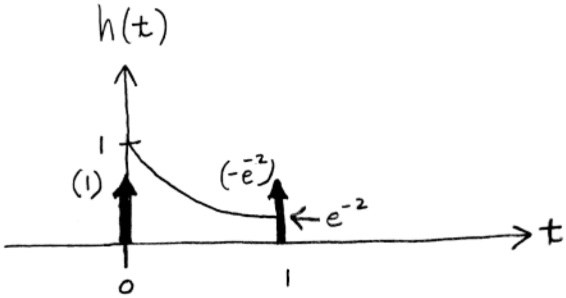
$$= \begin{cases} 0 & , \; t<-1 \\ -t-1 & , \; -1 \le t < 1 \\ -2 & , \; 1 \le t < 2 \\ t-4 & , \; 2 \le t < 4 \\ 0 & , \; t \ge 4 \end{cases}$$

9.5 $\quad x(t) = \delta(t) \Rightarrow w(t) = h_1(t) \Rightarrow y(t) = h_1(t) * h_2(t)$

$$h_1(t) = e^{-2t}\left(u(t)-u(t-1)\right)$$

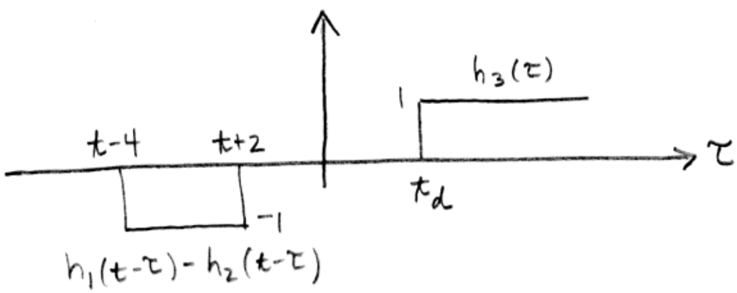
$$h_2(t) = \frac{d}{dt}\delta(t) + 3\delta(t)$$

$$\begin{aligned} h(t) &= h_1(t) * h_2(t) \\ &= \left[e^{-2t}\left(u(t)-u(t-1)\right)\right] * \left[\frac{d}{dt}\delta(t) + 3\delta(t)\right] \\ &= \frac{d}{dt}\left[e^{-2t}\left(u(t)-u(t-1)\right)\right] + 3e^{-2t}\left(u(t)-u(t-1)\right) \\ &= e^{-2t}\left(\delta(t)-\delta(t-1)\right) - 2e^{-2t}\left(u(t)-u(t-1)\right) + 3e^{-2t}\left(u(t)-u(t-1)\right) \\ &= e^{-2t}\left(\delta(t)-\delta(t-1) + u(t) - u(t-1)\right) \end{aligned}$$



9.6

(a) $x(t) = \delta(t) \Rightarrow v(t) = h_1(t) - h_2(t) \Rightarrow y(t) = \left(h_1(t) - h_2(t)\right) * h_3(t)$

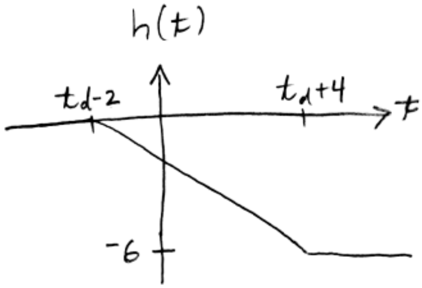


$$h(t) = \left(h_1(t) - h_2(t)\right) * h_3(t)$$

$$= \begin{cases} 0, & t+2 < t_d \\ \int_{t_d}^{t+2} -1 \, d\tau, & t+2 \geq t_d \text{ and } t-4 < t_d \\ \int_{t-4}^{t+2} -1 \, d\tau, & t-4 \geq t_d \end{cases}$$

$$= \begin{cases} 0, & t < t_d - 2 \\ -\tau \Big|_{t_d}^{t+2}, & t_d - 2 \leq t < t_d + 4 \\ -\tau \Big|_{t-4}^{t+2}, & t \geq t_d + 4 \end{cases}$$

$$= \begin{cases} 0, & t < t_d - 2 \\ -t + t_d - 2, & t_d - 2 \leq t < t_d + 4 \\ -6, & t \geq t_d + 4 \end{cases}$$



(b) $t_d - 2 \geq 0 \quad \Rightarrow \quad h(t) = 0$ for all $t < 0$

$t_d \geq 2$ guarantees causality

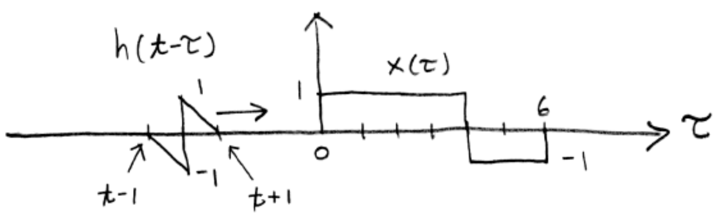
(c) $\int_{-\infty}^{\infty} |h_1(t)|\,dt = \int_{4}^{\infty} dt = t\Big|_4^{\infty} = \infty \quad \Rightarrow$ system 1 not stable

$\int_{-\infty}^{\infty} |h_2(t)|\,dt = \int_{-2}^{\infty} dt = t\Big|_{-2}^{\infty} = \infty \quad \Rightarrow$ System 2 not stable

$\int_{-\infty}^{\infty} |h_3(t)|\,dt = \int_{t_d}^{\infty} dt = t\Big|_{t_d}^{\infty} = \infty \quad \Rightarrow$ system 3 not stable

$\int_{-\infty}^{\infty} |h(t)|\,dt > \int_{t_d+4}^{\infty} 6\,dt = 6t\Big|_{t_d+4}^{\infty} = \infty \quad \Rightarrow$ overall system not stable

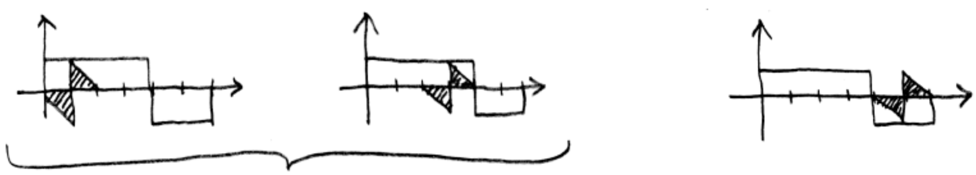
9.7



(a) $t = 0 \Rightarrow$

$y(0) = \frac{1}{2}$ (area of triangle)

(b) $y(t) = 0$ if:
1. $t + 1 \leq 0$
2. $t - 1 \geq 6$
3. whenever triangle areas cancel



$t - 1 \geq 0$ and $t + 1 \leq 4$ $\qquad\qquad$ $t + 1 = 6$

$\Rightarrow \quad y(t) = 0$ for all $t \in (-\infty, -1] \cup [1, 3] \cup \{5\} \cup [7, \infty)$