EE-2025 Fall-2001

Lecture 23
IIR Filters: Feedback & H(z)
26-Nov-01

Info: HW and Labs

- Reading Assignment:
  - Chapter 8 in DSP First
- Prob Set #11 - due this week
  - Prob Set #12 next week
- Lab #11 due this week of 26-Nov
  - Lab #12 due next week (3 — 6 Dec)
  - ALL Labs must be turned in by 5pm on 7-Dec-01

Final Exam Info

- Calendar: Final Exam(s)
  - Period 3, Monday, 10-Dec @ 2:50 pm
    - NOON Lecture
    - **Must take exam with your assigned section**
  - Period 11, Thurs, 13-Dec @ 11:30am
    - 11 AM Lecture
- **Report CONFLICTS immediately !!!!**
  - e.g., 3 exams in one day
- Reviews will be held on Sunday & Wednesday
  - 6:00 PM in ECE Auditorium

Grades

- Quiz #3 Results
  - **Median = 78**; average = 74.8
  - Nine 100's
- Check your own grades for accuracy
- Cumulative Averages are now posted
  - Lowest HW is dropped now.
  - Max = 70 (at this moment)
    - Final will be 25 (max)
    - Recitation Points will be 5 (max)

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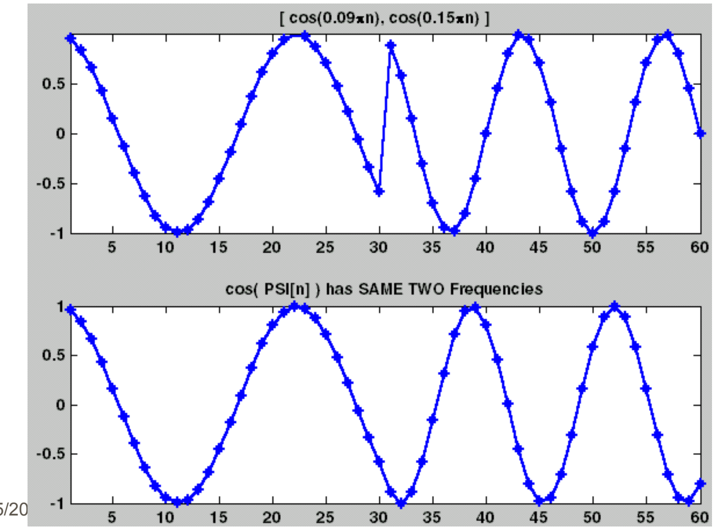
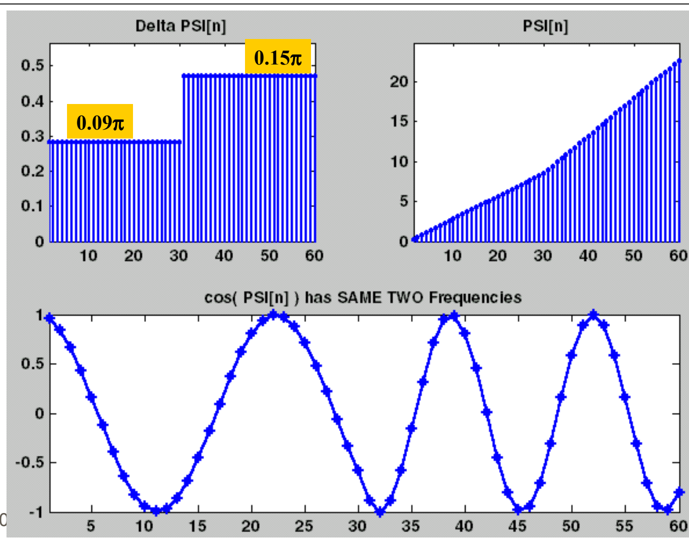
Labs 11 & 12
FSK synthesis & demodulation
26-Nov-01

