

Lecture 24
IIR Filters: 3 Domains
3-Dec-01

Final Exam Info

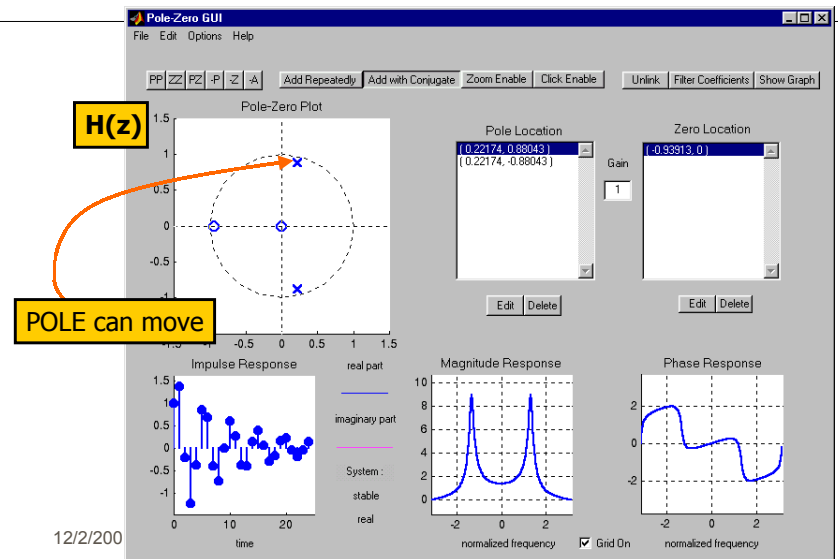
- Calendar: Final Exam(s)
 - Period 3, Monday, 11-Dec @ 2:50 pm
 - NOON Lecture
 - Period 11, Thurs, 13-Dec @ 11:30am
 - 11 AM Lecture
- Report **CONFLICTS** immediately !!!!
 - e.g., 3 exams in one day
- ID check will be done at Final Exam
- Reviews will be held on Sunday & Wednesday
 - 6:00pm in ECE Auditorium

LAST LAB This Week

- Evaluation of Lab #12 during Lab time
 - Run against pre-encoded messages
- **ALL Lab Reports due by Friday, 7-Dec:**
 - **5 PM at the latest (room VanLeer-475)**
- Course Evaluations during last week
 - THREE
 - Two for GT: Lecture & Recitation
 - ***** One on Web-CT *****
- HW #12 due this week: Friday in Lecture.

PeZ GUI for MATLAB

LECTURE



READING ASSIGNMENTS

■ This Lecture:

■ Chapter 8, pp. 279-300

■ Other Reading:

■ Recitation: Ch. 8, pp. 261-272

■ POLES & ZEROS

■ Next Lecture: Chapter 8, all

LECTURE OBJECTIVES

■ SECOND-ORDER IIR FILTERS

■ TWO FEEDBACK TERMS

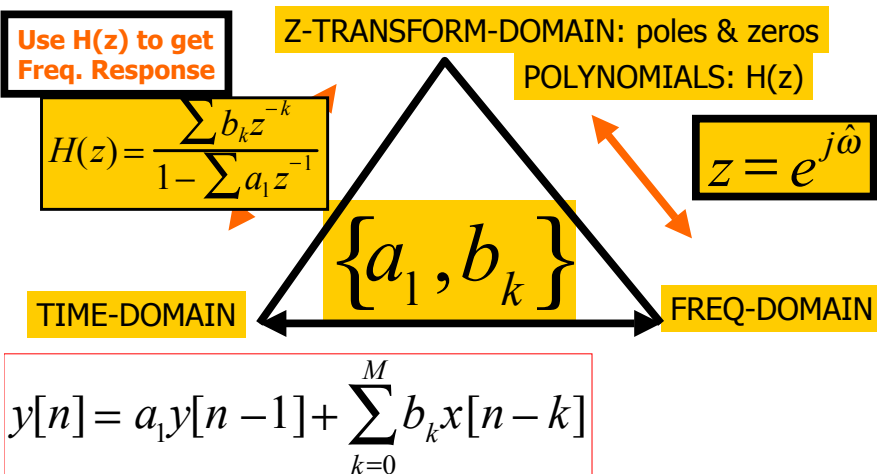
$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^2 b_k x[n-k]$$

