

EE-2025

Fall-2001

Lecture 17
Frequency Response of
Continuous-Time Systems
2-Nov-01

Info: Web-CT, Lab, HW

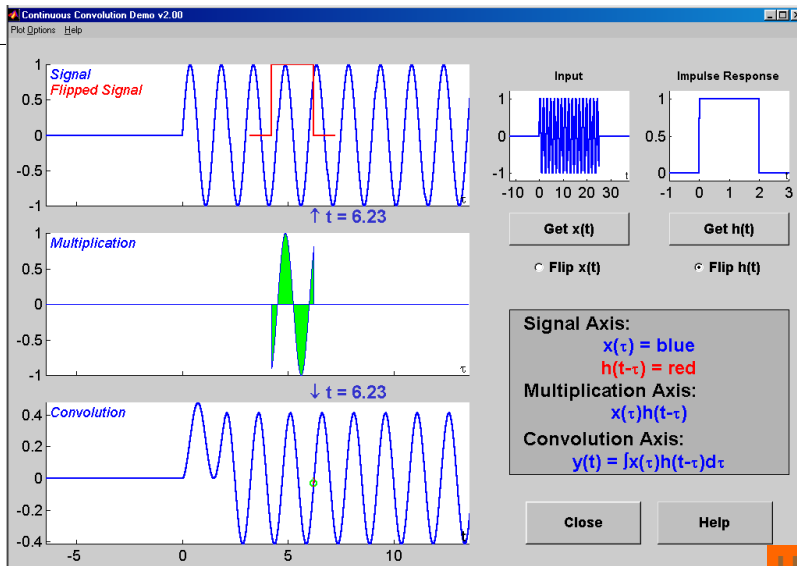
- Calendar:
 - Quiz #3 is 19-Nov (Monday)
- CHECK YOUR GRADES !!!
 - Web-CT is the OFFICIAL gradebook
- Prob Set #9 is due next week
- Lab #9 utilizes convolution GUIs
 - Animate the FLIP & SLIDE nature of convolution

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Convolution GUI: Sinusoid



READING ASSIGNMENTS

- This Lecture:
 - Chapter 11, pp. 1053-1072 (all)
 - In NEW Chapters
- Other Reading:
 - Recitation: Ch. 11 all, Ch 12, pp. 1079-1092
 - Next Lecture: Chapter 12, 1079-1092

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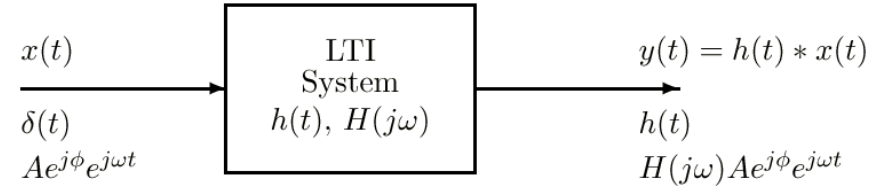
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LECTURE OBJECTIVES

- Review of convolution
 - **THE** operation for **LTI** Systems
- Complex exponential input signals
 - Frequency Response
 - Cosine signals
 - Real part of complex exponential
- Fourier Series thru $H(j\omega)$
 - These are Analog Filters