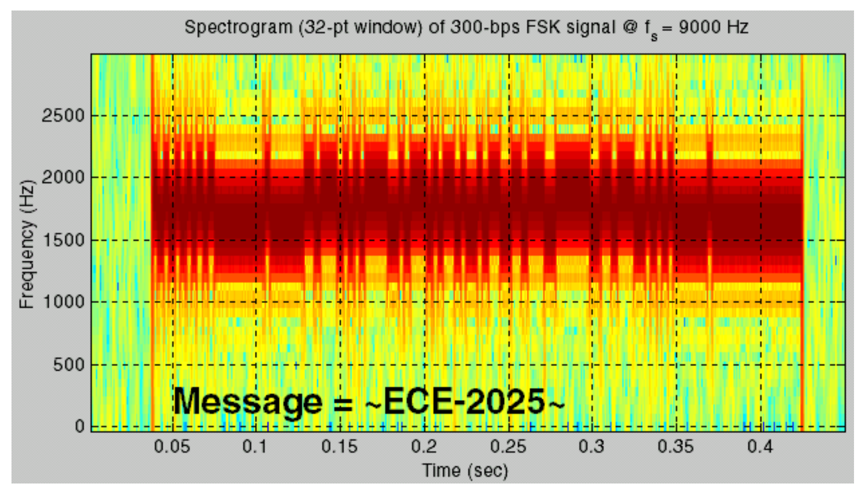
FSK: continuous phase



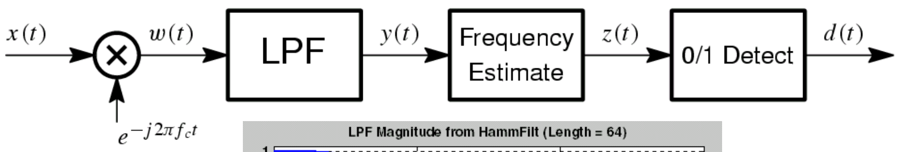
FSK: use cos(psi[n])

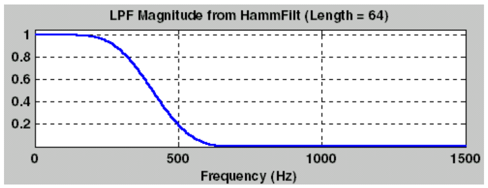


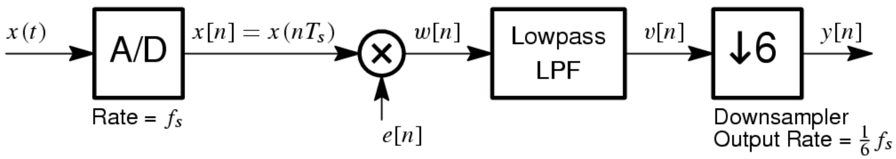
FSK Spectrogram



FSK demodulator





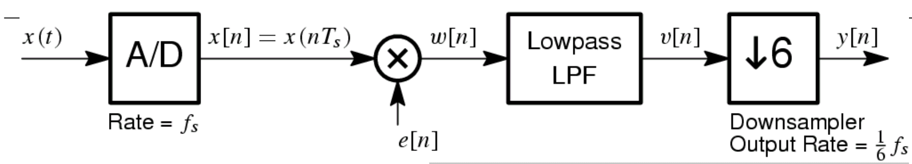


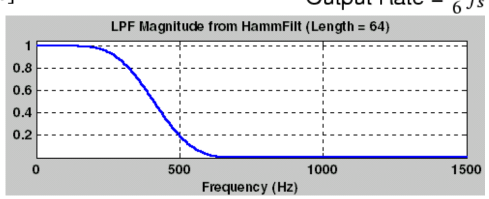
Anti-Aliasing Filter

- Given samples of w(t) at fs=9000 Hz, it is possible to downsample to a lower rate
  - Must obey **SAMPLING THEOREM**
  - NEW fs > twice the highest frequency
- w[n] is w(t) sampled at fs=9000 Hz
- Want w(t) sampled at fs=1500 Hz
  - Use **DIGITAL LPF** with cutoff at 750 Hz

$$\hat{\omega}_c = 2\pi \frac{f_c}{f_s} = 2\pi \frac{750}{9000} = \frac{\pi}{6}$$

