EE-2025 Fall-2001

Lecture 12
Digital Filtering of Analog Signals
8-Oct-01

Info: Web-CT, Lab, HW

- Quiz #2 on 22-Oct (Monday)
  - Coverage: HW #3, #4, #5, #6, and #7
- MATLAB Help on Monday & Tuesday
  - 6 PM, VL-261
  - Also, Tuesday at 11am in VL-261
- Lab Schedule:
  - No Labs the week of 15-Oct
  - Check Schedule for handing in HW & Labs

Schedule around Fall Break

	Wed Lab	Thurs Lab	Mon Lab	Tues Lab
	Mon Rec	Tues Rec	Wed Rec	Thur Rec
Lab 6 start	3-Oct	4-Oct	**8-Oct**	**9-Oct**
Lab 6 report due	10-Oct	11-Oct	**22-Oct**	**23-Oct**
Lab 7 start	**10-Oct**	**11-Oct**	22-Oct	23-Oct
Lab 7 report due	**24-Oct**	**25-Oct**	29-Oct	30-Oct
Rec: FIR Filters	1-Oct	2-Oct	3-Oct	4-Oct
Rec: Freq Resp	8-Oct	9-Oct	10-Oct	11-Oct
Rec: Z-Trans	**17-Oct**	**18-Oct**	17-Oct	18-Oct
HW #6 due	8-Oct	9-Oct	10-Oct	11-Oct
HW #7 due	**17-Oct**	**18-Oct**	17-Oct	18-Oct
HW #8 due	**24-Oct**	23-Oct	24-Oct	25-Oct
Quiz #2	22-Oct	22-Oct	22-Oct	22-Oct

Perseverance

- **A** lowly virtue whereby mediocrity achieves a glorious success…A. Bierce
- **B**ear in mind, if you are going to amount to anything, that your success does not depend upon the brilliance and the impetuosity with which you take hold, but upon the ever lasting and sanctified bull doggedness with which you hang on after you have taken hold…Dr. A. B. Meldrum

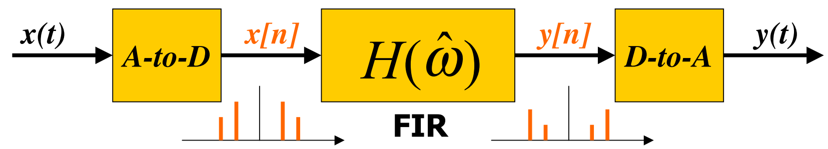
LECTURE

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, pp. 188-194
- Other Reading:
  - Recitation: Ch. 6, pp. 176-188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7, start

LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- **UNIFICATION**: How does Frequency Response affect x(t) to produce y(t) ?



TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

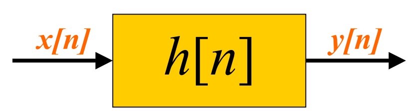
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(\hat{\omega}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

$$H(\hat{\omega}) = h[0]e^{-j0} + h[1]e^{-j\hat{\omega}} + h[2]e^{-j\hat{\omega}2} + h[3]e^{-j\hat{\omega}3} + \cdots$$

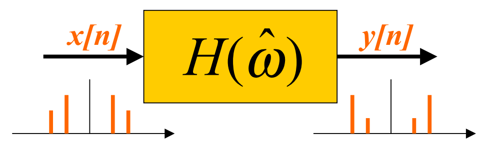
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$



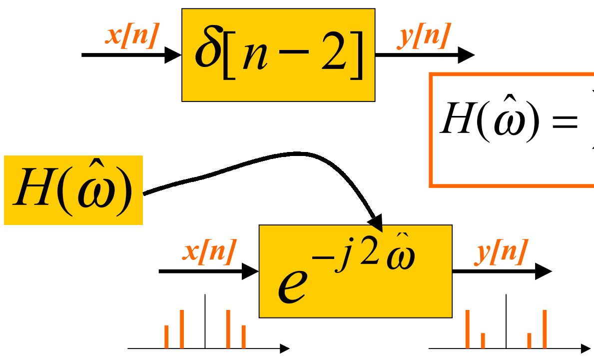
$$b_k = \{0,0,1\}$$

$$h[n] = \delta[n-2]$$



DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$



$$H(\hat{\omega}) = \sum_k \delta[k-2] e^{-j\hat{\omega}k}$$

k=2 ONLY

GENERAL DELAY PROPERTY

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-n_d]$

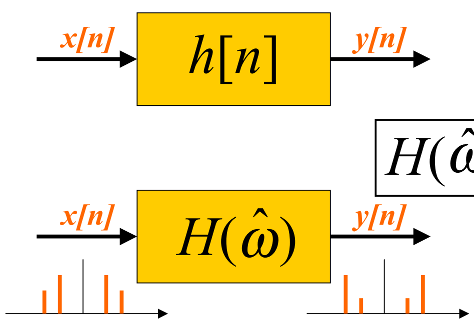
$$h[n] = \delta[n-n_d]$$

$$H(\hat{\omega}) = \sum_k \delta[k-n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for k at k = n_d

FREQ DOMAIN --> TIME ??