■ H(z) can have **COMPLEX POLES** & ZEROS

■ THREE-DOMAIN APPROACH

■ BPFs have POLES NEAR THE UNIT CIRCLE

THREE DOMAINS



Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS

	$x[n]$	\iff	$X(z)$
1.	$ax_1[n] + bx_2[n]$	\iff	$aX_1(z) + bX_2(z)$
2.	$x[n - n_0]$	\iff	$z^{-n_0} X(z)$
3.	$y[n] = x[n] * h[n]$	\iff	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	\iff	1
5.	$\delta[n - n_0]$	\iff	z^{-n_0}
6.	$a^n u[n]$	\iff	$\frac{1}{1 - az^{-1}}$

SECOND-ORDER FILTERS

- Two FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

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MORE POLES

- Denominator is QUADRATIC

- 2 Poles: REAL
- or COMPLEX CONJUGATES

$$\frac{a_1 \pm \sqrt{a_1^2 + 4a_2}}{2}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 - a_1 z - a_2}$$

PROPERTY OF REAL POLYNOMIALS

A polynomial of degree N has N roots. If all the coefficients of the polynomial are real, the roots either must be real, or must occur in complex conjugate pairs.

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TWO COMPLEX POLES

- Find Impulse Response ?

- Can OSCILLATE vs. n

- "RESONANCE"

$$(p_k)^n = (r e^{j\theta})^n = r^n e^{jn\theta}$$

- Find **FREQUENCY RESPONSE**

- Depends on Pole Location

- Close to the Unit Circle?

- Make **BANDPASS FILTER**

$$\text{pole} = r e^{j\theta}$$

$$r \rightarrow 1?$$

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2nd ORDER EXAMPLE

$$h[n] = 0.9^n \cos\left(\frac{\pi}{3}n\right) u[n] = 0.9^n \frac{1}{2} (e^{j\pi n/3} + e^{-j\pi n/3}) u[n]$$

$$H(z) = \frac{0.5}{1 - 0.9 e^{j\pi/3} z^{-1}} + \frac{0.5}{1 - 0.9 e^{-j\pi/3} z^{-1}}$$

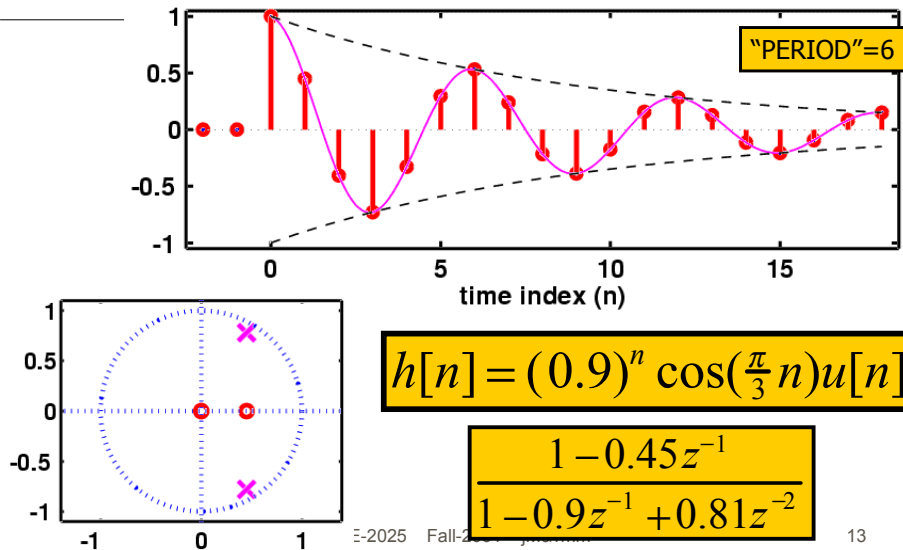
$$H(z) = \frac{1 - 0.9 \cos\left(\frac{\pi}{3}\right) z^{-1}}{(1 - 0.9 e^{j\pi/3} z^{-1})(1 - 0.9 e^{-j\pi/3} z^{-1})}$$