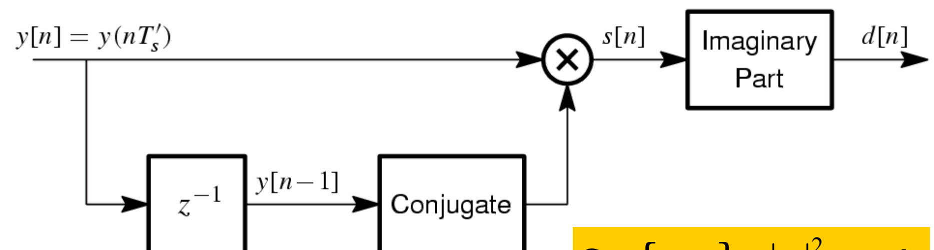
Anti-Aliasing Filter





- y[n] is every sixth sample of v[n]
  - i.e., samples of w(t) at fs=1500 Hz
- LPF must remove all freqs above 750 Hz

SLICER produces Frequency

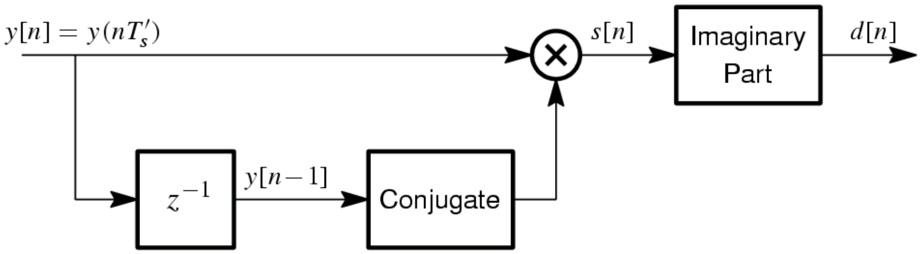


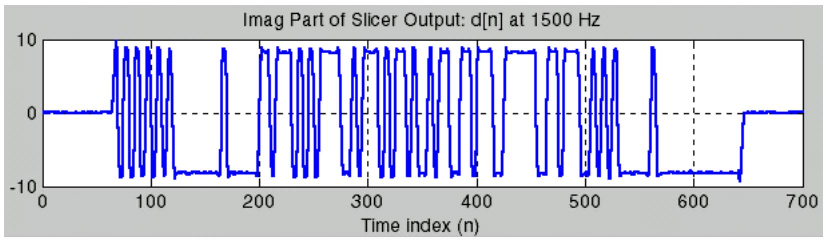
$$\Im m\{s[n]\} = |A|^2 \sin\hat{\omega}_1$$

$$s[n] = Ae^{j\hat{\omega}_1 n}\left(Ae^{j\hat{\omega}_1 (n-1)}\right)^*$$

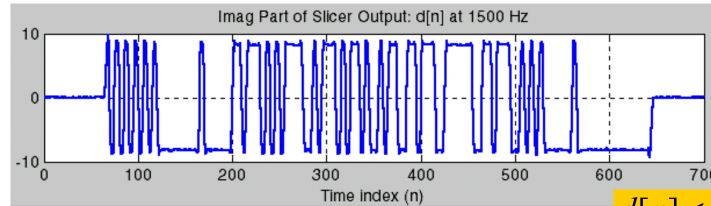
$$= |A|^2 e^{j\hat{\omega}_1 n} e^{-j\hat{\omega}_1 (n-1)} = |A|^2 e^{j\hat{\omega}_1}$$