- START with $H(\hat{\omega})$ and find $h[n]$ or b_k



$$h[n] = ?$$

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$

FREQ DOMAIN --> TIME

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$

EULER's Formula

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0,\ 3.5,\ 0,\ 3.5\}$$

PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING:

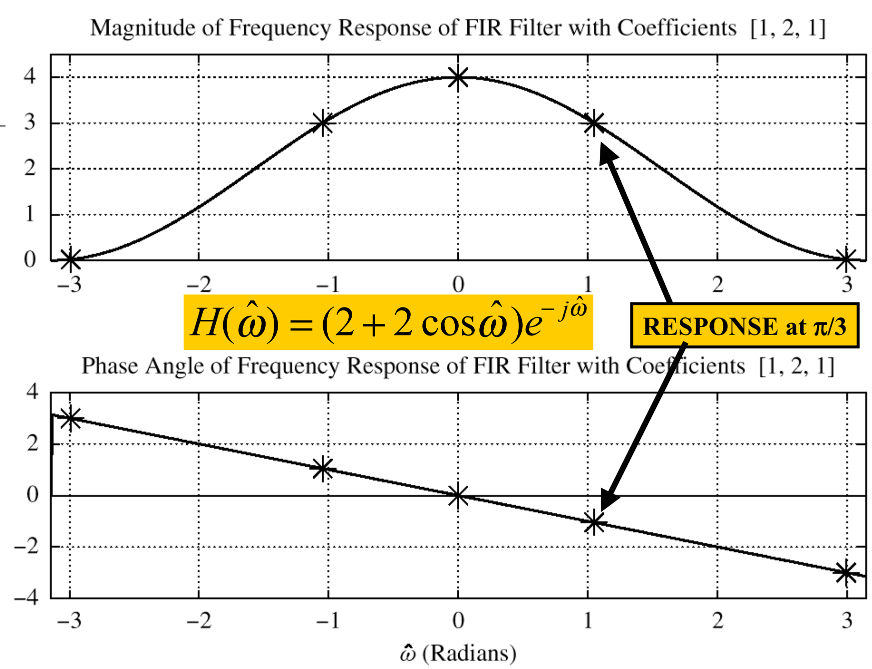
MAG PHASE

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\varphi(\hat{\omega})}$$

FREQ. RESPONSE PLOTS

- DENSE GRID `(ww)` from $-\pi$ to $+\pi$
  - `ww = -pi:(pi/100):pi;`
- `yy = freqz(bb,1,ww)`
  - VECTOR `bb` contains Filter Coefficients
  - DSP-First: `yy = freekz(bb,1,ww)`

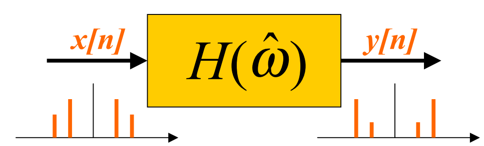
$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$



EXAMPLE 6.2

Find $y[n]$ when $H(\hat{\omega})$ is known

& $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(\hat{\omega}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

Answer: Eval $H(\hat{\omega})$ at $\hat{\omega} = \pi/3$.

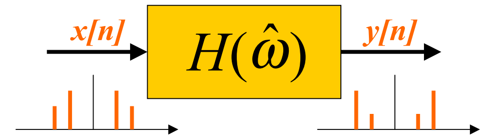
$$H(\hat{\omega}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(\hat{\omega}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(\hat{\omega})$ is known and $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(\hat{\omega}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EX: COSINE INPUT (ans-1)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

$$y_1[n] = H(\tfrac{\pi}{3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(-\tfrac{\pi}{3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(\hat{\omega}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(\tfrac{\pi}{3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(-\tfrac{\pi}{3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6\cos(\tfrac{\pi}{3}n - \tfrac{\pi}{12})$$

