GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

Final Exam

DATE: 13-Dec-01 COURSE: ECE 2025

NAME: ______________________________ gtxxx (login) # : ______________
LAST, FIRST

<u>Recitation Section</u>: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30am (Bordelon)	L02:Thur-9:30am (Casinovi)	L03:Tues-12:00pm (Casinovi)
L04:Thur-12:00pm (Ji)	L05:Tues-1:30pm (Bordelon)	L06:Thur-1:30pm (Ji)
L07:Tues-3:00pm (Casinovi)	L08:Thur-3:00pm (Bordelon)	L09:Tues-4:30pm (Lanterman)
L10:Thur-4:30pm (Bordelon)	L11:Tues-6:00pm (Lanterman)	L13:Mon -3:00pm (Yezzi)
L14:Weds-3:00pm (Taylor)	L15:Mon -4:30pm (Verriest)	L16:Weds-4:30pm (Taylor)
L17:Mon -6:00pm (Verriest)	L20:Mon -1:30pm (McClellan)	RPK: (Abler)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive any partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.

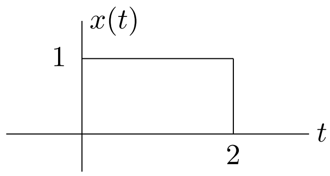
Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	

Problem Fall-01-F.1:

(a) A continuous-time linear, time-invariant system has the impulse response

$$h(t) = \delta(t) + A\delta(t - \Delta).$$

Find the output of the system, $y(t)$, when the input is the signal $x(t) = u(t) - u(t-2)$ sketched below. Let $A = -1$ and $\Delta = 3$. Express your answer as a plot.

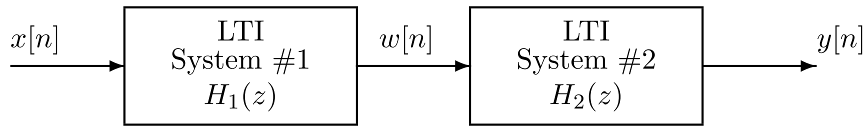


(b) Assuming that the input signal is the same as in part (a), find values for A and Δ that will create the output signal $y(t) = u(t) - 2u(t - 2) + u(t - 4)$.

$A =$	$\Delta =$

Problem Fall-01-F.2:

A cascade of two FIR discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 - z^{-2}) \quad \text{and} \quad h_2[n] = (0.9)^{n-1}u[n-1].$$

(a) If the input to the first system is

$$x[n] = u[n],$$

determine the output, $w[n]$, of the **first** system.

$w[n] =$

(b) Determine the system function $H(z)$ of the overall system.

$H(z) =$

(c) Determine the impulse response of the the overall system.

$h[n] =$

Problem Fall-01-F.3:

For each short question, pick a correct frequency[1] and enter its letter in the answer box[2]:

Frequency

(a) 4000 Hz

(b) 2500 Hz

(c) 2000 Hz

(d) 800 Hz

(e) 600 Hz

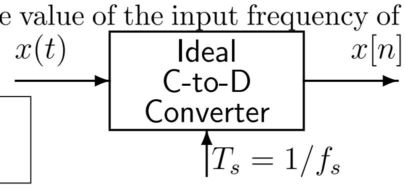
(f) 500 Hz

(g) 400 Hz

(h) 250 Hz

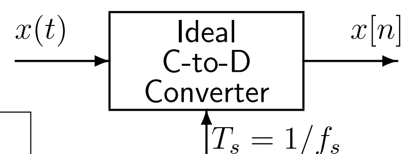
(i) 200 Hz

(a) If the output from an ideal C/D converter is $x[n] = A\cos(\pi n)$, and the sampling rate is 5000 samples/sec, then determine one possible value of the input frequency of $x(t)$:



ANS =

(b) If the output from an ideal C/D converter is $x[n] = A\cos(\pi n)$, and the input signal $x(t)$ defined by: $x(t) = A\cos(2500\pi t)$ then determine one possible value of the sampling frequency of the C-to-D converter:



ANS =

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = \Re e\{e^{j2000\pi t} + e^{j1500\pi t}\}$.