$$H(z) = \frac{1 - 0.45 z^{-1}}{1 - 0.9 z^{-1} + 0.81 z^{-2}}$$

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h[n]: Decays & Oscillates



2nd ORDER Z-transform PAIR

$$h[n] = r^n \cos(\theta n)u[n]$$

GENERAL ENTRY for
z-Transform TABLE

$$H(z) = \frac{1 - r \cos \theta z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

$$h[n] = Ar^n \cos(\theta n + \varphi)u[n]$$

$$H(z) = A \frac{\cos \varphi - r \cos(\theta - \varphi)z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

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2nd ORDER EX: n-Domain

$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

$$y[n] = 0.9y[n-1] - 0.81y[n-2] + x[n] - 0.45x[n-1]$$

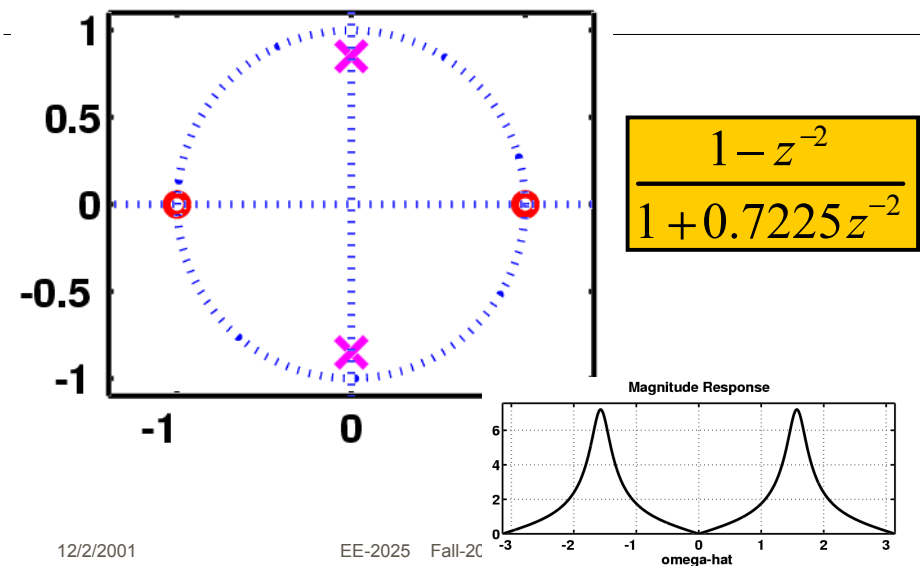
```
aa = [ 1, -0.9, 0.81 ];
bb = [ 1, -0.45 ];
nn = -2:19;
hh = filter( bb, aa, (nn==0) );
HH = freqz( bb, aa, [-pi,pi/100:pi] );
```

