ECE 2025
FALL '01

HOMEWORK # 10
SOLUTIONS

1)

a) $H(j\omega) = 1 - 100 \dfrac{1}{100 + j\omega} = \dfrac{j\omega}{100 + j\omega}$

b) $|H(j\omega)|^2 = \dfrac{\omega^2}{10,000 + \omega^2}$ (plot on next page)

c) $|H(j\omega)| = \max$ at $\omega = \infty$

$|H(j\infty)|^2 = 1$

$\dfrac{\omega^2}{\omega^2 + 10,000} = \dfrac{1}{2} \Rightarrow \omega^2 = \dfrac{1}{2}\omega^2 + 5,000$

$\Rightarrow \omega^2 = 10,000 \Rightarrow \omega = 100$ rad/sec.

d) $-2 \longrightarrow H(j0)\cdot(-2) = 0$

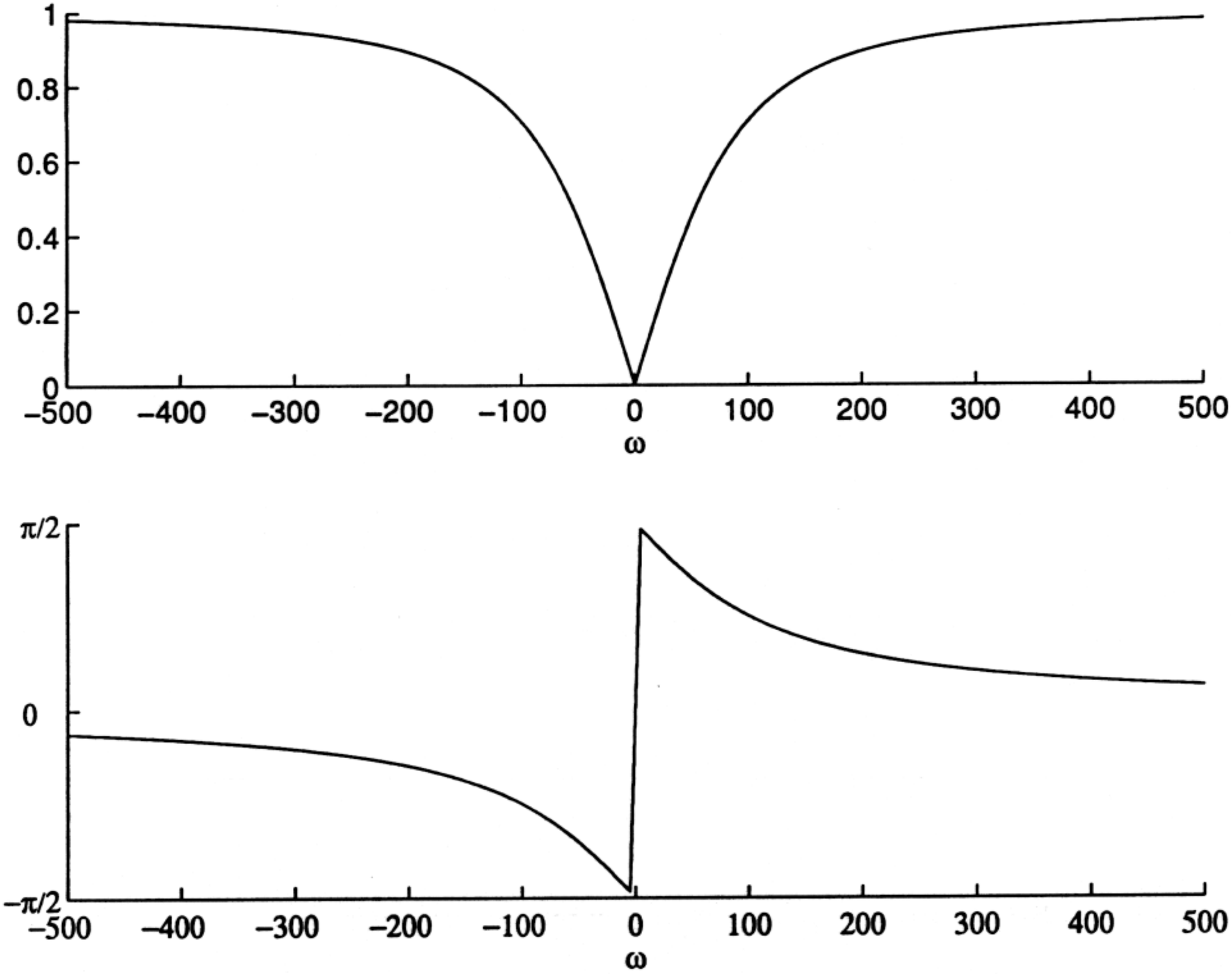
$5\cos(1000t) \rightarrow 5\,|H(j1000)|\cos(1000t + \angle H(j1000)) =$