ANS =

[1]Some questions have more than one answer, but you only need to pick one correct answer from the list.
[2]It is possible to use an answer more than once.

Problem Fall-01-F.4:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Evaluate $\Re e\{x[n+1]x^*[n-1]\}$ when $x[n] = -je^{-j(0.1)\pi n}$.

(b) Evaluate the following expression, $|e^{j\pi/6} + e^{-j\pi/6}|^2 =$ <table><tr><td>   </td></tr></table>

(c) Evaluate the following integral, $\int_{-\infty}^{\infty} \delta(t + \frac{1}{2}) \sin(2\pi t) e^{j\pi t/2} dt$

(d) Evaluate the following integral, $\int_{-\infty}^{\infty} e^{-\pi t/2} u(t-3) e^{-j\omega t/2} dt.$

Problem Fall-01-F.5:

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or plot. (The symbol $*$ denotes covolution.)

(a) Find $Y(j\omega)$ when $y(t) = h(t) * x(t) = \cos(\pi t) * (\delta(t-0.5) - \delta(t+0.5))$.

(b) Find $h(t)$ when $H(j\omega) = e^{j3\omega}\left(\frac{j\omega}{2+j\omega}\right)$.

(c) Find $v(t)$ when $V(j\omega) = \begin{cases} \cos(2\omega), & |\omega| < \pi \\ 0, & |\omega| > \pi \end{cases}$.

(d) Find $H(j\omega)$ when $h(t) = \frac{3\sin(2\pi(t-1))}{\pi(t-1)}$.

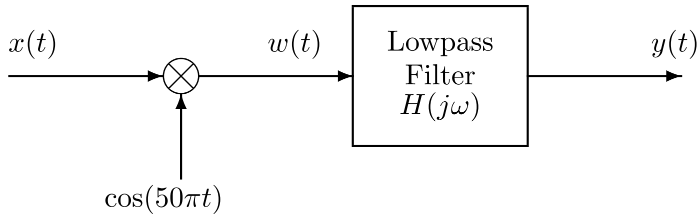
Problem Fall-01-F.6:

(a) The Fourier coefficients for the Fourier Series of a periodic signal $x(t)$ are defined using the following MATLAB code:

```
N = 2;
for k = -N:N
   if k == 0
      ak(k+N+1) = =0.5; % DC term
   else
      ak(k+N+1) = (-j/(pi*k))*(1 - exp(j*k*pi));
   end
end
```

If the fundamental frequency is 75 Hz, sketch the Fourier transform of the signal, $X(j\omega)$.

(b)



In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform, $X(j\omega)$, as depicted in Figure A below.

