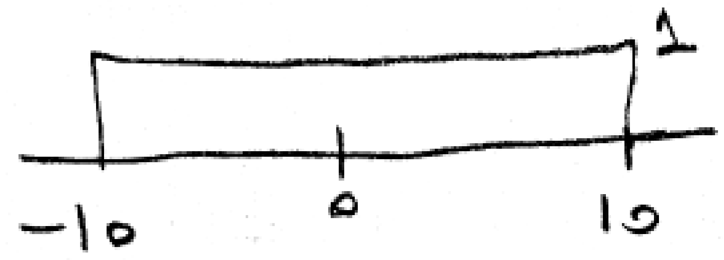
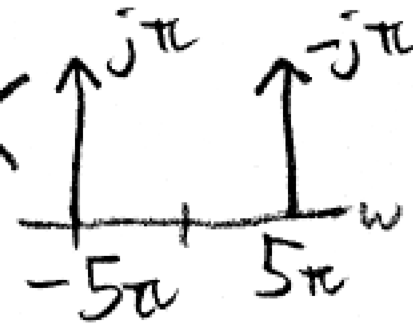
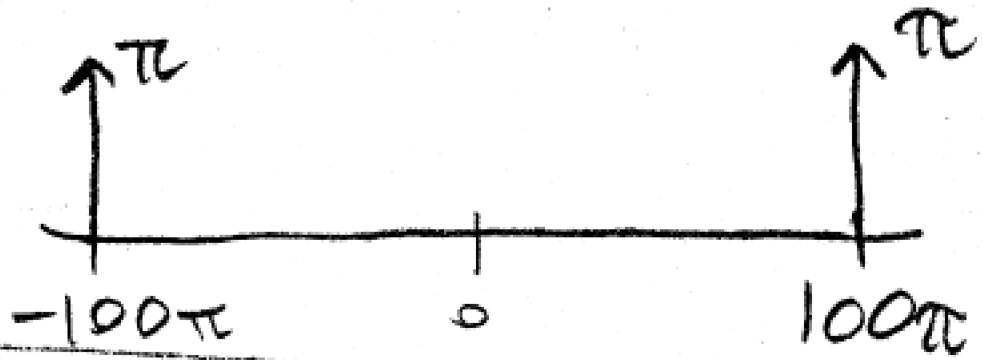
ECE 2025 Fall 2001 HW #11 Solutions