$= 5 \cdot 0.995 \cos(1000t + 0.1) =$

$= 4.975 \cos(1000t + 0.1)$

$\delta(t-3) \rightarrow h(t) * \delta(t-3) = h(t-3) =$

$= \delta(t-3) - 100\, e^{-100(t-3)} u(t-3)$


1
0.8
0.6
0.4
0.2
0
−500 −400 −300 −200 −100 0 100 200 300 400 500
ω
π/2
0
−π/2
−500 −400 −300 −200 −100 0 100 200 300 400 500
ω

$$y(t) = 4.975\cos(1000t+0.1) + \delta(t-3) +$$
$$-100\,e^{-100(t-3)}u(t-3)$$

2) a) $2\delta(\omega-2)e^{\omega} = 2e^{2}\delta(\omega-2)$

$$\Rightarrow x(t) = \frac{2e^{2}}{2\pi}e^{j2t} = \frac{e^{2}}{\pi}e^{j2t}$$

b) $X(j\omega) = e^{-j4\omega} + e^{j4\omega} = 2\cos(4\omega)$

c) $x(t) = 20\pi\,\dfrac{\sin(200t)}{\pi t} \Rightarrow X(j\omega) = 20\pi[u(\omega+200) +$
$$- u(\omega-200)]$$

d) $u(\omega+3)u(3-\omega) = u(\omega+3) - u(\omega-3)$

$$\Rightarrow x(t) = \frac{\sin 3t}{\pi t}$$

3) a) $\delta(t-2)\cos(2t) = \cos 4\,\delta(t-2)$

$$\Rightarrow X(j\omega) = \cos 4\, e^{-j2\omega}$$

b) $\delta(t-2) * \cos(2t) = \cos[2(t-2)]$

$$\Rightarrow X(j\omega) = e^{-j2\omega}\pi[\delta(\omega-2) + \delta(\omega+2)] =$$
$$= \pi[e^{-j4}\delta(\omega-2) + e^{j4}\delta(\omega+2)]$$

c) $X(j\omega) = -e^{j\omega} + 2 - e^{-j\omega} = 2 - 2\cos\omega$

d) $X(j\omega) = \dfrac{1}{3+j\omega} - e^{-9}\dfrac{1}{3+j\omega}e^{-j3\omega} =$

$$= \frac{1-e^{-(9+j3\omega)}}{3+j\omega} = \frac{1-e^{-3(3+j\omega)}}{3+j\omega}$$

e) $u(t+2.5) - u(t-2.5) \longleftrightarrow \dfrac{\sin\left(\frac{5\omega}{2}\right)}{\omega/2}$

$u(t) - u(t-5) \longleftrightarrow e^{-j\frac{5\omega}{2}} \dfrac{\sin\left(\frac{5\omega}{2}\right)}{\omega/2} =$

$$= \frac{1-e^{-j5\omega}}{j\omega}$$

$[u(t)-u(t-5)]\cos(10\pi t) =$

$= \frac{1}{2}[u(t)-u(t-5)]e^{j10\pi t} +$

$+ \frac{1}{2}[u(t)-u(t-5)]e^{-j10\pi t}$

$\Rightarrow X(j\omega) = \dfrac{1}{2}\,\dfrac{1-e^{-j5(\omega-10\pi)}}{j(\omega-10\pi)} +$

$+ \dfrac{1}{2}\,\dfrac{1-e^{-j5(\omega+10\pi)}}{j(\omega+10\pi)} =$

$$= \frac{1}{2}\,\frac{1-e^{-j5\omega}}{j(\omega-10\pi)} + \frac{1}{2}\,\frac{1-e^{-j5\omega}}{j(\omega+10\pi)} =$$

$$= \frac{1-e^{-j5\omega}}{2j}\left(\frac{1}{\omega-10\pi} + \frac{1}{\omega+10\pi}\right) =$$

$$= e^{-j\frac{5\omega}{2}}\,\frac{e^{j\frac{5\omega}{2}}-e^{-j\frac{5\omega}{2}}}{2j}\,\frac{2\omega}{\omega^2-(10\pi)^2} =$$

$$= e^{-j\frac{5\omega}{2}}\,\sin\left(\frac{5\omega}{2}\right)\cdot\frac{2\omega}{\omega^2-(10\pi)^2}$$