LTI Systems

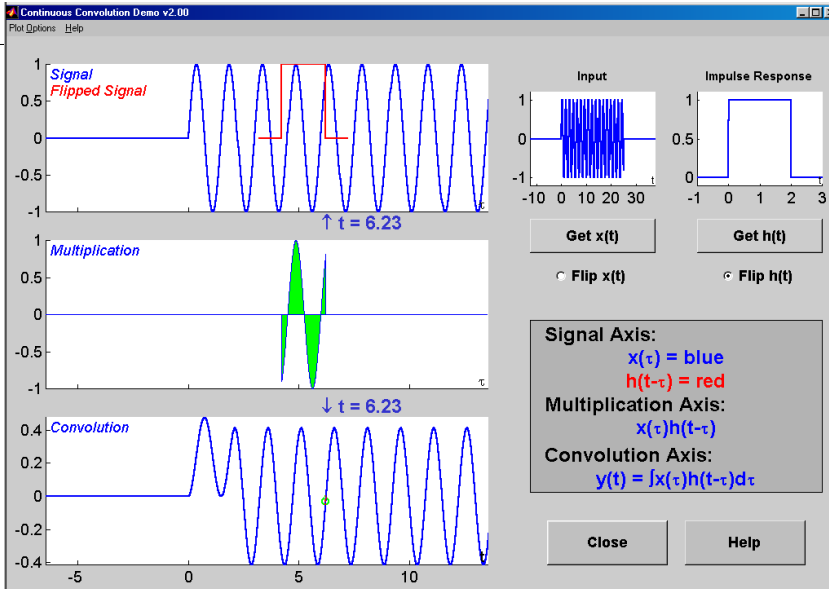


- Convolution defines LTI system

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

- Response to a complex exponential gives frequency response $H(j\omega)$

Convolution GUI: Sinusoid



Thought Process #1

- **SUPERPOSITION (Linearity)**
 - Make $x(t)$ a weighted sum of signals
 - Then $y(t)$ is also a sum—different weights
 - DIFFERENT OUTPUT SIGNALS usually
- Use **SINUSOIDS**
 - Make $x(t)$ a weighted sum of sinusoids
 - Then $y(t)$ is also a sum of sinusoids
 - Different Magnitudes and Phase
- **LTI SYSTEMS: Sinusoidal Response**

Thought Process #2

- **SUPERPOSITION (Linearity)**
 - ▮ Make $x(t)$ a weighted sum of signals
- **Use SINUSOIDS**
 - ▮ **Any $x(t)$ = weighted sum of sinusoids**
 - ▮ **HOW? Use FOURIER ANALYSIS INTEGRAL**
 - ▮ **To find the weights from $x(t)$**
- **LTI SYSTEMS:**
 - ▮ Frequency Response changes each sinusoidal component

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Complex Exponential Input

$$x(t) = Ae^{j\phi} e^{j\omega t} \mapsto y(t) = H(j\omega) Ae^{j\phi} e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) Ae^{j\phi} e^{j\omega(t-\tau)} d\tau$$

$$y(t) = \left(\int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \right) Ae^{j\phi} e^{j\omega t}$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

Frequency Response

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When does $H(j\omega)$ Exist?

- When is $|H(j\omega)| < \infty$?

$$|H(j\omega)| = \left| \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \right| \leq \int_{-\infty}^{\infty} |h(\tau)| |e^{-j\omega\tau}| d\tau$$

$$|H(j\omega)| \leq \int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

- Thus the frequency system is a **stable** system.

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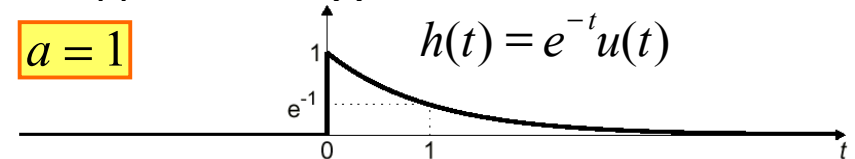
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$$h(t) = e^{-at} u(t) \Leftrightarrow H(j\omega) = \frac{1}{a + j\omega}$$

- Suppose that $h(t)$ is:

$$a = 1$$

$$h(t) = e^{-t} u(t)$$



$$H(j\omega) = \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) e^{-j\omega\tau} d\tau = \int_0^{\infty} e^{-(a+j\omega)\tau} d\tau$$

$$a > 0$$

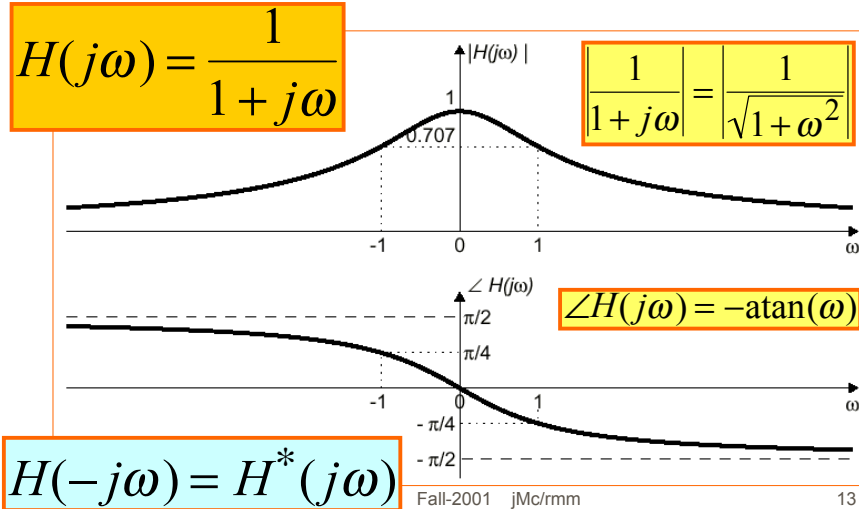
$$H(j\omega) = \frac{e^{-(a+j\omega)\tau}}{-(a+j\omega)} \Big|_0^{\infty} = \frac{e^{-a\tau} e^{-j\omega\tau}}{-(a+j\omega)} \Big|_0^{\infty} = \frac{1}{a+j\omega}$$