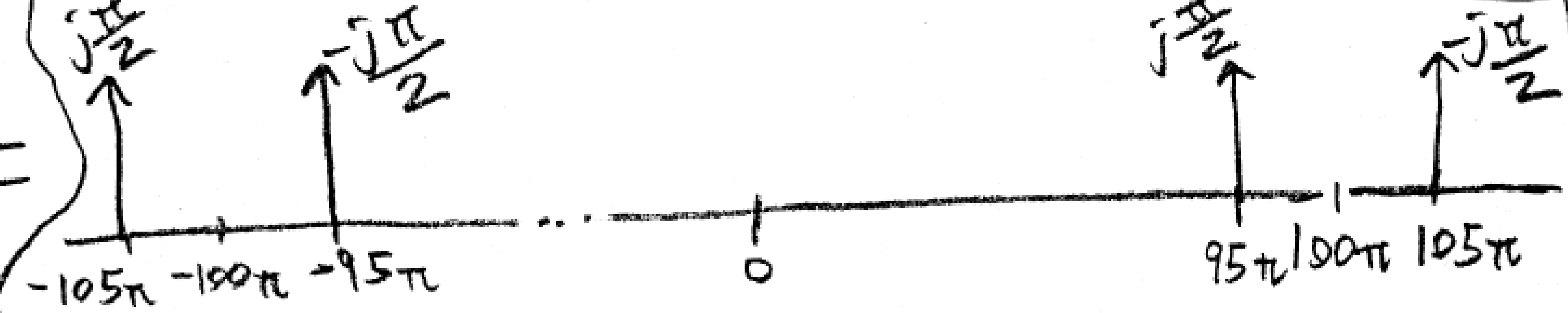
11.1) a) $X(j\omega) =$ 

$$\boxed{x(t) = \frac{\sin 10t}{\pi t}}$$

b) $X(j\omega) = \frac{1}{2\pi} \times$ 

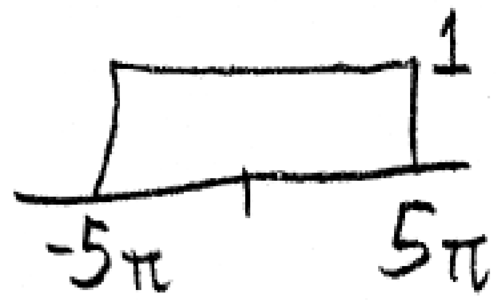
$*$ 

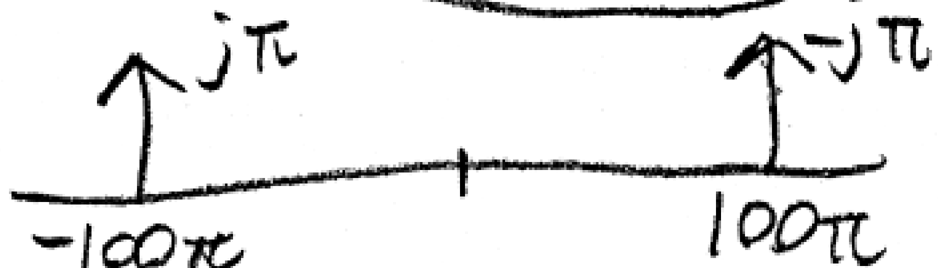
Either way of expressing the answer is perfectly valid

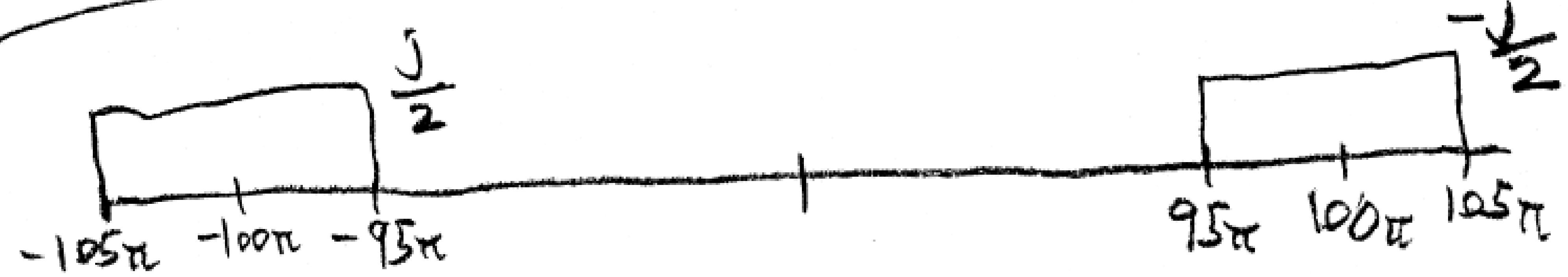
$=$ 

or

$$= \frac{j\pi}{2}\Big[\delta(\omega+105\pi) - \delta(\omega-105\pi) - \delta(\omega+95\pi) + \delta(\omega-95\pi)\Big]$$

