EE-2025 Fall-2001

Lecture 25
Review: Potpourri
7-Dec-01

LAST LAB This Week

- ALL Lab Reports due by Friday (TODAY)
  - 5pm in VanLeer-475, or directly to TA
    - **In PERSON**
- Course Evaluations during last week
  - THREE
    - Two for GT: Lecture & Recitation
    - ******** One on Web-CT ***********
- HW #12 solution will be posted tonight

Final Exam Info

- Calendar: Final Exam(s)
  - Noon Lecture: Monday, 10-Dec @ 2:50 pm
  - 11 Am Lecture: Thurs, 13-Dec @ 11:30am
- **ID check will be done at Final Exam**
- **Report CONFLICTS immediately !!!!**
  - E.g., 3 exams in one day
- Reviews will be held on Sunday & Wednesday
  - 6pm in ECE Auditorium

FINAL EXAM

- FORMULA PAGES ?
  - Students bring **ONE** page **HAND-WRITTEN**
    - Tables 12.1 & 12.2 will be supplied with the exam.
- COVERAGE / EMPHASIS?
  - Fourier Transform
    - Sampling, Filtering & Spectrum
  - Digital Filters: IIR & FIR & H(z)
  - Hard problems from Quizzes #2, #3.
  - Homework & Old Quizzes

LECTURE OBJECTIVES

- SINUSOIDAL RESPONSE
  - Transients and Stability
- RE-UNIFICATION:
  - How does Frequency Response affect x(t) to produce y(t) ?

THREE INPUTS

- Given: $H(z) = \dfrac{5}{1 + 0.8z^{-1}}$
- Find the output, y[n]
  - When

$$x[n] = \cos(0.2\pi n)$$

$$x[n] = u[n]$$

$$x[n] = \cos(0.2\pi n)u[n]$$

SINUSOID ANSWER

- Given: $H(z) = \dfrac{5}{1 + 0.8z^{-1}}$
- The input: $x[n] = \cos(0.2\pi n)$
- Then y[n] $y[n] = M\cos(0.2\pi n + \psi)$

$$H(e^{j0.2\pi}) = \frac{5}{1 + 0.8e^{-j0.2\pi}} = 2.919e^{j0.089\pi}$$

Step Response: x[n] = u[n]

$$Y(z) = H(z)X(z) = \left(\frac{5}{1 + .8z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right)$$

$$Y(z) = \frac{A}{1 + .8z^{-1}} + \frac{B}{1 - z^{-1}} = \frac{(A + B) + (.8B - A)z^{-1}}{(1 + .8z^{-1})(1 - z^{-1})}$$

$$\Rightarrow \quad (A + B) = 5 \quad \text{and} \quad (.8B - A) = 0$$

$$Y(z) = \frac{A}{1 + .8z^{-1}} + \frac{B}{1 - z^{-1}}$$

Step Response

$$Y(z) = \frac{\frac{20}{9}}{1+.8z^{-1}} + \frac{\frac{25}{9}}{1-z^{-1}}$$

$$y[n] = \frac{20}{9}(-.8)^n u[n] + \frac{25}{9}u[n]$$

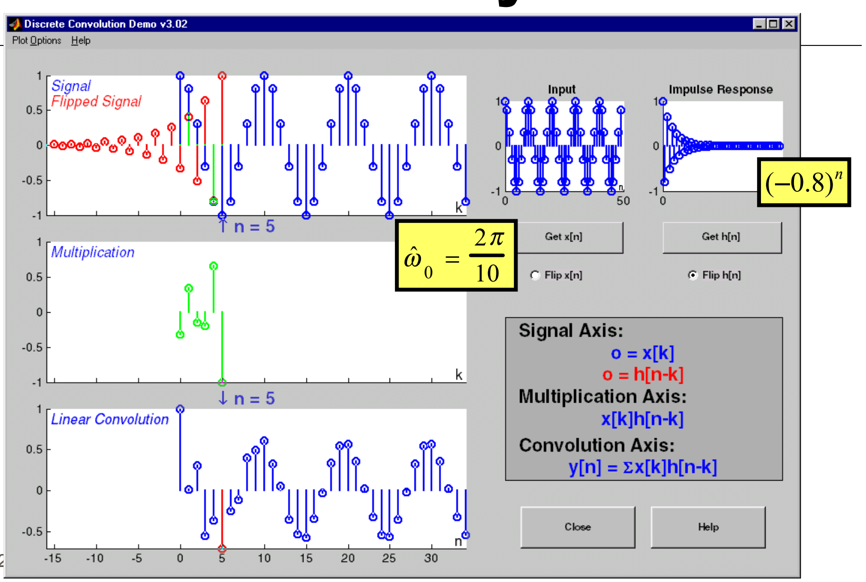
Transient → $(-.8)^n$ **Steady-State** → $u[n]$