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Magnitude and Phase Plots



Freq Response of Integrator?

- Impulse Response
 - $h(t) = u(t)$
- NOT a Stable System
 - Frequency response $H(j\omega)$ does NOT exist

$h(t) = e^{-at} u(t) \Leftrightarrow H(j\omega) = \frac{1}{a+j\omega} \rightarrow \frac{1}{j\omega}?$

Need another term

- Leaky Integrator (a is small)
 - Cannot build a perfect Integral $a \rightarrow 0$

Ideal Delay:

$y(t) = x(t - t_d)$

$H(j\omega) = \int_{-\infty}^{\infty} \delta(\tau - t_d) e^{-j\omega\tau} d\tau = e^{-j\omega t_d}$

$H(j\omega) = e^{-j\omega t_d}$

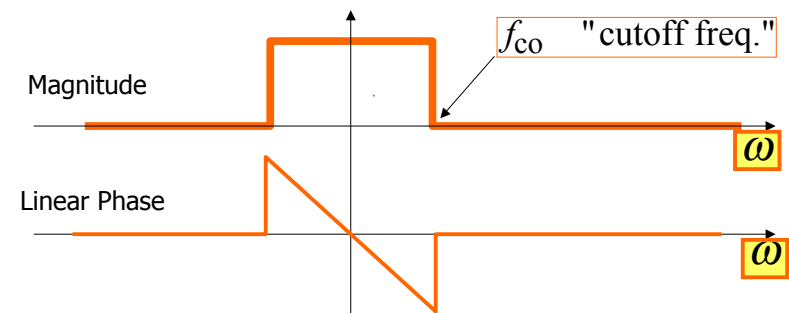
$x(t) = e^{j\omega t} \mapsto H(j\omega)$

$y(t) = e^{j\omega(t-t_d)} = e^{-j\omega t_d} e^{j\omega t}$

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Ideal Lowpass Filter w/ Delay

$H_{LP}(j\omega) = \begin{cases} e^{-j\omega t_d} & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$



Example: Ideal Low Pass

$$H_{LP}(j\omega) = \begin{cases} e^{-j3\omega} & |\omega| < 2 \\ 0 & |\omega| > 2 \end{cases}$$

$$x(t) = 10e^{j\pi/3} e^{j1.5t} \mapsto y(t) = H(j1.5)10e^{j\pi/3} e^{j1.5t}$$

$$y(t) = (e^{-j4.5})10e^{j\pi/3} e^{j1.5t} = 10e^{j\pi/3} e^{j1.5(t-3)}$$

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Cosine Input

$$x(t) = A \cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

$$y(t) = H(j\omega_0) \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + H(-j\omega_0) \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

Since $H(-j\omega_0) = H^*(j\omega_0)$

$$y(t) = A |H(j\omega_0)| \cos(\omega_0 t + \phi + \angle H(j\omega_0))$$

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Review Fourier Series

ANALYSIS

- Get representation from the signal
- Works for PERIODIC Signals

Fourier Series

- INTEGRAL over one period

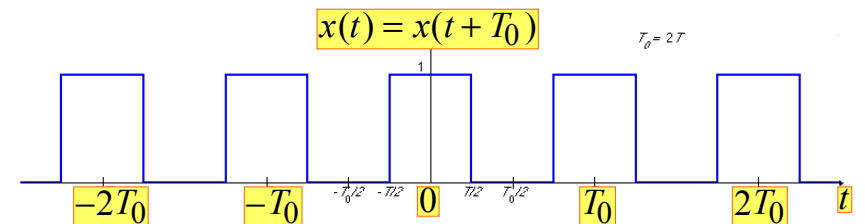
$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\omega_0 k t} dt$$