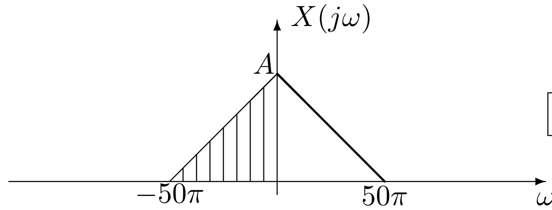


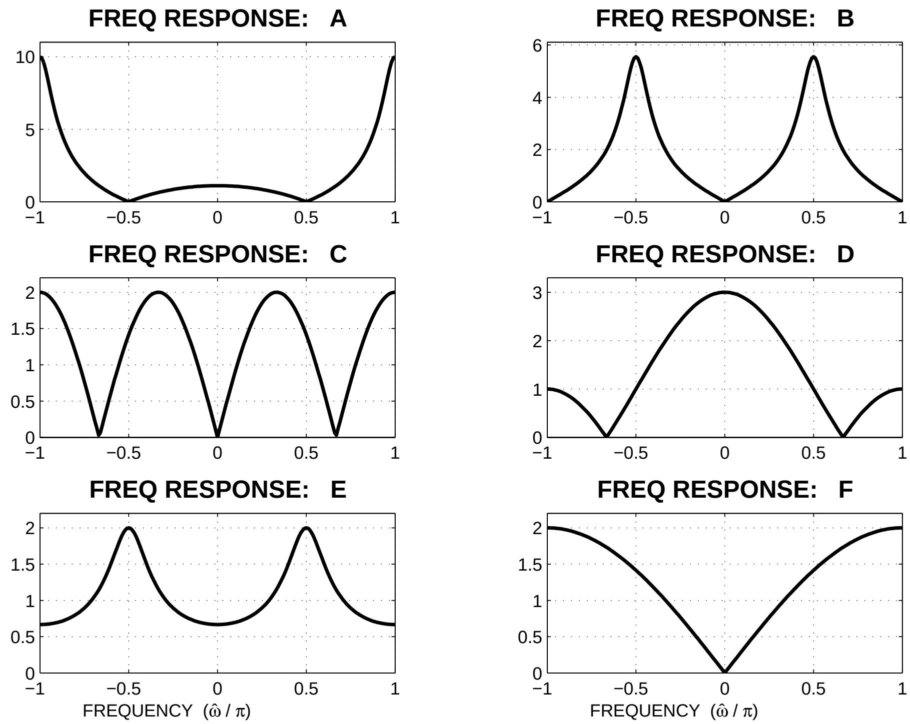
Figure A

The frequency response of the lowpass filter is

$$H(j\omega) = \begin{cases} 2 & |\omega| \leq 50\pi \\ 0 & |\omega| > 50\pi \end{cases}$$

Draw $Y(j\omega)$.

Problem Fall-01-F.7:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: the frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1$: $y[n] = 0.8y[n-1] + 0.5x[n]$

$\mathcal{S}_2$: $y[n] = -0.5y[n-2] + x[n-1]$

$\mathcal{S}_3$: $y[n] = -0.8y[n-1] + x[n] + x[n-2]$

$\mathcal{S}_4$: $y[n] = x[n] - x[n-1]$

$\mathcal{S}_5$: $H(z) = z^{-1} - z^{-4}$

$\mathcal{S}_6$: $H(z) = \dfrac{1 + z^{-1}}{1 - 0.9z^{-1}}$

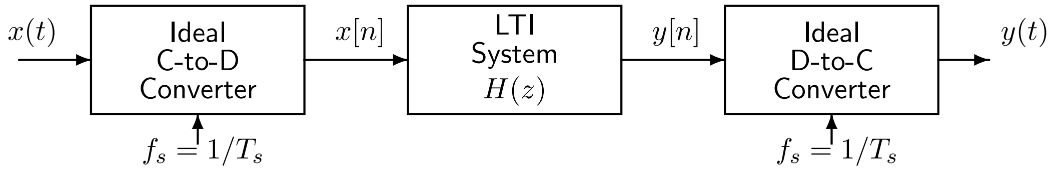
$\mathcal{S}_7$: $H(z) = 1 + z^{-1} + z^{-2}$

$\mathcal{S}_8$: $H(z) = \dfrac{1 - z^{-2}}{1 + 0.64z^{-2}}$

Mark your answers in the following table:

FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)
A		B	
C		D	
E		F	

Problem Fall-01-F.8:
Consider the following system for sampling, filtering, and reconstruction of a continuous-time signal:



where the LTI system function is $H(z) = 4z^{-3}$, and the continuous-time input signal is

$$x(t) = 5\cos(5000\pi t - 3\pi/5).$$

(a) Plot the complete frequency spectrum for $x[n]$ in the region $-\pi < \hat{\omega} \leq \pi$ for the case where $f_s = 3000$ samples/second.

(b) Determine an expression for the output $y(t)$ of this system for the input $x(t)$ indicated for the case where $f_s = 3000$ samples/second.

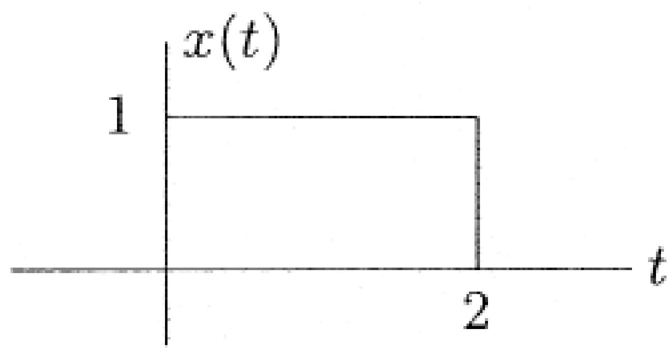
(c) Determine an expression for the output $y(t)$ for the same input signal if the sampling frequency is increased to $f_s = 6000$ samples/second.