4) a) $h(t) = \delta(t+1) + 2\delta(t) + \delta(t-2)$

b) $$H(j\omega) = \int_{-\infty}^{+\infty}[\delta(t+1)+2\delta(t)+\delta(t-2)]e^{-j\omega t}dt =$$

$$= e^{j\omega} + 2 + e^{-j2\omega}$$

c) $$y(t) = e^{j\omega(t+1)} + 2e^{j\omega t} + e^{j\omega(t-2)} =$$

$$= e^{j\omega}e^{j\omega t} + 2e^{j\omega t} + e^{-j2\omega}e^{j\omega t} =$$

$$= (e^{j\omega}+2+e^{-j2\omega})e^{j\omega t} =$$

$$= H(j\omega)\,e^{j\omega t}$$

5) a) $$\frac{1}{1+j0.2\omega} = \frac{5}{5+j\omega} \leftrightarrow 5e^{-5t}u(t)$$

$$\frac{e^{-j0.2\omega}}{1+j0.2\omega} \leftrightarrow 5e^{-5(t-0.2)}u(t-0.2) =$$

$$= 5e^{-(5t-1)}u(t-0.2)$$

Alternatively, one can use the scaling property:

$$\frac{e^{-j\omega}}{1+j\omega} \leftrightarrow e^{-(t-1)}u(t-1)$$

$$\frac{e^{-j\frac{\omega}{5}}}{1+j\frac{\omega}{5}} \leftrightarrow 5e^{-(5t-1)}u(5t-1)$$

The two answers are the same because

$u(t-0.2) = u(5t-1)$ (try it!)

b) $$j\,10\sin\omega = 5e^{j\omega} - 5e^{-j\omega} \leftrightarrow 5\delta(t+1) + \\ -5\delta(t-1)$$

c) $\dfrac{1+j\omega}{2+j\omega} = \dfrac{2+j\omega-1}{2+j\omega} = 1 - \dfrac{1}{2+j\omega}$

$$\leftrightarrow \delta(t) - e^{-2t}u(t)$$

d) $x(t) = \dfrac{1}{2\pi}\left(e^{j50\pi t} + e^{-j50\pi t}\right) =$

$$= \frac{1}{\pi}\cos(50\pi t)$$

6) a)

b) $u(t+\frac{1}{2}) - u(t-\frac{1}{2}) \leftrightarrow \dfrac{\sin\frac{\omega}{2}}{\omega/2}$

$$u(t) - u(t-1) \leftrightarrow e^{-j\frac{\omega}{2}}\frac{\sin\frac{\omega}{2}}{\omega/2} = \frac{1-e^{-j\omega}}{j\omega}$$

$$x(t) = [u(t)-u(t-1)]\sin(\pi t) =$$

$$= \frac{1}{2j}[u(t)-u(t-1)]e^{j\pi t} - \frac{1}{2j}[u(t)-u(t-1)]e^{-j\pi t}$$

$$\Rightarrow X(j\omega) = \frac{1}{2j}\,\frac{1-e^{-j(\omega-\pi)}}{j(\omega-\pi)} - \frac{1}{2j}\,\frac{1-e^{-j(\omega+\pi)}}{j(\omega+\pi)} =$$

$$= -\frac{1}{2}\,\frac{1+e^{-j\omega}}{\omega-\pi} + \frac{1}{2}\,\frac{1+e^{-j\omega}}{\omega+\pi} =$$

$$= \frac{1+e^{-j\omega}}{2}\left(\frac{1}{\omega+\pi} - \frac{1}{\omega-\pi}\right) =$$

$$= e^{-j\frac{\omega}{2}}\,\frac{e^{j\frac{\omega}{2}}+e^{-j\frac{\omega}{2}}}{2}\,\frac{-2\pi}{\omega^2-\pi^2} =$$

$$= -2\pi\, e^{-j\frac{\omega}{2}}\,\frac{\cos\frac{\omega}{2}}{\omega^2-\pi^2} = 2\pi\, e^{-j\frac{\omega}{2}}\,\frac{\cos\frac{\omega}{2}}{\pi^2-\omega^2}$$

c) See plots on last page

d) $x(t+0.5) \longleftrightarrow -2\pi\,\dfrac{\cos\frac{\omega}{2}}{\omega^2-\pi^2}$ (time shift property)

$q(t) = x(100t+0.5) \longleftrightarrow$

$$Q(j\omega) = \frac{-2\pi}{100}\;\frac{\cos\frac{\omega}{200}}{\left(\frac{\omega}{100}\right)^2-\pi^2}$$