c) $X(j\omega) = \frac{1}{2\pi} \times$ 

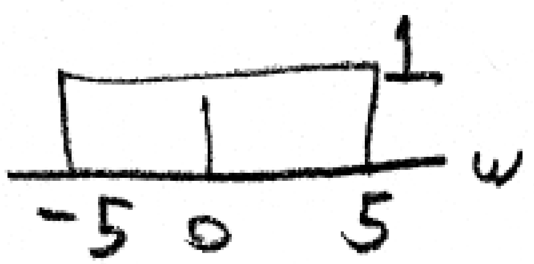
$*$ 

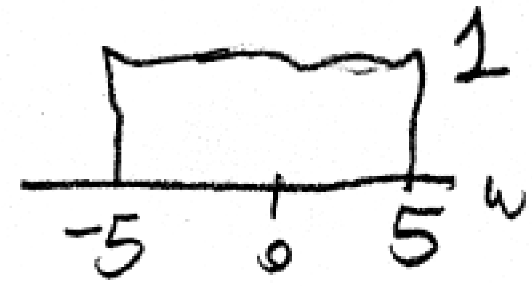
$=$ 

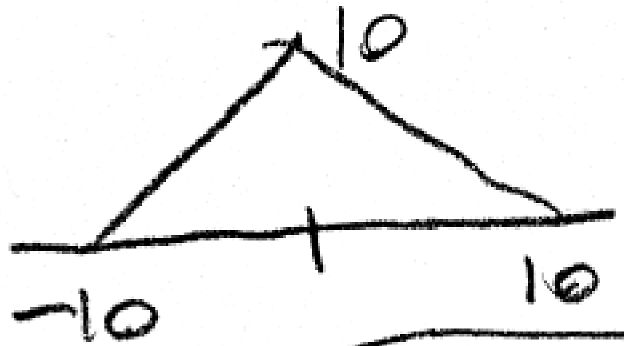
or

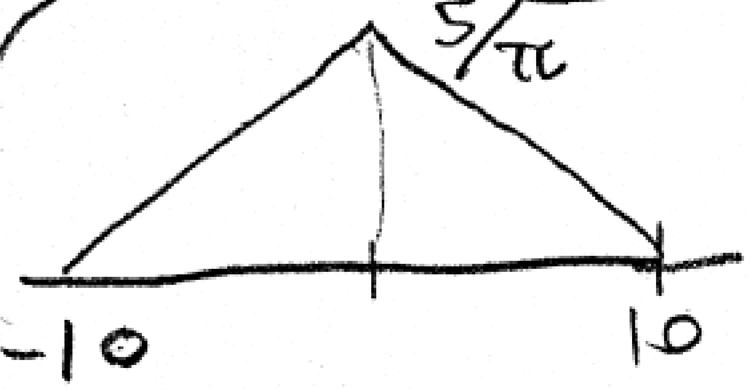
$$= \begin{cases} -\frac{j}{2} & \text{for } 95\pi < \omega < 105\pi \\ \frac{j}{2} & \text{for } -105\pi < \omega < -95\pi \\ 0 & \text{otherwise} \end{cases}$$

(could also write solution using unit step functions)

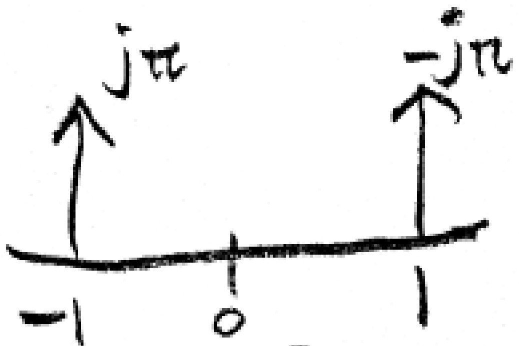
11.1) d) $X(j\omega) = \frac{1}{2\pi} \times$ 