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Complex POLE-ZERO PLOT

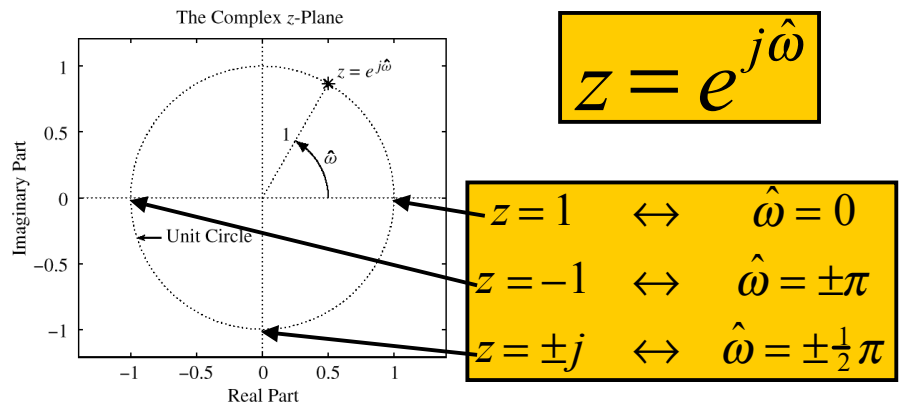


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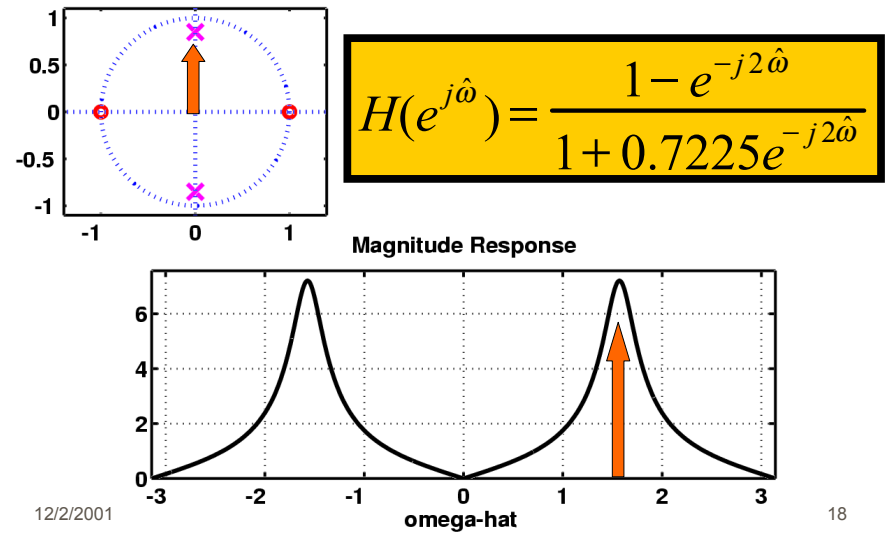
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UNIT CIRCLE