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General Periodic Signals



$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \quad \leftarrow \text{Fourier Synthesis}$$

Fundamental Freq.
 $\omega_0 = 2\pi / T_0 = 2\pi f_0$

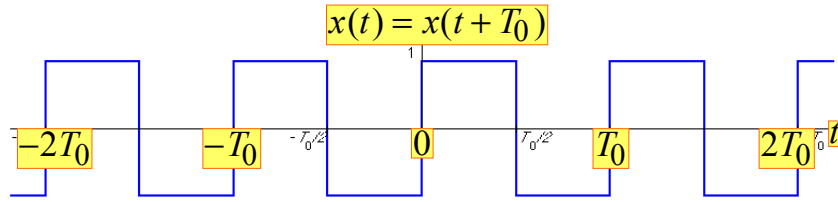
$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\omega_0 k t} dt \quad \leftarrow \text{Fourier Analysis}$$

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Square Wave Signal



$$a_k = \frac{1}{T_0} \int_0^{T_0/2} (1)e^{-j\omega_0 kt} dt + \frac{1}{T_0} \int_{T_0/2}^{T_0} (-1)e^{-j\omega_0 kt} dt$$

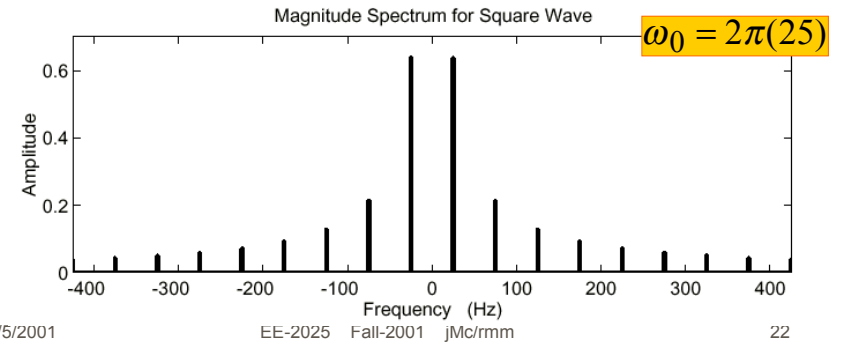
$$a_k = \frac{e^{-j\omega_0 kt} \Big|_0^{T_0/2}}{-j\omega_0 k T_0} - \frac{e^{-j\omega_0 kt} \Big|_{T_0/2}^{T_0}}{-j\omega_0 k T_0} = \frac{1 - e^{-j\pi k}}{j\pi k}$$

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Spectrum from Fourier Series

$$a_k = \frac{1 - e^{-j\pi k}}{j\pi k} = \begin{cases} \frac{2}{j\pi k} & k = \pm 1, \pm 3, \dots \\ 0 & k = 0, \pm 2, \pm 4, \dots \end{cases}$$

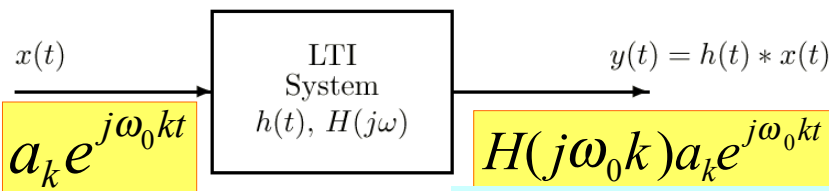


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LTI Systems with Periodic Inputs



By superposition,

Output has *same* frequencies

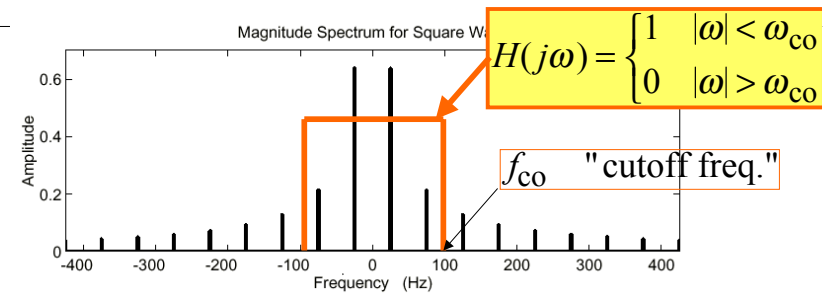
$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(j\omega_0 k) e^{j\omega_0 kt} = \sum_{k=-\infty}^{\infty} b_k e^{j\omega_0 kt}$$