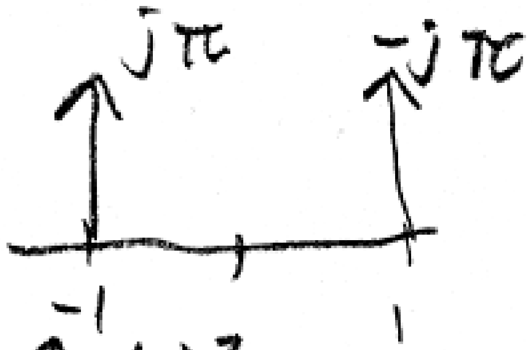
$*$ 

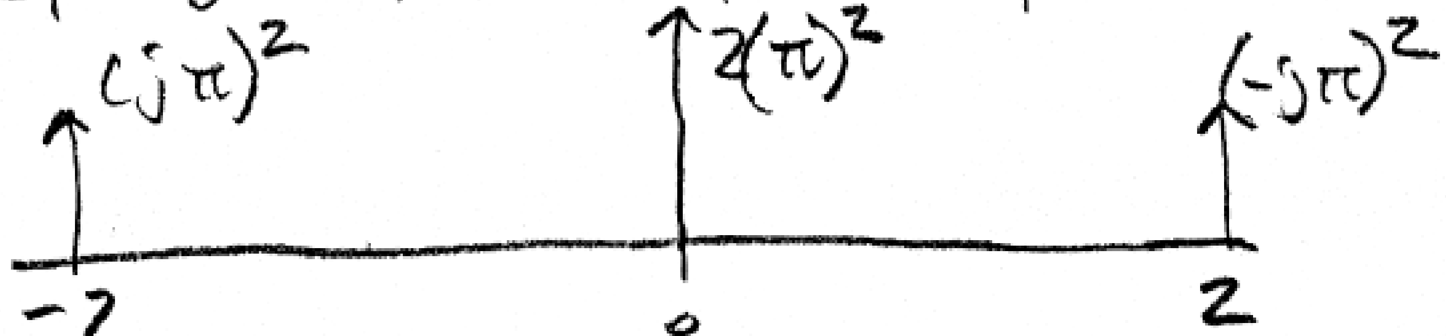
$= \frac{1}{2\pi} \times$ 

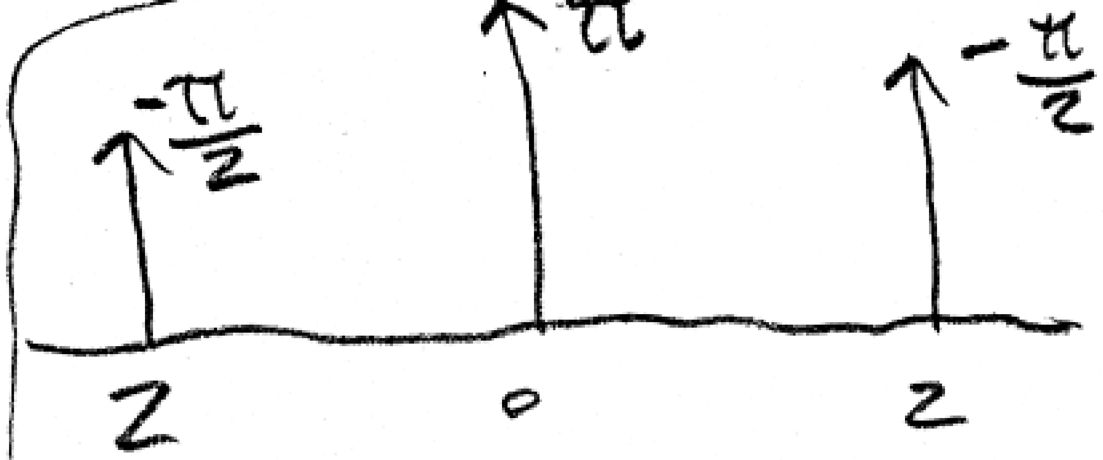
$=$ 

$= \begin{cases} \frac{5}{\pi}[1-|t|] & \text{for } |t|<1 \\ 0 & \text{otherwise} \end{cases}$

e) $X(j\omega) = \frac{1}{2\pi} \times$ 

$*$ 

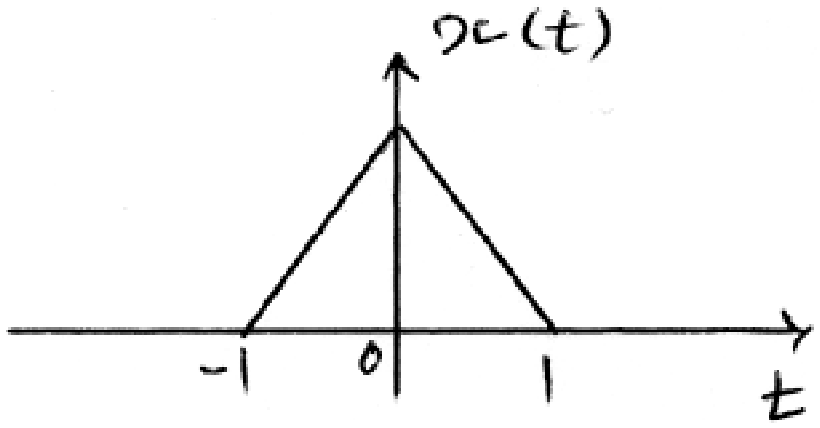
$= \frac{1}{2\pi} \times$ 

$=$ 

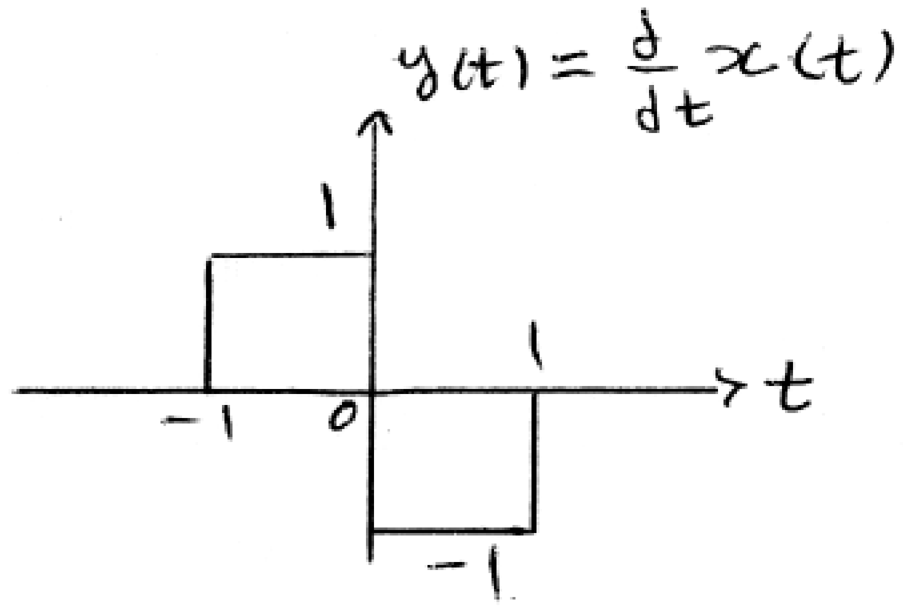
$= \pi[\delta(\omega) - \frac{1}{2}\delta(\omega-2) - \frac{1}{2}\delta(\omega+2)]$