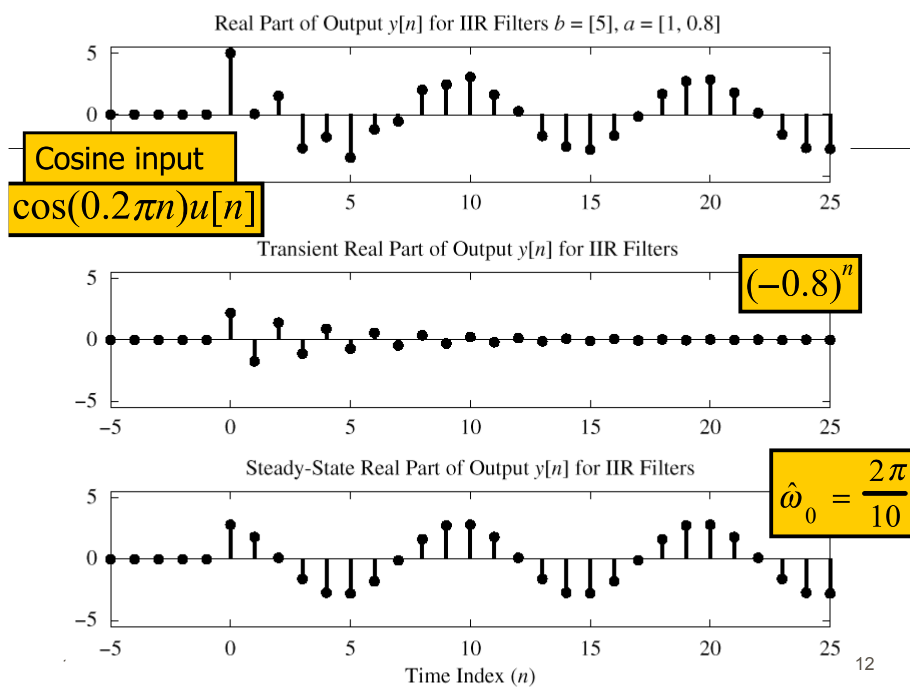
$$y[n] \to \frac{25}{9} \quad \text{as} \quad n \to \infty$$

SINUSOID starting at n=0

- We'll look at an example in MATLAB
  - **$x[n] = \cos(0.2\pi n)u[n]$**
  - H(z) has a POLE at –0.8, so a^n is $(-0.8)^n$
- There are two components:
  - TRANSIENT
    - Start-up region just after n=0; **$(-0.8)^n$**
  - STEADY-STATE
    - Eventually, y[n] looks sinusoidal.
    - **Magnitude & Phase from Frequency Response**

Transient & Steady State





STABILITY

- When Does the TRANSIENT DIE OUT ?

STEADY-STATE RESPONSE AND STABILITY

A stable system is one that does not "blow up." This intuitive statement can be formalized by saying that the output of a stable system can always be bounded ($|y[n]| < M_y$) whenever the input is bounded ($|x[n]| < M_x$).[3]

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

$$h[n] = b_0 a_1^n u[n]$$

need $|a_1| < 1$

Stability

- Nec. & suff. condition: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

$$h[n] = b(a)^n u[n] \Leftrightarrow H(z) = \frac{b}{1 - az^{-1}}$$

$$\sum_{n=0}^{\infty} |b||a|^n < \infty \text{ if } |a| < 1 \Rightarrow$$

Pole must be Inside unit circle

STABILITY CONDITION

- ALL POLES INSIDE the UNIT CIRCLE
- UNSTABLE EXAMPLE:

POLE @ z=1.1

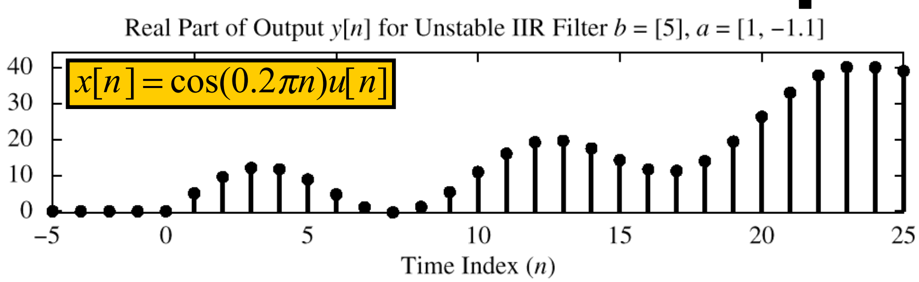


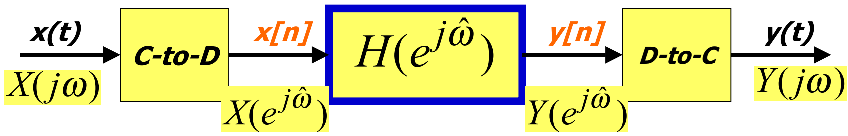
Figure 8.15 Illustration of an unstable IIR system. Pole is at $z = 1.1$.

Lectures

- A Lecture is the process in which the notes of the professor become the notes of the students …

  without passing through the minds of either.

DT Filtering of CT Signals

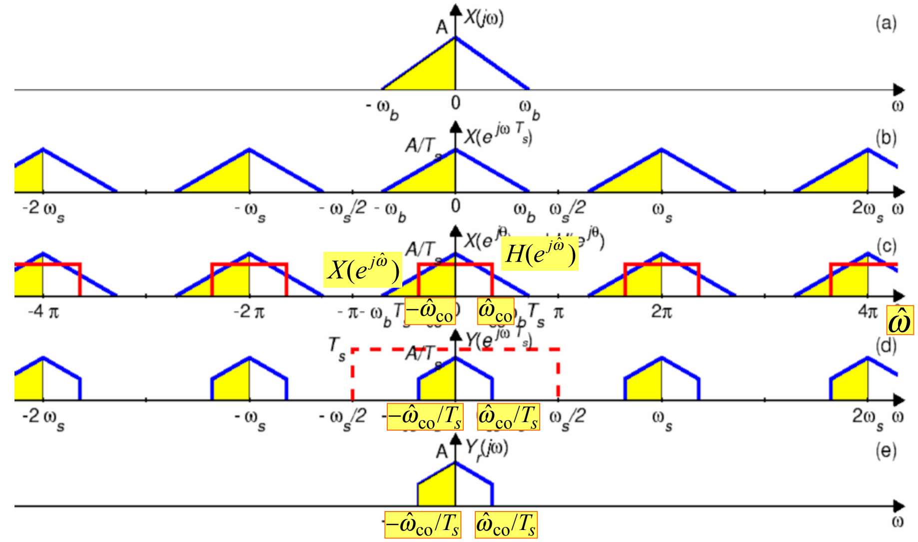


If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ 0 & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

Illustration of DT Filtering of a CT Signal



EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <--> DIGITAL FREQ

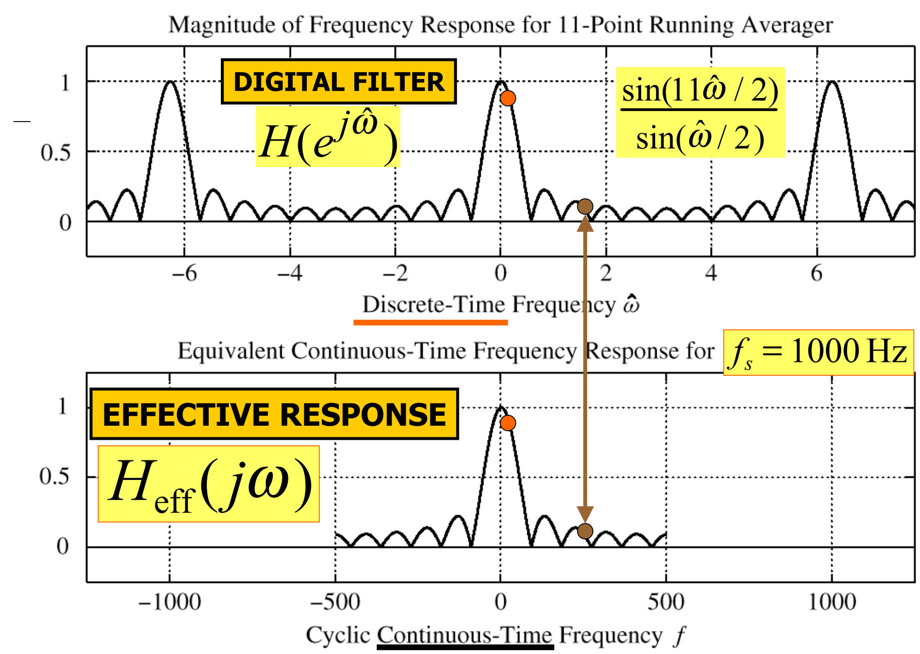
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