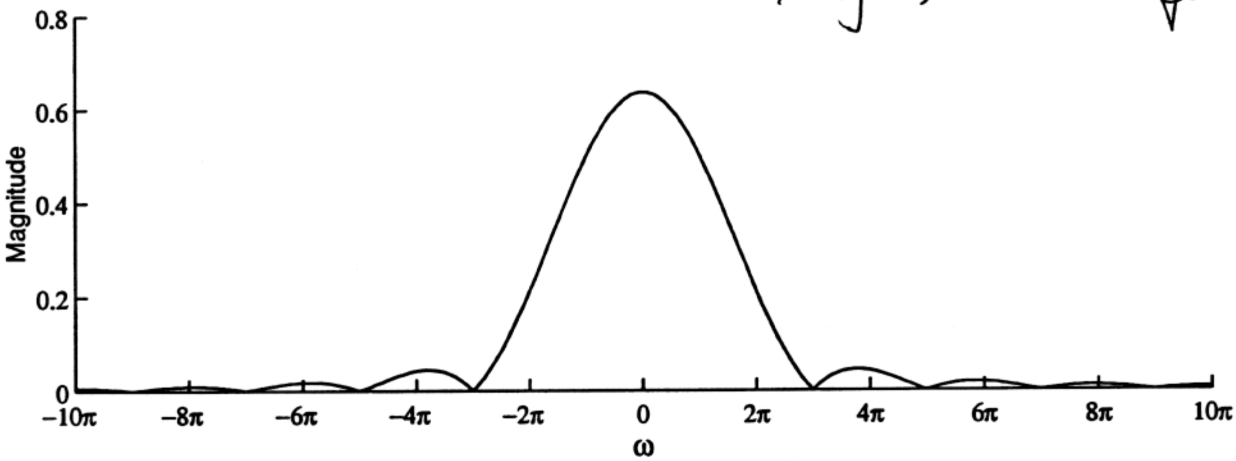
e) Obviously, $Q(j\omega)$ is a real function. More generally, every real signal that is an even function of t (i.e. $x(-t) = x(t)$) has a real Fourier transform, because:

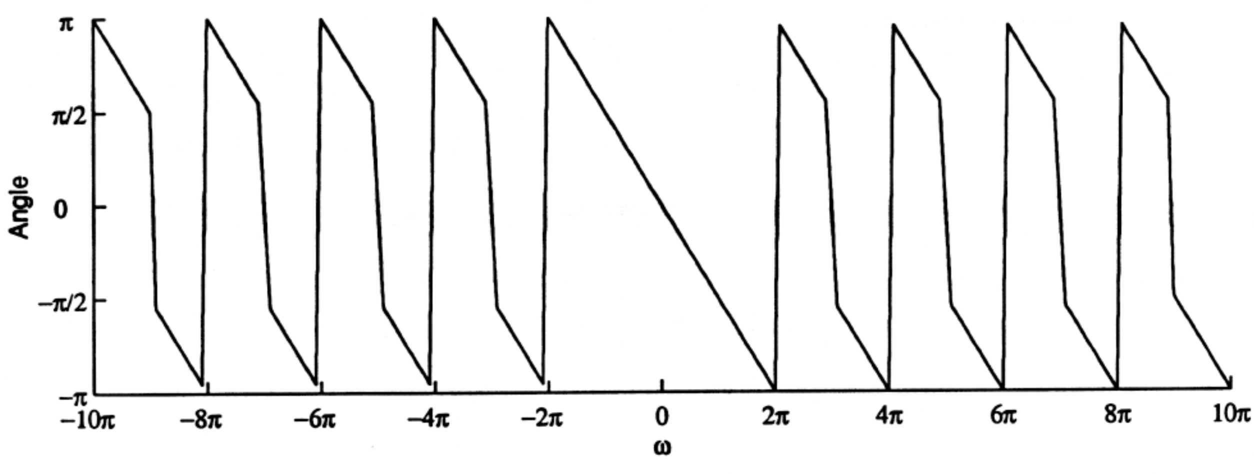
$$x(t) \longleftrightarrow X(j\omega)$$

$$x(-t) \longleftrightarrow X(-j\omega) \quad \text{(time reversal property)}$$

For real signals, $X(-j\omega) = X^*(j\omega)$, so:

$$x(t) = x(-t) \Longrightarrow X(j\omega) = X(-j\omega) = X^*(j\omega),$$ i.e. $X(j\omega)$ is a real function.





(10)

7) a) $\omega_0 = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$ rad/sec.

$$a_0 = \frac{1}{8}\int_{-2}^{2} 10\,dt = 5$$

$$a_k = \frac{10}{8}\int_{-2}^{2} e^{-jk\omega_0 t}\,dt = \frac{10}{8}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-2}^{2} =$$

$$= -\frac{10}{j8k\omega_0}\left(e^{-j2k\omega_0} - e^{j2k\omega_0}\right) =$$

$$= -\frac{10}{jk2\pi}\left(-2j\sin\left(k\frac{\pi}{2}\right)\right) = \frac{10}{k\pi}\sin\left(k\frac{\pi}{2}\right)$$

b)