SLICER output



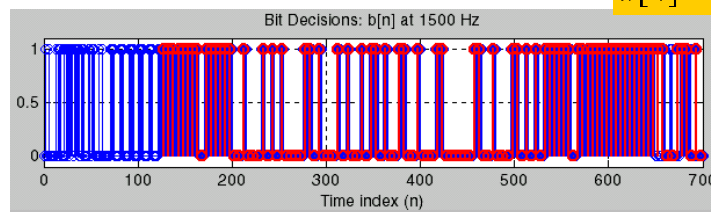


FSK Bit Detection

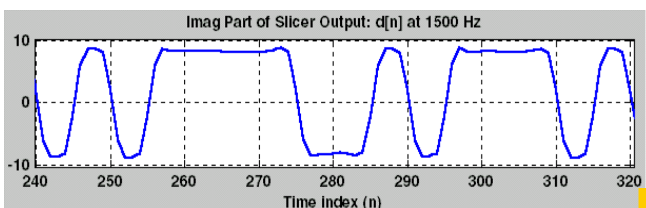


$d[n] < 0 \Rightarrow b[n] = 1$

$d[n] > 0 \Rightarrow b[n] = 0$

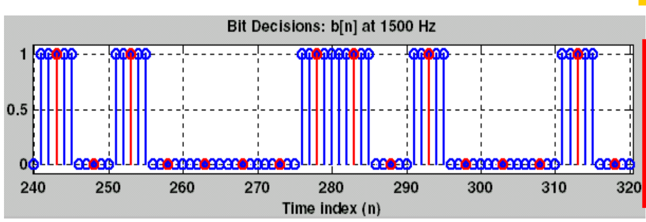


FSK Bits (Close-up)



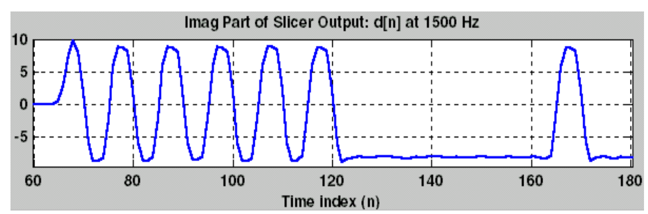
$d[n] < 0 \Rightarrow b[n] = 1$

$d[n] > 0 \Rightarrow b[n] = 0$

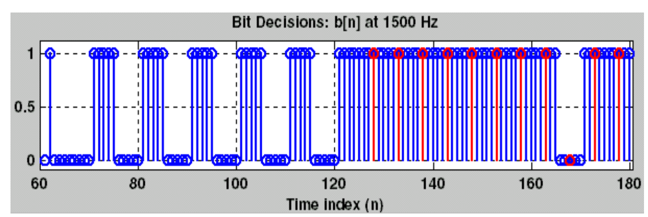


RED denotes index where decision is made.
1500Hz = 5*300bps

FSK: SYNCH on Preamble



101010101010 11111111



RED denotes index where decision is made.
Estimate the **middle** of the 5-element groups
Find the transitions of the 01010101… sequence
Eight 1's 11111111 marks the beginning of the message

READING ASSIGNMENTS

- This Lecture:
  - Chapter 8, pp. 249-263
- Other Reading:
  - Recitation: Ch. 8, pp. 261-272
    - POLES & ZEROS
  - Next Lecture: Chapter 8, pp. 269-282

LECTURE OBJECTIVES

- INFINITE IMPULSE RESPONSE FILTERS
  - Define **IIR** DIGITAL Filters
  - Have **FEEDBACK**: use PREVIOUS OUTPUTS

$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

- Show how to compute the output y[n]
  - FIRST-ORDER CASE (N=1)
  - Z-transform: Impulse Response h[n] <--> H(z)

