EE-2025 Fall-2001

Lecture 11
Frequency Response of FIR
5-Oct-01

Info: Web-CT, Lab, HW

- Lab #6: FIR Filtering of Images
- Prob Set #6 due next week
- Quiz #2 on 22-Oct (Monday)
  - Coverage: HW #3, #4, #5, #6, and #7

Schedule around Fall Break

	Wed Lab	Thurs Lab	Mon Lab	Tues Lab
	Mon Rec	Tues Rec	Wed Rec	Thur Rec
Lab 6 start	3-Oct	4-Oct	**8-Oct**	**9-Oct**
Lab 6 report due	10-Oct	11-Oct	**22-Oct**	**23-Oct**
Lab 7 start	**10-Oct**	**11-Oct**	22-Oct	23-Oct
Lab 7 report due	**24-Oct**	**25-Oct**	29-Oct	30-Oct
Rec: FIR Filters	1-Oct	2-Oct	3-Oct	4-Oct
Rec: Freq Resp	8-Oct	9-Oct	10-Oct	11-Oct
Rec: Z-Trans	**17-Oct**	**18-Oct**	17-Oct	18-Oct
HW #6 due	8-Oct	9-Oct	10-Oct	11-Oct
HW #7 due	**17-Oct**	**18-Oct**	17-Oct	18-Oct
HW #8 due	**24-Oct**	23-Oct	24-Oct	25-Oct
Quiz #2	22-Oct	22-Oct	22-Oct	22-Oct

DEBUGGING

- "Any Fool" can write code
- Debugging is the interesting part
  - It takes talent and **DEDICATION** !!!
- HOWEVER,
  - Assume the **stupid** mistake is the problem

LECTURE

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, pp. 157-165, 169-176
- Other Reading:
  - Recitation: Ch. 6, pp. 176-188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 6, pp. 188-194

LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG PHASE

$$H(\hat{\omega}) = \left|H(\hat{\omega})\right| e^{j\varphi(\hat{\omega})}$$

DOMAINS: Time & Frequency

- **Time-Domain: "n" = time**
  - x[n] discrete-time signal
  - x(t) continuous-time signal
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs. f (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs. omega-hat
- Move back and forth QUICKLY

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **IMPULSE RESPONSE** h[n]
  - **CONVOLUTION**: y[n] = x[n]*h[n]
    - The "rule" defining the system can ALWAYS be re-written as convolution
- FIR Example: h[n] is same as b_k

POP QUIZ

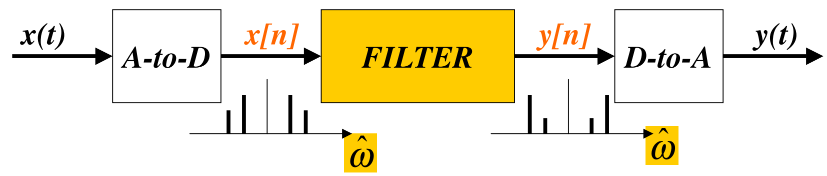
- FIR Filter is "FIRST DIFFERENCE"
  - y[n] = x[n] - x[n-1]
- Write output as a convolution
  - Need impulse response

$$h[n] = \delta[n] - \delta[n-1]$$

  - Then, another way to compute the output:

$$y[n] = (\delta[n] - \delta[n-1]) * x[n]$$

DIGITAL "FILTERING"



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

FILTERING EXAMPLE

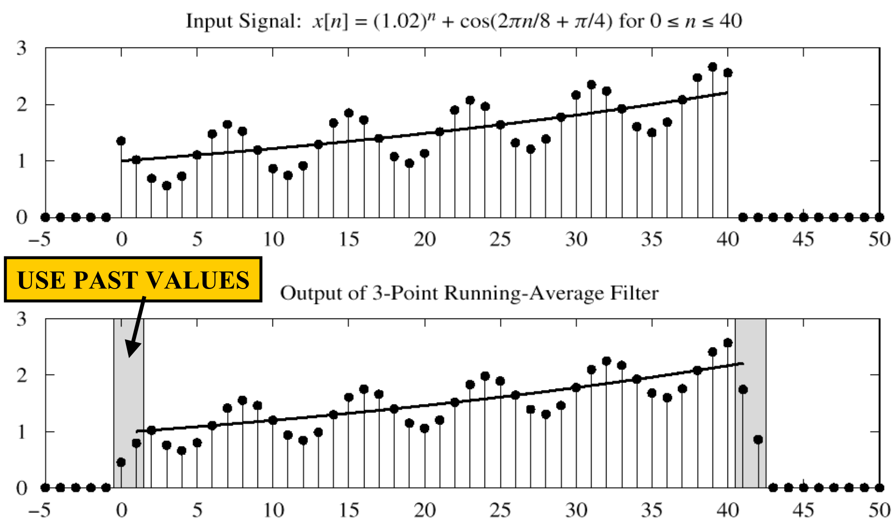
- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^{6} (\tfrac{1}{7}) x[n-k]$$