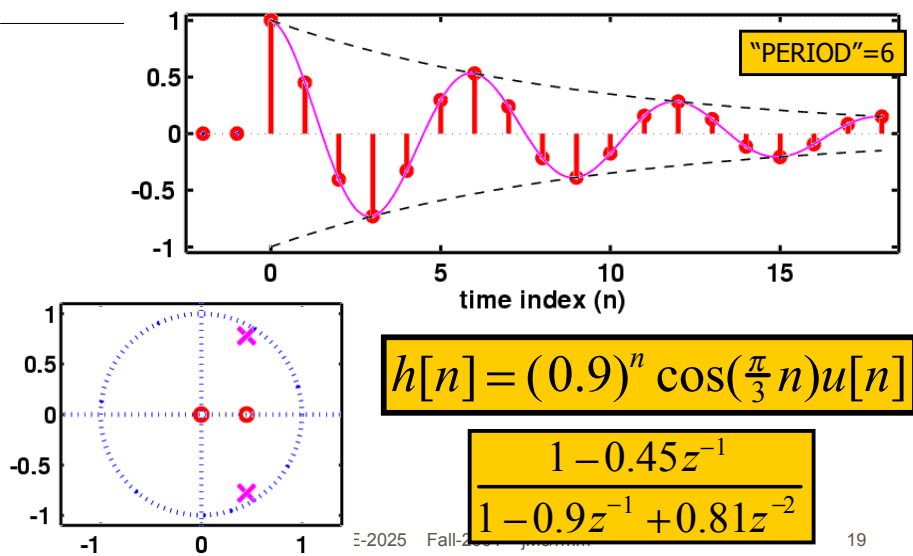
MAPPING BETWEEN z and $\hat{\omega}$



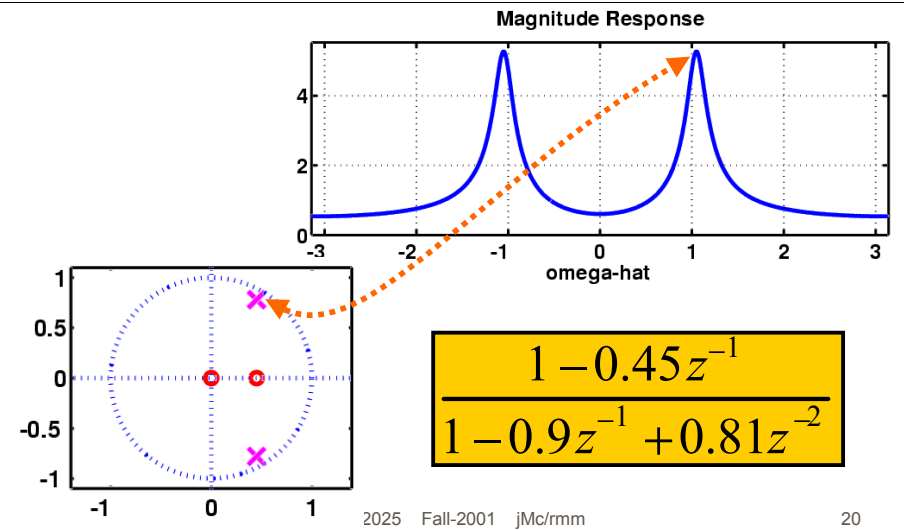
FREQUENCY RESPONSE from POLE-ZERO PLOT



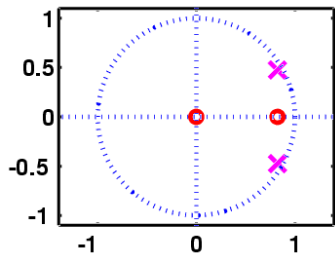
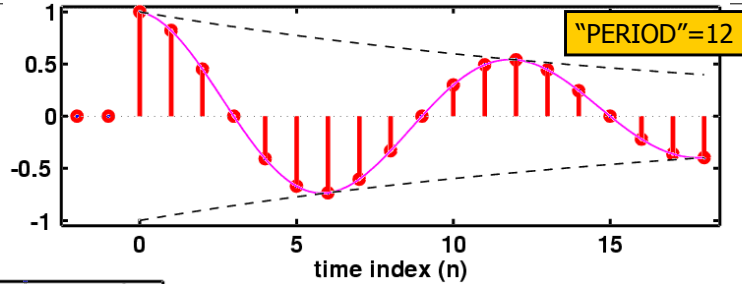
$h[n]$: Decays & Oscillates



Complex POLE-ZERO PLOT



h[n]: Decays & Oscillates



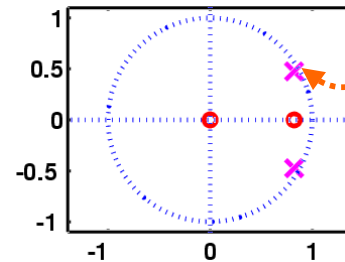
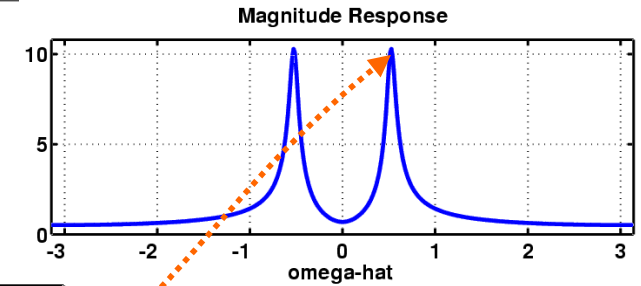
$$h[n] = (0.95)^n \cos\left(\frac{\pi}{6} n\right) u[n]$$