11.2

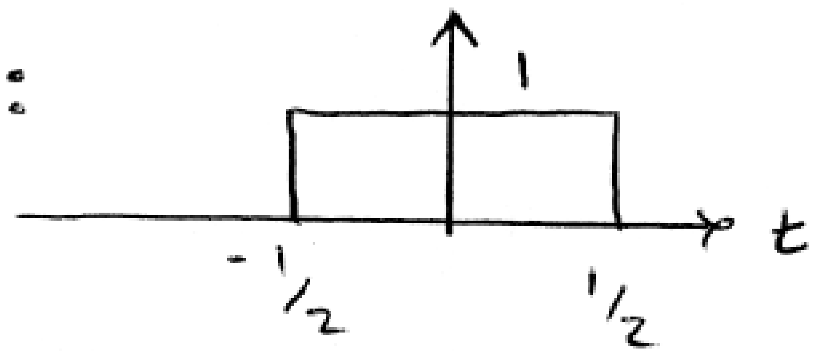
(a)



$\Longrightarrow$

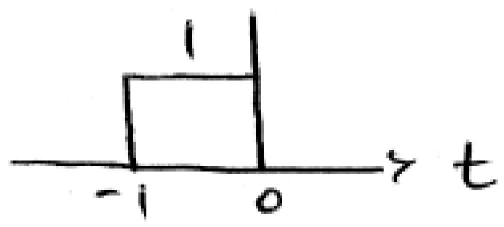


since:

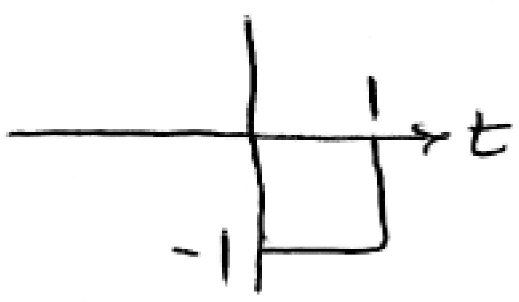


$$\longleftrightarrow \quad \frac{\sin\left(\frac{w}{2}\right)}{\left(\frac{w}{2}\right)}$$

then



$$\longleftrightarrow \quad e^{j\frac{w}{2}} \frac{\sin\left(\frac{w}{2}\right)}{\left(\frac{w}{2}\right)}$$



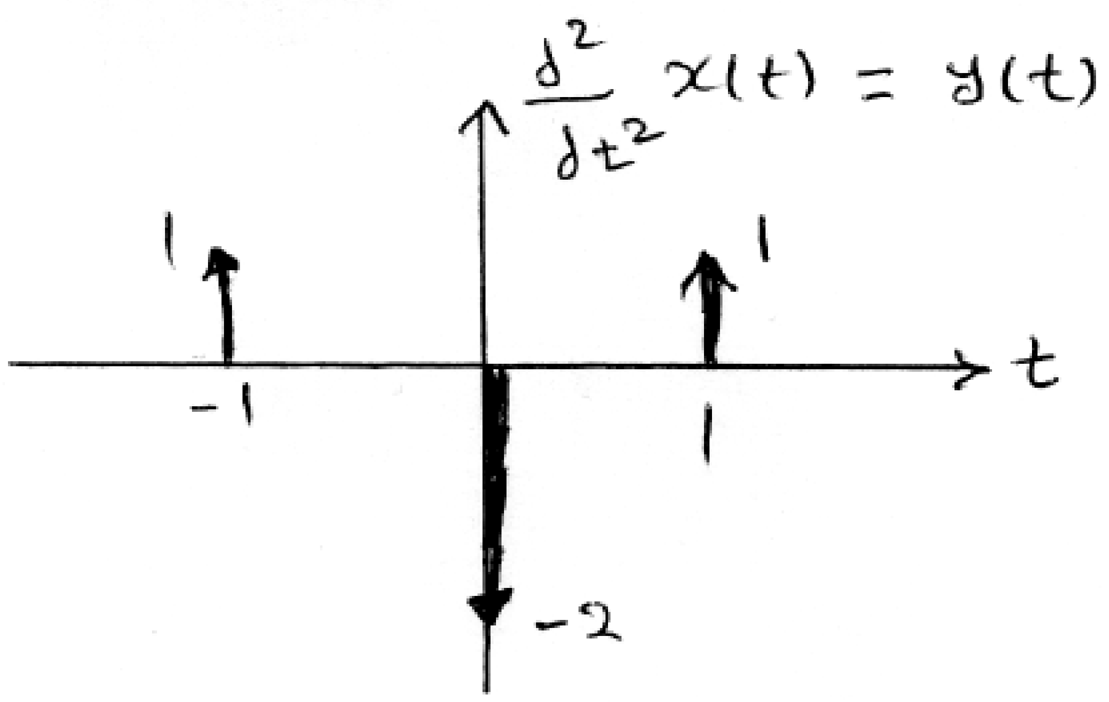
$$\longleftrightarrow \quad (-1)e^{-j\frac{w}{2}} \frac{\sin\left(\frac{w}{2}\right)}{\left(\frac{w}{2}\right)}$$