$$b_k = a_k H(j\omega_0 k)$$

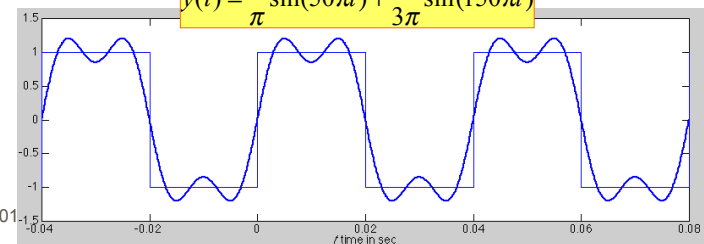
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Ideal Lowpass Filter



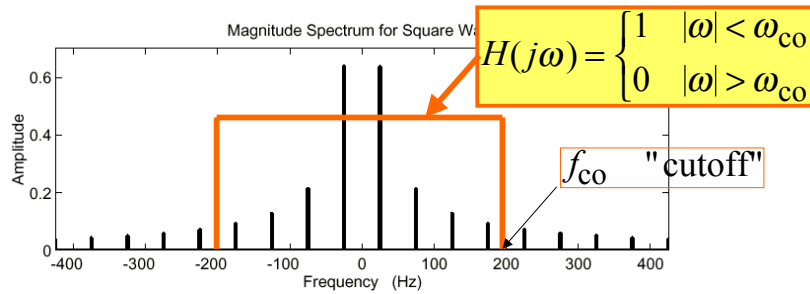
$$y(t) = \frac{4}{\pi} \sin(50\pi t) + \frac{4}{3\pi} \sin(150\pi t)$$



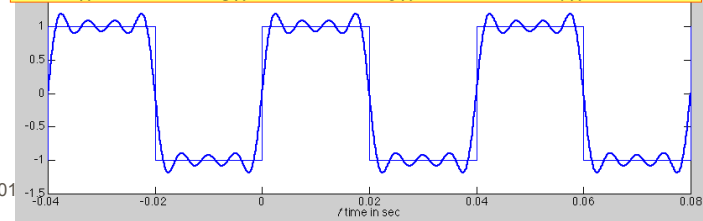
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Ideal Lowpass Filter



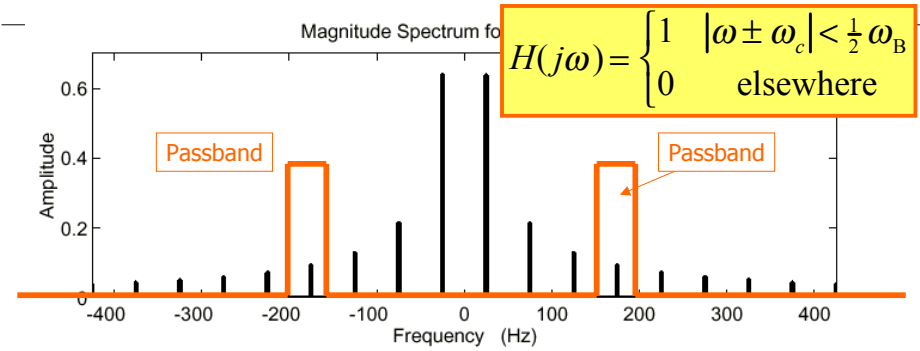
$$y(t) = \frac{4}{\pi} \sin(50\pi t) + \frac{4}{3\pi} \sin(150\pi t) + \frac{4}{5\pi} \sin(250\pi t) + \frac{4}{7\pi} \sin(350\pi t)$$



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Ideal Bandpass Filter



What is the output signal ?

$$y(t) = \frac{2}{j7\pi} e^{j2\pi(175)t} - \frac{2}{j7\pi} e^{-j2\pi(175)t} = \frac{4}{7\pi} \cos(2\pi(175)t - \frac{1}{2}\pi)$$

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Example

$$H(j\omega) = e^{-j\omega t_d}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \mapsto y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j\omega_0 k t}$$

$$b_k = a_k H(j\omega_0 k) = a_k e^{-j\omega_0 k t_d}$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k e^{-j\omega_0 k t_d} e^{j\omega_0 k t} = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k (t - t_d)}$$

$$\therefore y(t) = x(t - t_d)$$

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