$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$

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Complex POLE-ZERO PLOT



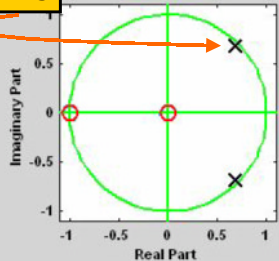
$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$

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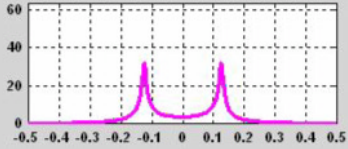
3 DOMAINS MOVIE: IIR

POLE MOVES

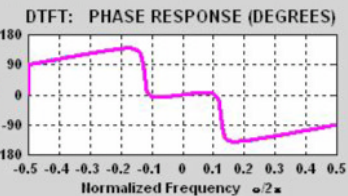
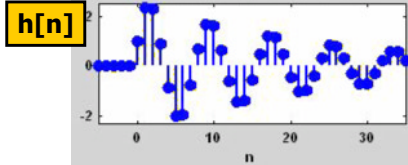


$$\frac{1 + z^{-1}}{1 - 1.36z^{-1} + 0.918z^{-2}}$$

H(z)



H(omega)



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THREE INPUTS

Given:

$$H(z) = \frac{5}{1 + 0.8z^{-1}}$$

Find the output, y[n]

When

$$x[n] = \cos(0.2\pi n)$$

$$x[n] = u[n]$$

$$x[n] = \cos(0.2\pi n)u[n]$$

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SINUSOID ANSWER

■ Given: $H(z) = \frac{5}{1 + 0.8z^{-1}}$
 ■ The input: $x[n] = \cos(0.2\pi n)$
 ■ Then $y[n] = M \cos(0.2\pi n + \psi)$
 $H(e^{j0.25\pi}) = \frac{5}{1 + 0.8e^{-j0.2\pi}} = 2.919e^{j0.089\pi}$

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Step Response

$$Y(z) = H(z)X(z) = \left(\frac{5}{1 + .8z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right)$$

$$Y(z) = \frac{A}{1 + .8z^{-1}} + \frac{B}{1 - z^{-1}} = \frac{(A + B) + (.8B - A)z^{-1}}{(1 + .8z^{-1})(1 - z^{-1})}$$

$$\Rightarrow (A + B) = 5 \quad \text{and} \quad (.8B - A) = 0$$

$$Y(z) = \frac{A}{1 + .8z^{-1}} + \frac{B}{1 - z^{-1}}$$

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Step Response

$$Y(z) = \frac{20}{1 + .8z^{-1}} + \frac{25}{1 - z^{-1}}$$

$$y[n] = \frac{20}{9}(-.8)^n u[n] + \frac{25}{9}u[n]$$

$$y[n] \rightarrow \frac{25}{9} \quad \text{as} \quad n \rightarrow \infty$$

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Stability

■ Nec. & suff. condition: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

$$h[n] = b(a)^n u[n] \Leftrightarrow H(z) = \frac{b}{1 - az^{-1}}$$

$$\sum_{n=0}^{\infty} |b||a|^n < \infty \quad \text{if} \quad |a| < 1 \Rightarrow$$