SINUSOID thru FIR

$$x[n] = X_0 + \sum_{k=1}^{N}\left(\frac{X_k}{2}e^{j\hat{\omega}_k n} + \frac{X_k^*}{2}e^{-j\hat{\omega}_k n}\right)$$

$$= X_0 + \sum_{k=1}^{N}|X_k|\cos(\hat{\omega}_k n + \angle X_k)$$

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

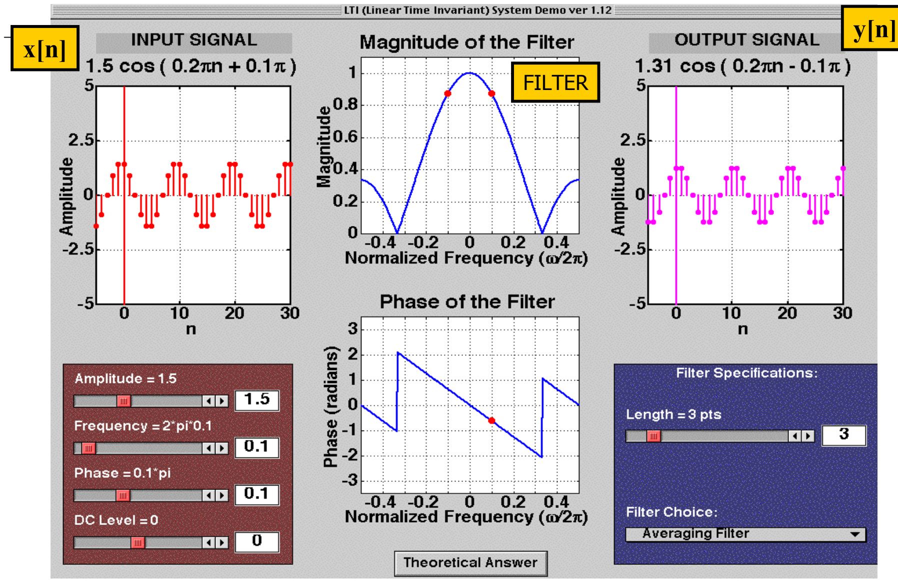
$$y[n] = \mathcal{H}(0)X_0 + \sum_{k=1}^{N}\left(\mathcal{H}(\hat{\omega}_k)\frac{X_k}{2}e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k)\frac{X_k^*}{2}e^{-j\hat{\omega}_k n}\right)$$

$$= \mathcal{H}(0)X_0 + \sum_{k=1}^{N}|\mathcal{H}(\hat{\omega}_k)||X_k|\cos\left(\hat{\omega}_k n + \angle X_k + \angle\mathcal{H}(\hat{\omega}_k)\right)$$

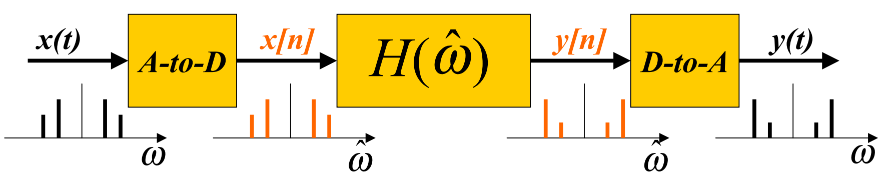
MULTIPLY MAGS

ADD PHASES

LTI Demo with Sinusoids

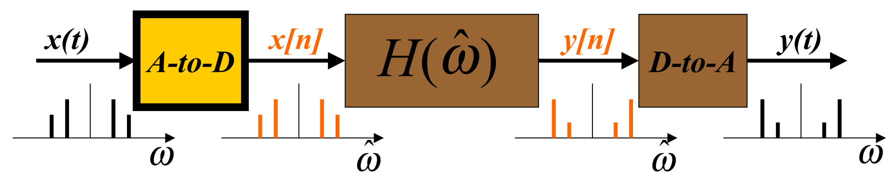


DIGITAL "FILTERING"



- ω SPECTRUM of x(t) (SUM of SINUSOIDS)
- SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?
- $\hat{\omega}$ SPECTRUM y[n] (FIR Gain or Nulls)
- ω Then, OUTPUT y(t) = SUM of SINUSOIDS

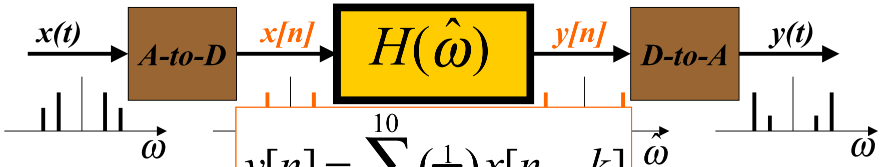
FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s$
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example



$$y[n] = \sum_{k=0}^{10} (\tfrac{1}{11}) x[n-k]$$

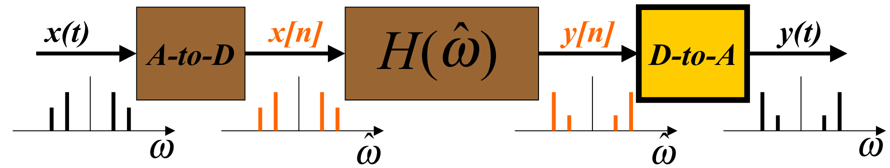
- 250 Hz
- 25 Hz

$$H(\hat{\omega}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

?