Thus: $Y(jw) = \dfrac{\sin\frac{w}{2}}{\frac{w}{2}} \left(e^{-j\frac{w}{2}} - e^{-j\frac{w}{2}} \right) = \dfrac{4j}{w} \sin^2\left(\dfrac{w}{2}\right)$

$Y(jw) = jw\, X(jw) \quad \Longrightarrow \quad X(jw) = \dfrac{4}{w^2} \sin^2\dfrac{w}{2} = \left(\dfrac{\sin\frac{w}{2}}{w/2} \right)^2$

11.2

(b)



$Y(jw) = e^{jw} + e^{-jw} - 2 = 2\cos w - 2 = -4\sin^2\left(\dfrac{w}{2}\right)$

$X(jw) = \left(\dfrac{1}{jw}\right)^2 Y(jw) = \dfrac{4}{w^2} \sin^2\left(\dfrac{w}{2}\right)$

11.3) a) $\mathcal{F}^{-1}\left\{\frac{1}{3+j\omega}\right\} = e^{-3t}u(t)$; use delay property

$$\boxed{x(t) = e^{-3(t-2)}u(t-2)}$$

b) Differentiation property:

$$x(t) = \frac{d}{dt}\left[e^{-3t}u(t)\right] = \boxed{e^{-3t}\delta(t) - 3e^{-3t}u(t)}$$

c) Just a delay of (b):

$$\boxed{x(t) = e^{-3(t-2)}\delta(t-2) - 3e^{-3(t-2)}u(t-2)}$$

d) $X(j\omega) = \frac{10}{2}\left[u(\omega+200\pi) - u(\omega-200\pi)\right]$

$$= \boxed{5\left[u(\omega+200\pi) - u(\omega-200\pi)\right]}$$

e) $T_s = \frac{1}{5}$, $\omega_0 = \frac{2\pi}{T_s} = \frac{2}{5}\pi$

$$\boxed{X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2}{5}\pi k\right)}$$

(using results from 11.5 problem statement)

11.4) a) Only DC gets through, so

$y(t) = 2 \times \frac{1}{2} = \boxed{1}$

b) Complex modulation in freq. corresponds to time shift:

$$\boxed{y(t) = 0.5x(t - \tfrac{4}{3})}$$

c) Only middle three lines get through. Also, delay by $\frac{1}{3}$:

$$y(t) = \frac{1}{2} + \frac{2}{\pi}\cos(w_o t - \tfrac{1}{3})$$

d) High pass filter kills DC term, so $\boxed{y(t) = x(t) - \frac{1}{2}}$

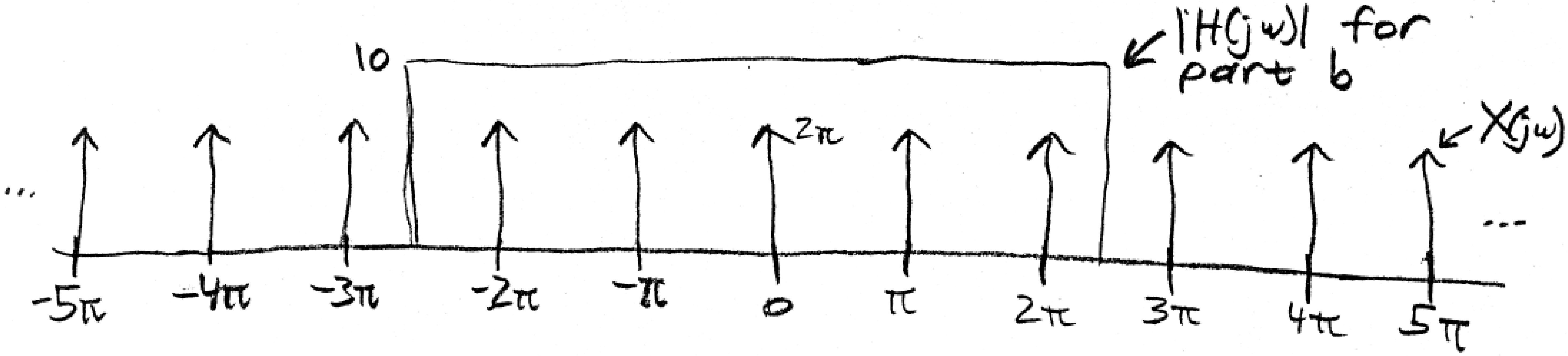
e) Middle three lines get through, but lines at w_o are attenuated by $\frac{1}{2}$.