**Pole must be
Inside unit circle**

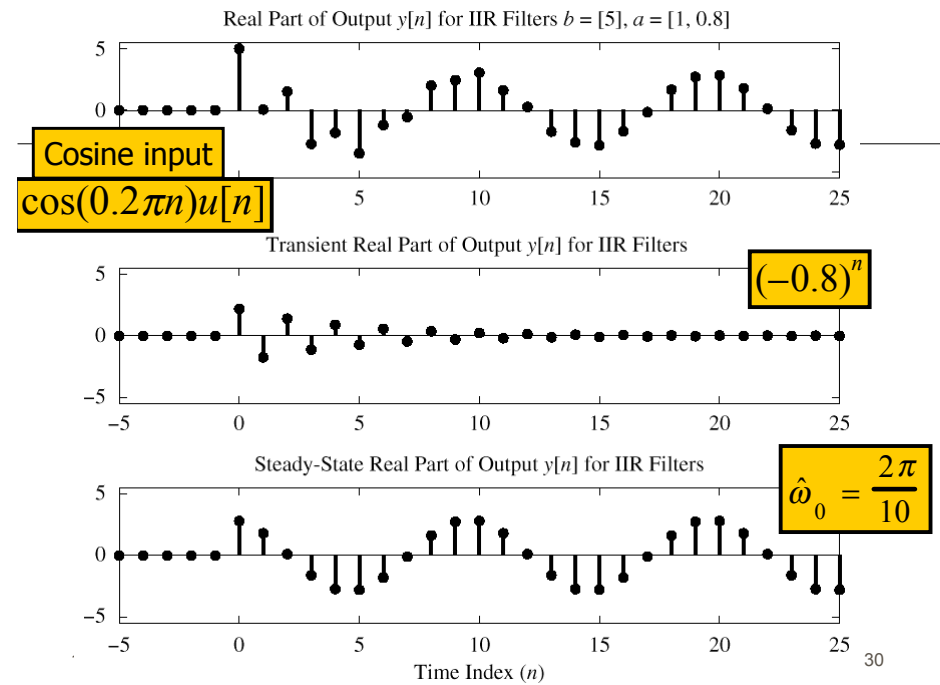
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SINUSOID starting at n=0

- We'll look at an example in MATLAB
 - $\cos(0.2\pi n)$
 - Pole at -0.8 , so a^n is $(-0.8)^n$
- There are two components:
 - TRANSIENT
 - Start-up region just after $n=0$; $(-0.8)^n$
 - STEADY-STATE
 - Eventually, $y[n]$ looks sinusoidal.
 - **Magnitude & Phase from Frequency Response**



STABILITY

- When Does the TRANSIENT DIE OUT ?

STEADY-STATE RESPONSE AND STABILITY

A stable system is one that does not “blow up.” This intuitive statement can be formalized by saying that the output of a stable system can always be bounded ($|y[n]| < M_y$) whenever the input is bounded ($|x[n]| < M_x$).³

$$y[n] = a_1 y[n - 1] + b_0 x[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

$$h[n] = b_0 a_1^n u[n]$$

need $|a_1| < 1$

STABILITY CONDITION

- ALL POLES INSIDE the UNIT CIRCLE
- UNSTABLE EXAMPLE: POLE @ $z=1.1$

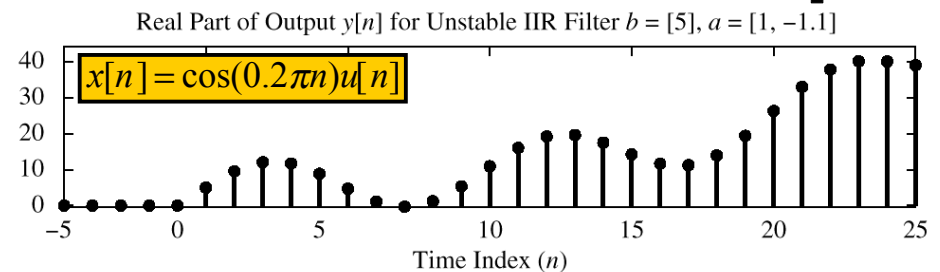


Figure 8.15 Illustration of an unstable IIR system. Pole is at $z = 1.1$.