Quick Review: L-pt Averager

$$y[n] = \sum_{k=0}^{L-1} \tfrac{1}{L} x[n-k]$$

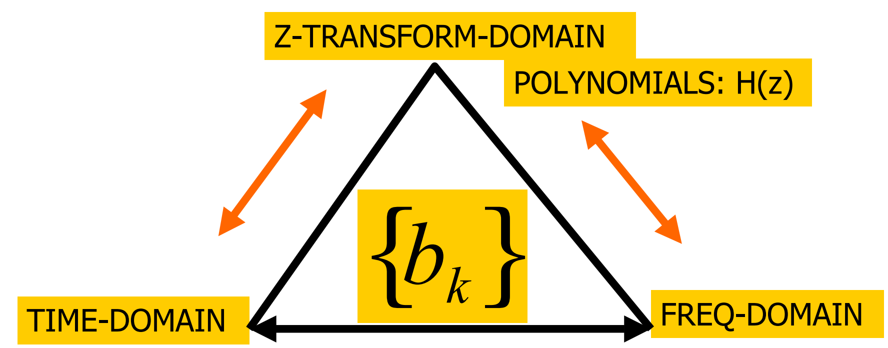
IMPULSE RESPONSE

$$h[n] = \sum_{k=0}^{L-1} \tfrac{1}{L} \delta[n-k]$$

SYSTEM FUNCTION

$$H(z) = \sum_{n=0}^{L-1} \tfrac{1}{L} z^{-n}$$

THREE DOMAINS



$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega} k}$$

LOGICAL THREAD

- FIND the IMPULSE RESPONSE, h[n]
  - INFINITELY LONG
  - **IIR** Filters

$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n}$$

- EXPLOIT THREE DOMAINS:
  - Show Relationship for IIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

ONE FEEDBACK TERM

- ADD PREVIOUS OUTPUTS

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

PREVIOUS FEEDBACK

FIR PART of the FILTER FEED-FORWARD

- CAUSALITY
  - NOT USING FUTURE OUTPUTS or INPUTS

FILTER COEFFICIENTS

- ADD PREVIOUS OUTPUTS

$$y[n] = 0.8 y[n-1] + 3x[n] - 2x[n-1]$$

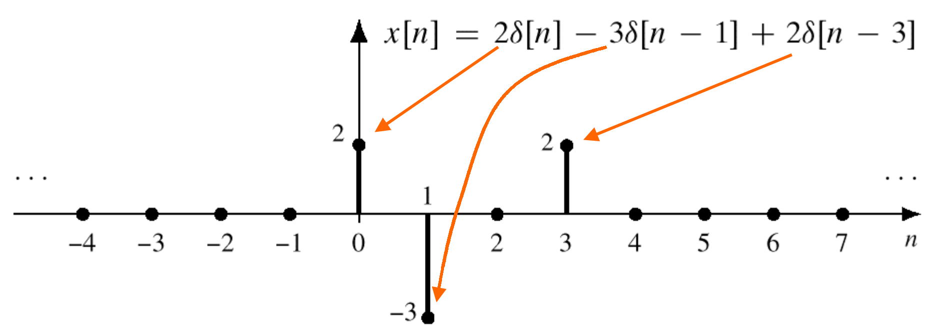
FEEDBACK COEFFICIENT

SIGN CHANGE

- MATLAB
  - `yy = filter([3,-2],[1,-0.8],xx)`

COMPUTE OUTPUT

$$y[n] = 0.8y[n-1] + 5x[n]$$



COMPUTE y[n]

- FEEDBACK DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 5x[n]$$

- NEED y[-1] to get started

$$y[0] = 0.8y[-1] + 5x[0]$$

AT REST CONDITION

- y[n] = 0, for n<0
- BECAUSE x[n] = 0, for n<0

INITIAL REST CONDITIONS

1. The input must be assumed to be zero prior to some starting time n_0, i.e., $x[n] = 0$ for $n < n_0$. We say that such inputs are *suddenly applied.*
2. The output is likewise assumed to be zero prior to the starting time of the signal, i.e., $y[n] = 0$ for $n < n_0$. We say that the system is *initially at rest* if its output is zero prior to the application of a suddenly applied input.