  - So, we can plot:
  - Scaled Freq. Axis

DIGITAL FILTER

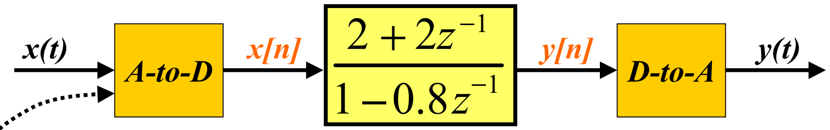
$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

ANALOG FREQUENCY



POP QUIZ

- Given:



- Find the output, y(t)
  - When $x(t) = \cos(2000\pi t)$

$f_s = 5000\,\text{Hz}$

POP QUIZ BECOMES

- Given: $H(z) = \frac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the output, y[n]
  - When $x[n] = \cos(0.4\pi n)$
  - Because $\omega T_s = 2000\pi / 5000 = 0.4\pi$

NO Aliasing

SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if } \; x[n] = e^{j\hat{\omega}n} \; \text{ then}$$
$$y[n] = H(e^{j\hat{\omega}})e^{j\hat{\omega}n}$$
$$\text{where } \; H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

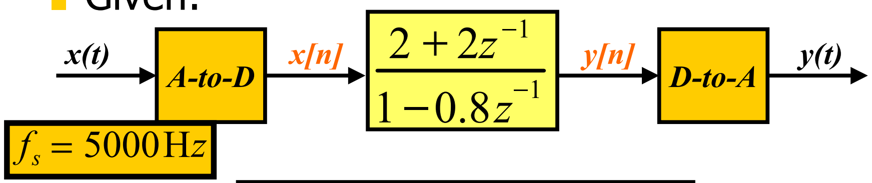
POP QUIZ INSIDE ANSWER

- Given: $H(z) = \frac{2+2z^{-1}}{1-0.8z^{-1}}$
- The input: $x[n] = \cos(0.4\pi n)$
- Then y[n] $y[n] = M\cos(0.4\pi n + \psi)$

$$H(e^{j0.4\pi}) = \frac{2+2e^{-j0.4\pi}}{1-0.8e^{-j0.4\pi}} = 3.02e^{-j0.452\pi}$$

POP QUIZ ANSWER

- Given:

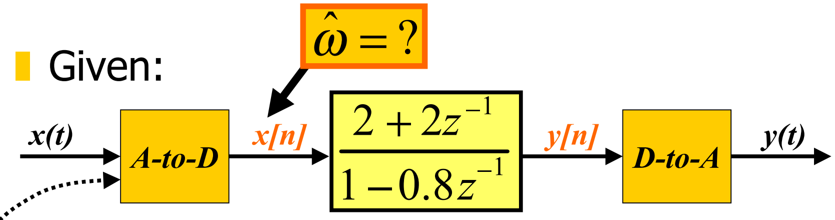


- When $x(t) = \cos(2000\pi t)$
- The output is

$$y(t) = 3.02\cos(2000\pi t - 0.452\pi)$$

ANOTHER INPUT FREQ

- Given:



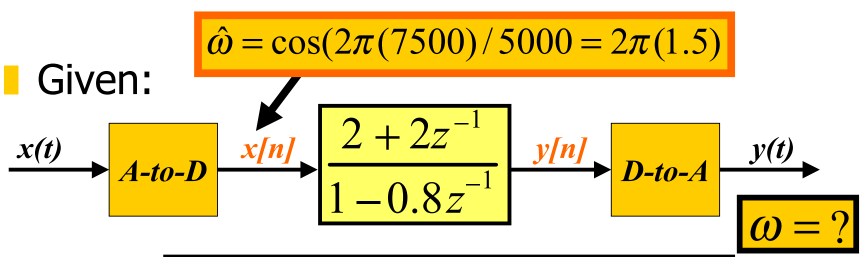
- Find the output, y(t)
  - When $x(t) = \cos(2\pi(7500)t)$

$f_s = 5000\,\text{Hz}$

$$\hat{\omega} = ?$$

2nd POP QUIZ ANSWER

- Given:



- When $x(t) = \cos(2\pi(7500)t)$

$f_s = 5000\,\text{Hz}$ → $\hat{\omega} = 3\pi$ → $y(t) = ?$

Superficial Knowledge

- It depends how carefully you think about it. If you don't think very carefully it's obvious; but if you think about it in depth, you'll get confused and it won't be obvious.

THREE DOMAINS

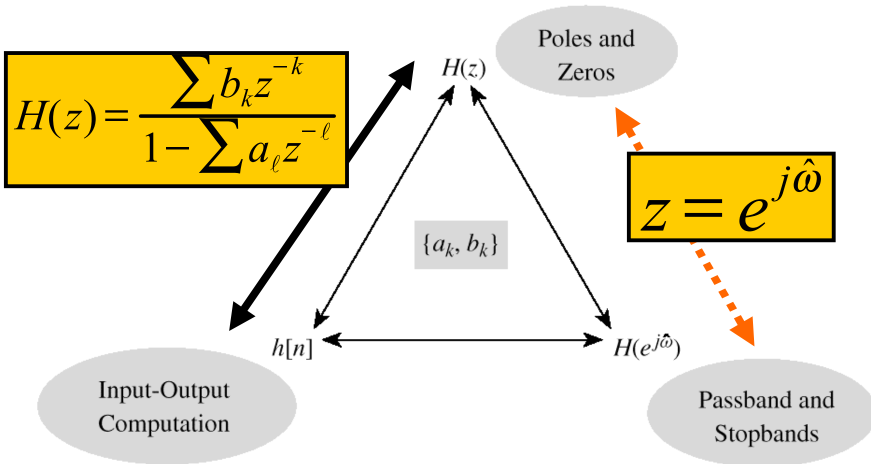
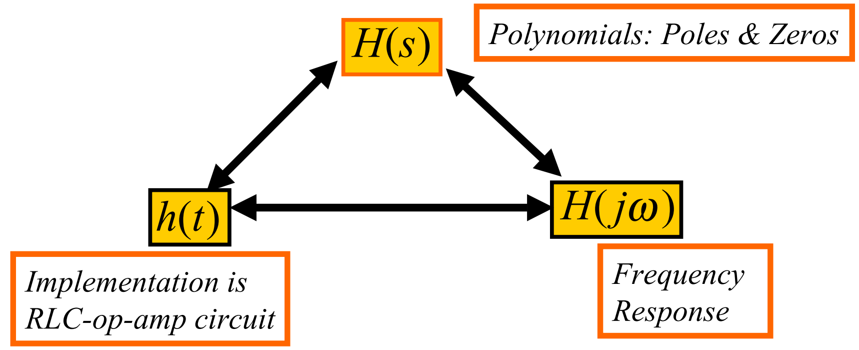


Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k,\ b_k\}$ play a central role.

THE FUTURE

- Circuits & Laplace Transforms



Mathematical Elegance

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

Fourier S... (Inverse...

$$X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Fourier Analysis (Forward Transform)

$$\text{Time-domain} \Leftrightarrow \text{Frequency-domain}$$
$$x(t) \Leftrightarrow X(j\omega)$$

IMPORTANT CONCEPTS

- ALL Signals have Frequency Content
  - Sum of Sinusoids
  - Complex Exponentials
  - Impulses, Square Pulses
- FILTERS alter the Frequency Content
  - Image Processing Example: Blur
  - Linear Time-Invariant Processing
- 3 Domains for Analysis