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

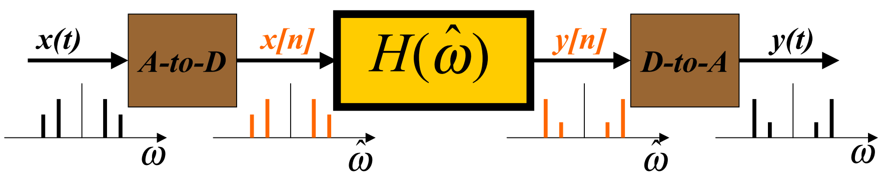
D-A FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow t f_s$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

$$\omega = \hat{\omega} f_s$$

TRACK the FREQUENCIES

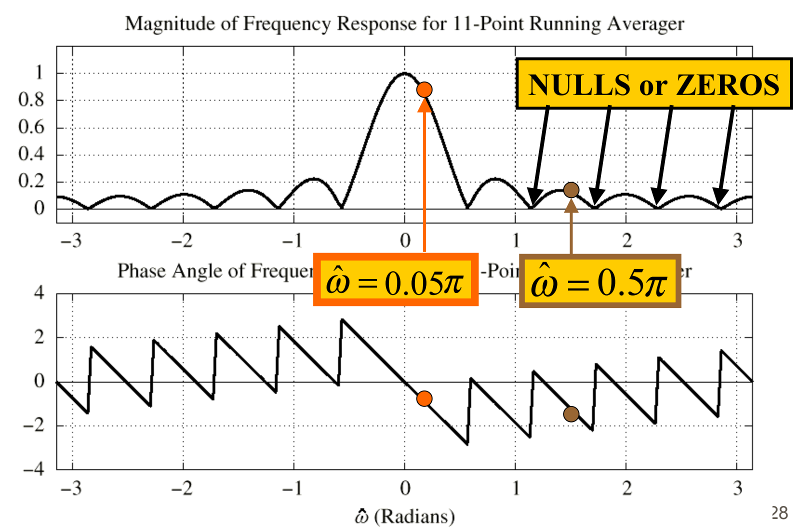


250 Hz	0.5π	$H(0.5\pi)$	0.5π	250 Hz
25 Hz	$.05\pi$	$H(0.05\pi)$	$.05\pi$	25 Hz

Fs = 1000 Hz

NO new freqs

11-pt AVERAGER



EVALUATE Freq. Response

$$H(\hat{\omega}) = \frac{\sin(\hat{\omega}11/2)}{11\sin(\hat{\omega}/2)} e^{-j\hat{\omega}5}$$

At $\hat{\omega} = 0.5\pi$

$$H(\hat{\omega}) = \frac{\sin((0.5\pi)11/2)}{11\sin(0.5\pi/2)} e^{-j(0.5\pi)5}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$f_s = 1000$

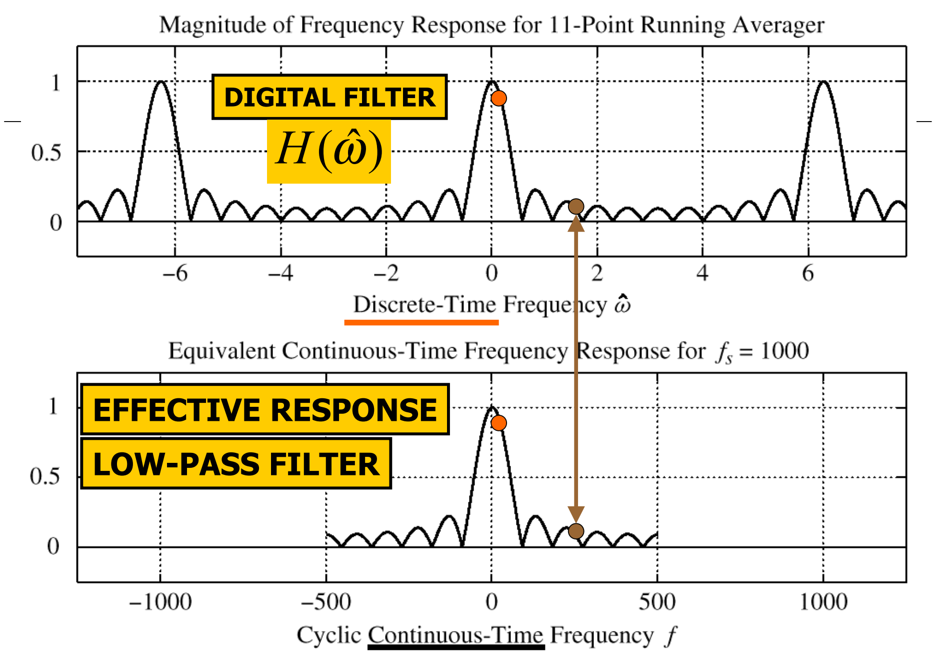
$$\mathcal{H}(2\pi(25)/1000) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000} = 0.8811 e^{-j\pi/4}$$