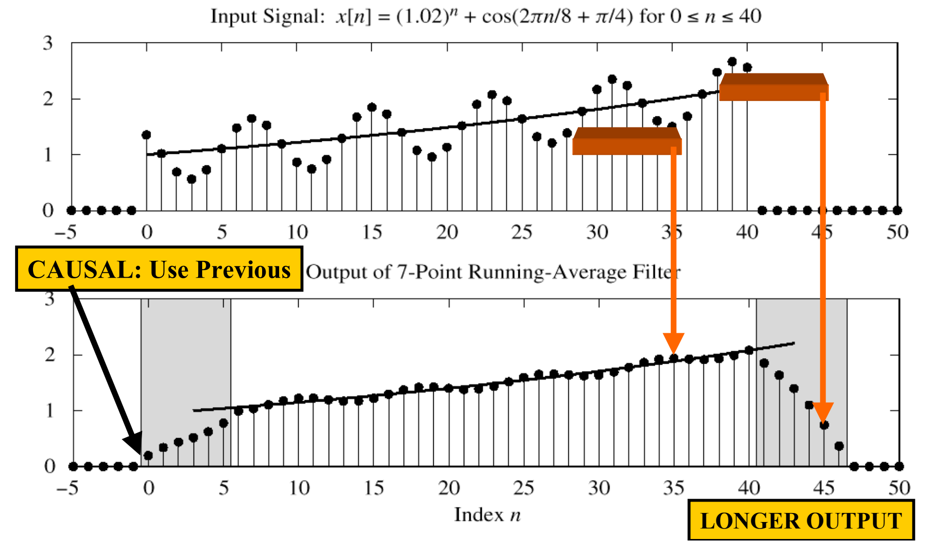
- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} (\tfrac{1}{3}) x[n-k]$$

3-pt AVG EXAMPLE



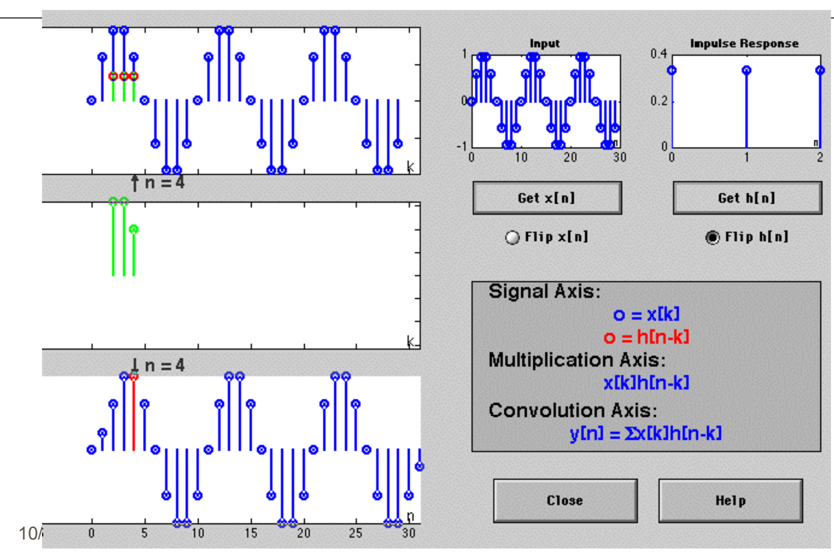
7-pt FIR EXAMPLE (AVG)



SINUSOIDAL RESPONSE

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - **SAME** Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the **FREQUENCY RESPONSE**