COMPUTE y[0]

- THIS STARTS THE RECURSION:

With the initial rest assumption, $y[n] = 0$ for $n < 0$,

$$y[0] = 0.8y[-1] + 5(2) = 0.8(0) + 5(2) = 10$$

- SAME with MORE FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^{2} b_k x[n-k]$$

COMPUTE MORE y[n]

- CONTINUE THE RECURSION:

$$y[1] = 0.8y[0] + 5x[1] = 0.8(10) + 5(-3) = -7$$

$$y[2] = 0.8y[1] + 5x[2] = 0.8(-7) + 5(0) = -5.6$$

$$y[3] = 0.8y[2] + 5x[3] = 0.8(-5.6) + 5(2) = 5.52$$

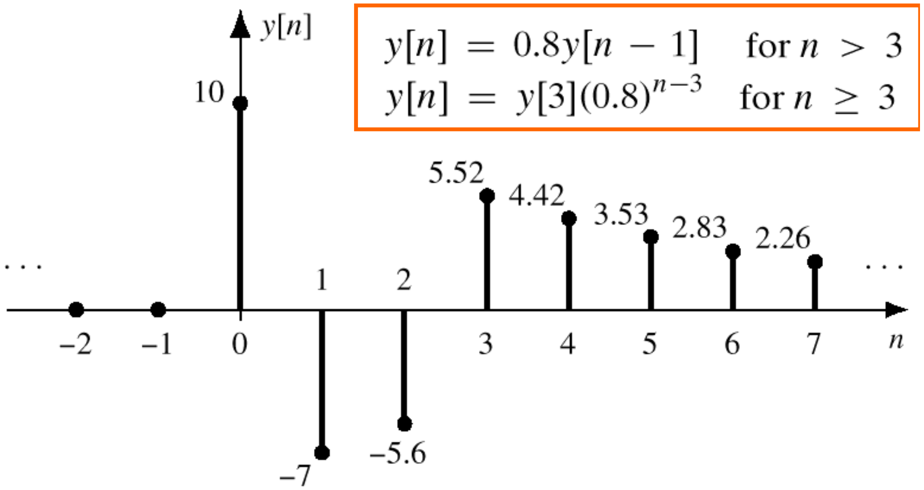
$$y[4] = 0.8y[3] + 5x[4] = 0.8(5.52) + 5(0) = 4.416$$

$$y[5] = 0.8y[4] + 5x[5] = 0.8(4.416) + 5(0) = 3.5328$$

$$y[6] = 0.8y[5] + 5x[6] = 0.8(3.5328) + 5(0) = 2.8262$$

PLOT y[n]

$$y[n] = 0.8y[n-1] \quad \text{for } n > 3$$
$$y[n] = y[3](0.8)^{n-3} \quad \text{for } n \geq 3$$



IMPULSE RESPONSE

$$h[n] = a_1 h[n-1] + b_0 \delta[n]$$

n	$n<0$	0	1	2	3	4
$\delta[n]$	0	1	0	0	0	0
$h[n-1]$	0	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$
$h[n]$	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$	$b_0(a_1)^4$

From this table it is obvious that the general formula is

$$h[n] = \begin{cases} b_0(a_1)^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$h[n] = b_0(a_1)^n u[n]$$

$$u[n] = 1, \quad \text{for } n \geq 0$$

IMPULSE RESPONSE

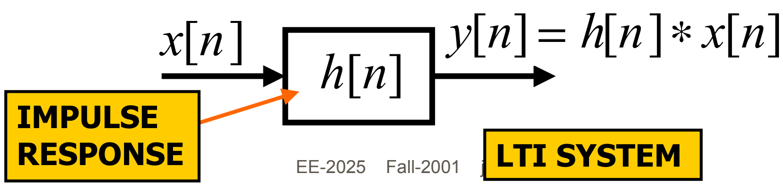
- DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n]$$

- Find h[n]

$$h[n] = 3(0.8)^n u[n]$$

- **CONVOLUTION** in TIME-DOMAIN



PLOT IMPULSE RESPONSE

$$h[n] = b_0(a_1)^n u[n] = 3(0.8)^n u[n]$$