$$\mathcal{H}(2\pi(250)/1000) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000} = 0.0909 e^{-j\pi/2}$$

MAG SCALE

PHASE CHANGE

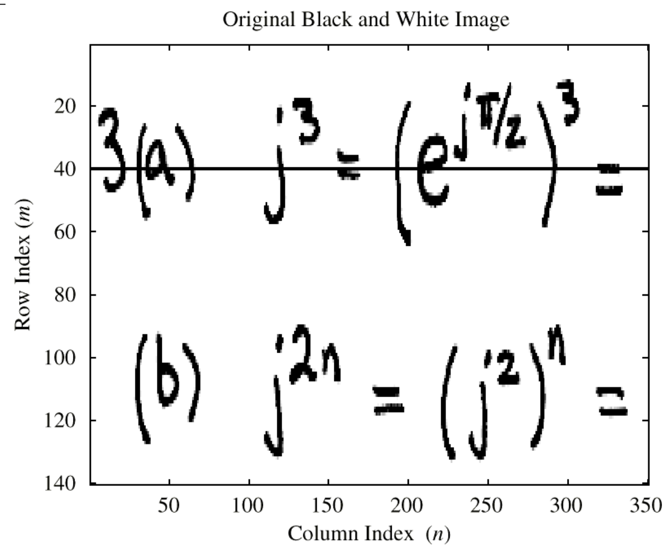
$$y(t) = 0.8811\cos(2\pi(25)t - \pi/4) + 0.0909\sin(2\pi(250)t - \pi/2)$$



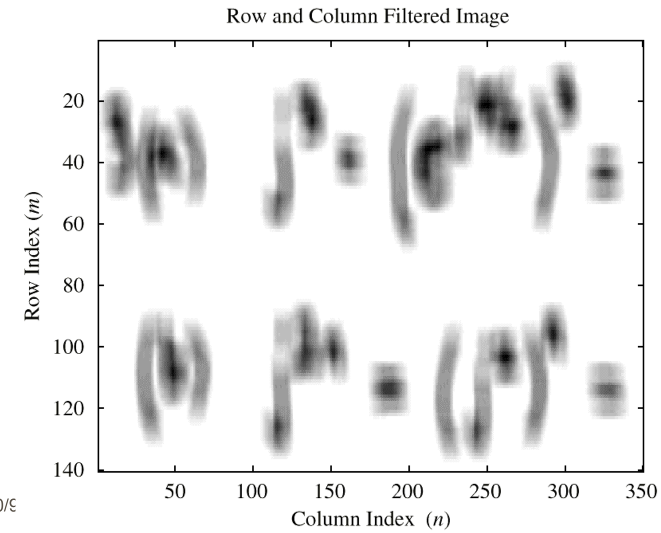
FILTER TYPES

- LOW-PASS FILTER (LPF)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (HPF)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (BPF)

B & W IMAGE

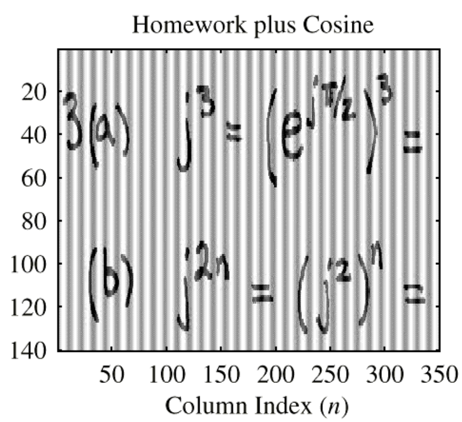


FILTERED B&W IMAGE



LPF: BLUR

B&W IMAGE with COSINE



FILTERED: 11-pt AVG