CONVDEMO: MATLAB GUI



COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\phi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k Ae^{j\phi}e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)}\right) Ae^{j\phi}e^{j\hat{\omega}(n)}$$

$$= H(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}(n)}$$

FREQUENCY REPONSE

- At each frequency, we can DEFINE

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

FREQUENCY RESPONSE

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency

EXAMPLE 6.1

Example 6.1

$$\{b_k\} = \{1,\ 2,\ 1\}$$

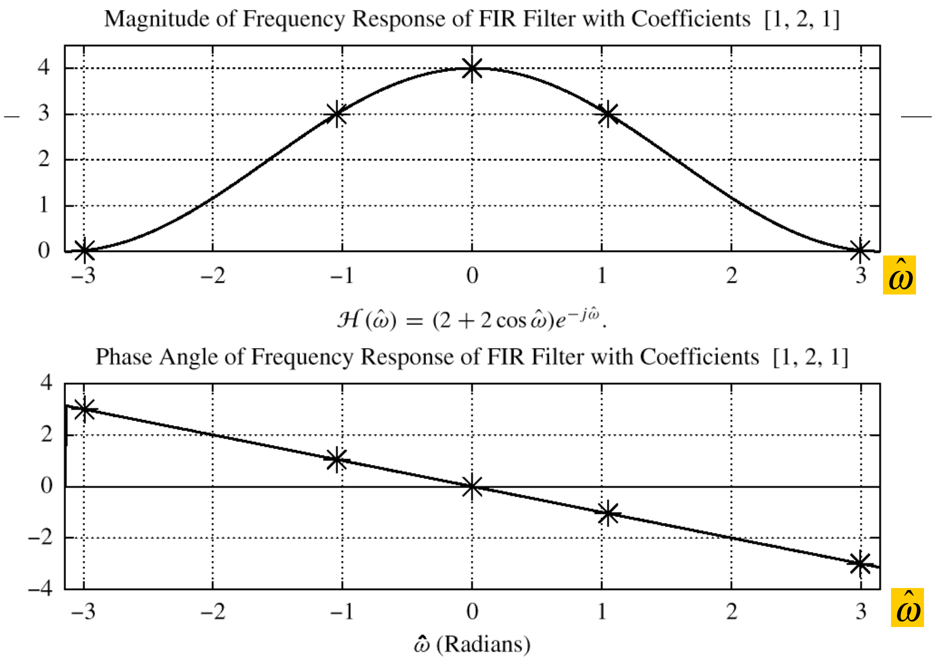
$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

To obtain formulas for the magnitude and phase of the frequency response

$$\begin{aligned}\mathcal{H}(\hat{\omega}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}\\ &= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}\right)\\ &= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})\end{aligned}$$

EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$ for frequencies $-\pi < \hat{\omega} \leq \pi$, the magnitude is $|\mathcal{H}(\hat{\omega})| = (2 + 2\cos\hat{\omega})$ and the phase is $\angle\mathcal{H}(\hat{\omega}) = -\hat{\omega}$.



MATLAB: FREQUENCY RESPONSE

- **HH = freqz(bb,1,ww)**
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: **HH = freekz(bb,1,ww)**
- FILTER COEFFICIENTS $\{b_k\}$

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - FREQUENCY RESPONSE, or
  - IMPULSE RESPONSE h[n]
- Sinusoid IN -----> Sinusoid OUT
  - At the SAME Frequency

Time & Frequency Relation

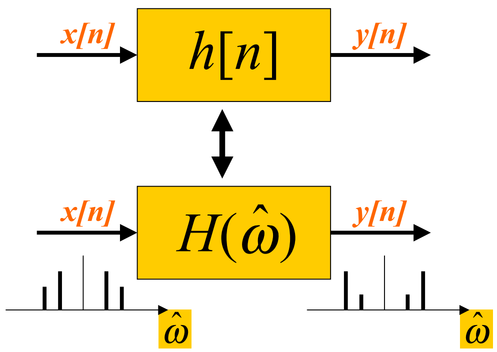
- Get Frequency Response from h[n]
  - Here is the FIR case:

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

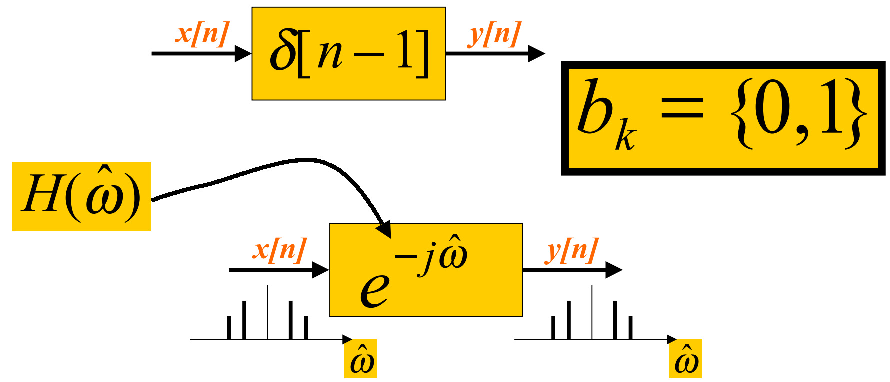
BLOCK DIAGRAMS

- Equivalent Representations



UNIT-DELAY SYSTEM

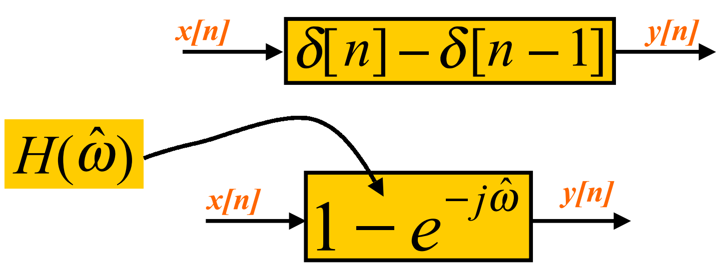
Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-1]$



FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for the Diff. Eqn:

$$y[n] = x[n] - x[n-1]$$



EXAMPLE 6.2

- Find y[n] when x[n] = complex exp.

Example 6.2 Consider the complex input $x[n] = 2e^{j\pi/4}e^{j\pi n/3}$.

$|\mathcal{H}(\pi/3)| = 2 + 2\cos(\pi/3) = 3$ and $\angle\mathcal{H}(\hat{\omega}) = -\pi/3$.

Therefore, the output of the system for the given input is

$$\begin{aligned} y[n] &= 3e^{-j\pi/3} \cdot 2e^{j\pi/4}e^{j\pi n/3} \\ &= (3 \cdot 2) \cdot e^{(j\pi/4 - j\pi/3)}e^{j\pi n/3} \\ &= 6e^{-j\pi/12}e^{j\pi n/3} = 6e^{j\pi/4}e^{j\pi(n-1)/3} \end{aligned}$$

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?

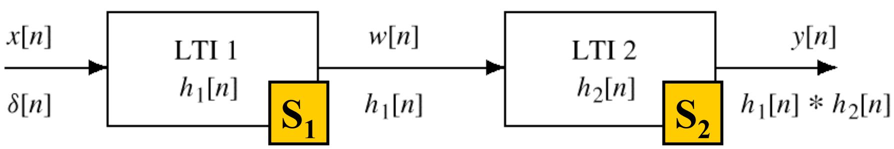
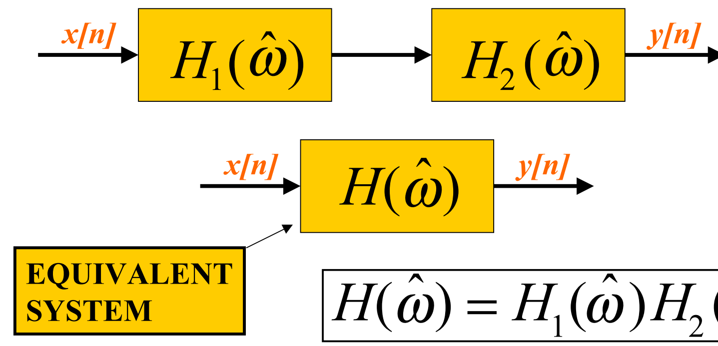


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- MULTIPLY the Frequency Responses



$$H_1(\hat{\omega}) \quad H_2(\hat{\omega}) \quad H(\hat{\omega})$$

$$H(\hat{\omega}) = H_1(\hat{\omega})H_2(\hat{\omega})$$

DLTI Demo with Sinusoids