$$y(t) = \frac{1}{2} + \frac{1}{\pi}\cos(w_o t)$$

f) Bandpass filter only lets lines at w_o through:

$$y(t) = \frac{2}{\pi}\cos(w_o t)$$

11.5) a) $\boxed{X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi\, \delta(\omega - k\pi)}$



b) $\boxed{H(j\omega) = 10[u(\omega + 2.5\pi) - u(\omega - 2.5\pi)]}$

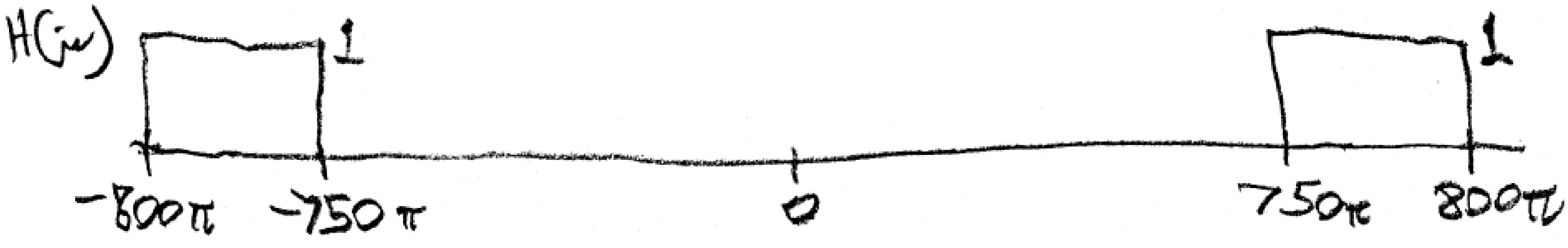
c) Middle 5 lines get through:

$\boxed{y(t) = 10[1 + 2\cos(\pi t) + 2\cos(2\pi t)]}$

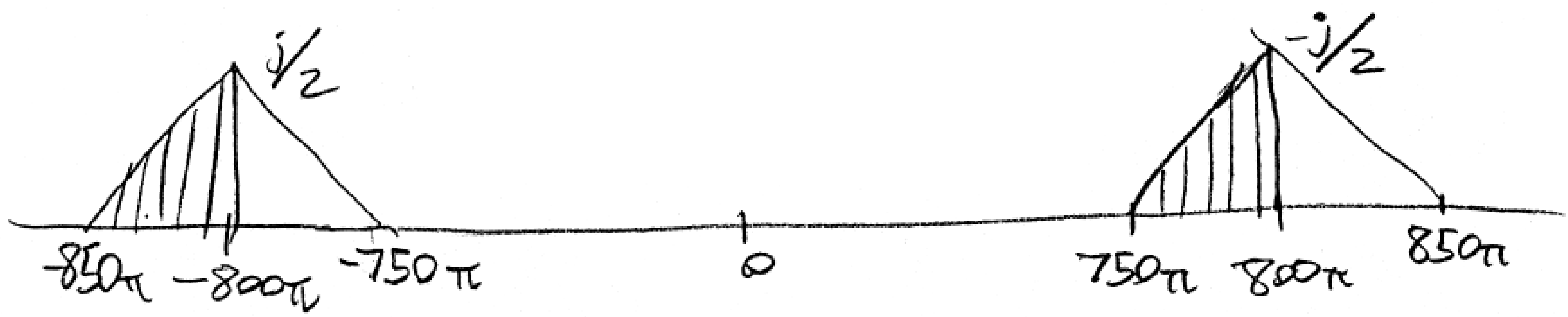
d) Choose $\boxed{0 < \omega_{co} < \pi}$

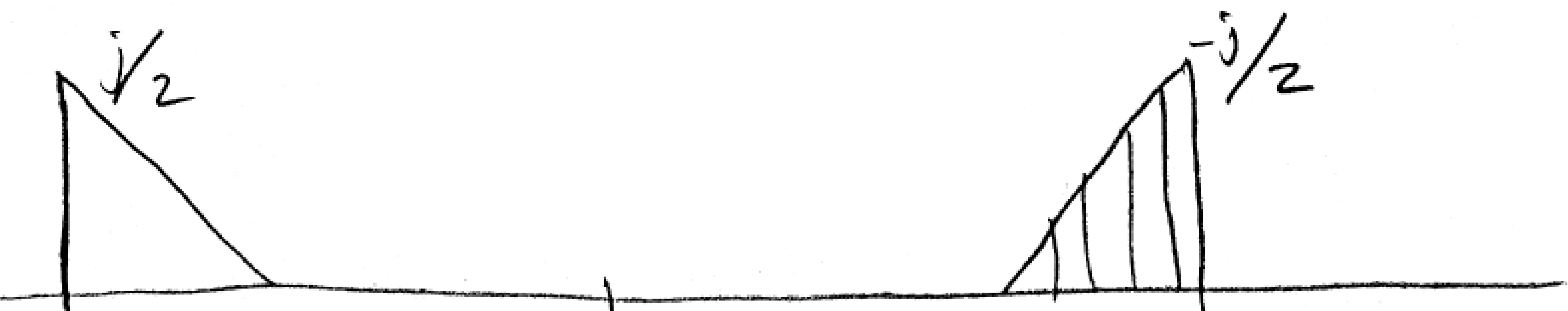
then $\boxed{C = 10}$

11.6) a)



b) $V(j\omega) = \frac{1}{2\pi} \times X(j\omega) *$ 

$=$ 

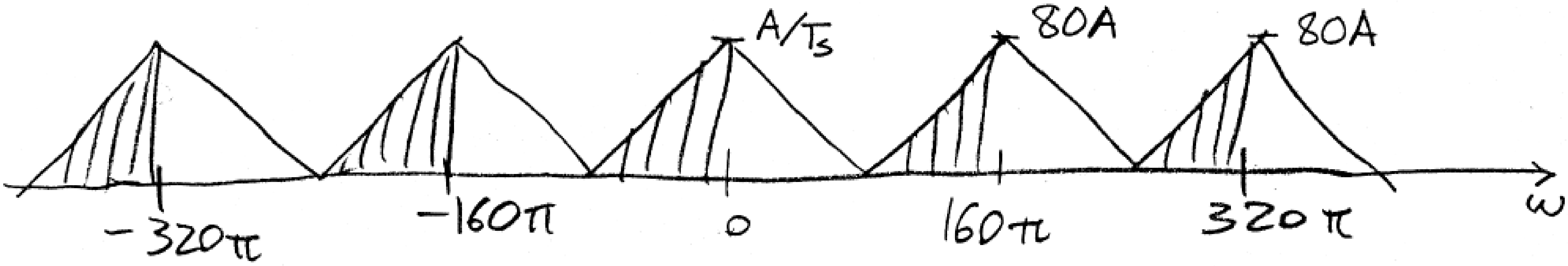
c) 

d) Each of the triangles has had a "side" lopped off, leaving only a single side for each triangle.

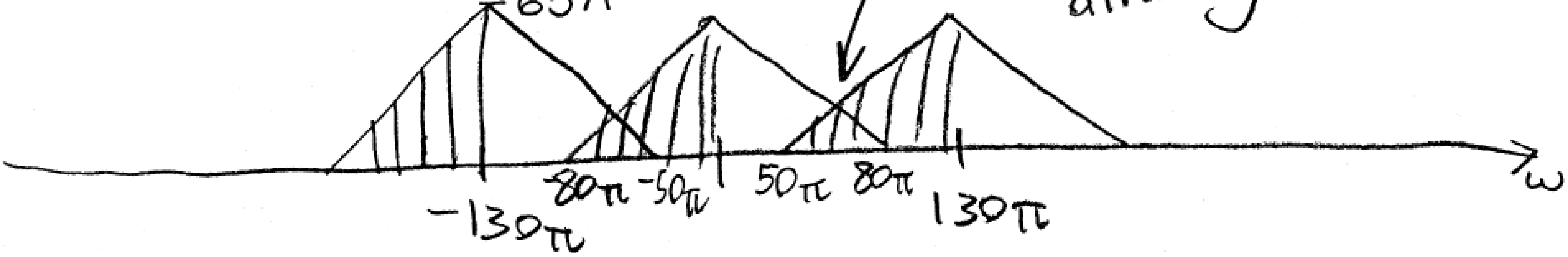
[Aside: Amateur radio operators often use SSB since it lets them occupy a smaller part of the spectrum.]

11.7) a) $w_s = 2 \times 80\pi = \boxed{160\pi}$ rad/sec

$f_s = 80\text{Hz}$



b) $f_s = 65\text{Hz}$



c) $\frac{\pi}{T_s} = \pi \frac{130\pi}{2\pi} = 65\pi$

$T_s = \frac{1}{65}$ sec