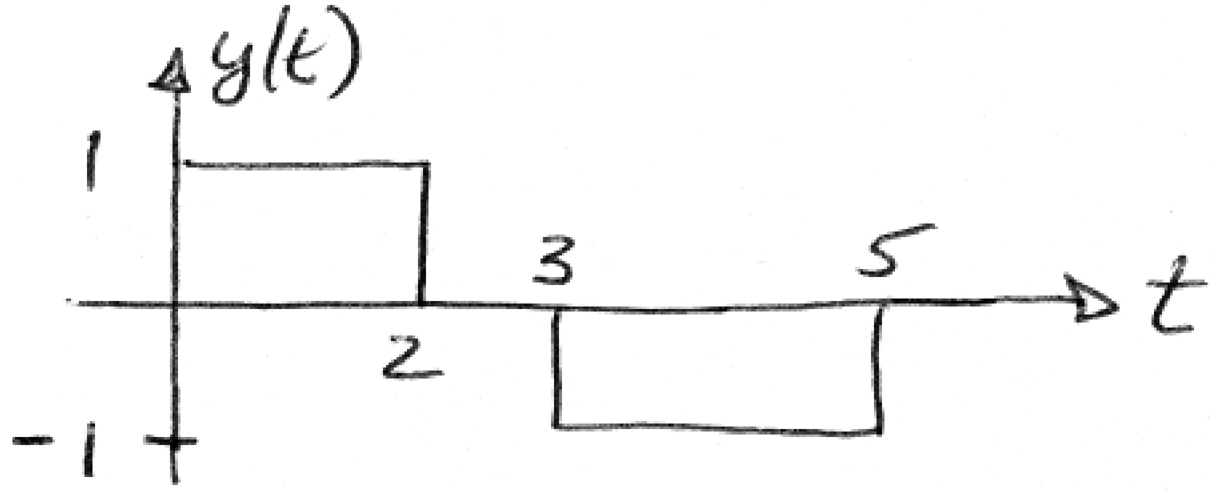
Problem Fall-01-F.1:

(a) A continuous-time linear, time-invariant system has the impulse response

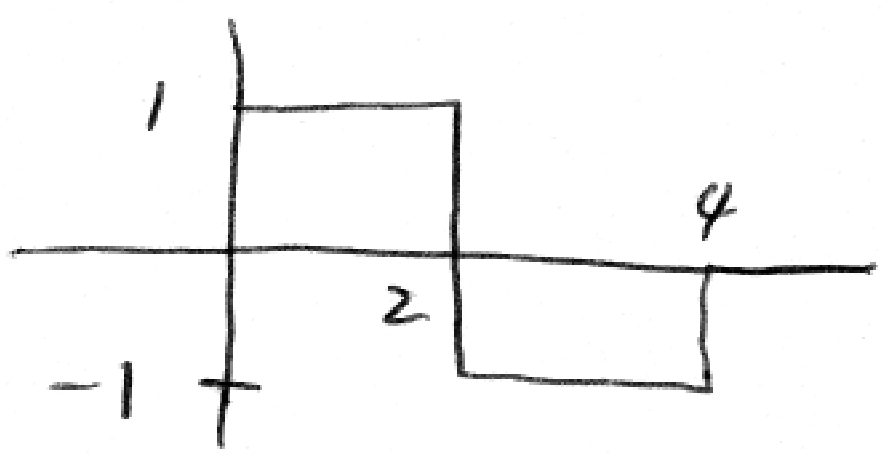
$$h(t) = \delta(t) + A\delta(t - \Delta).$$

Find the output of the system, $y(t)$, when the input is the signal $x(t) = u(t) - u(t-2)$ sketched below. Let $A = -1$ and $\Delta = 3$. Express your answer as a plot.





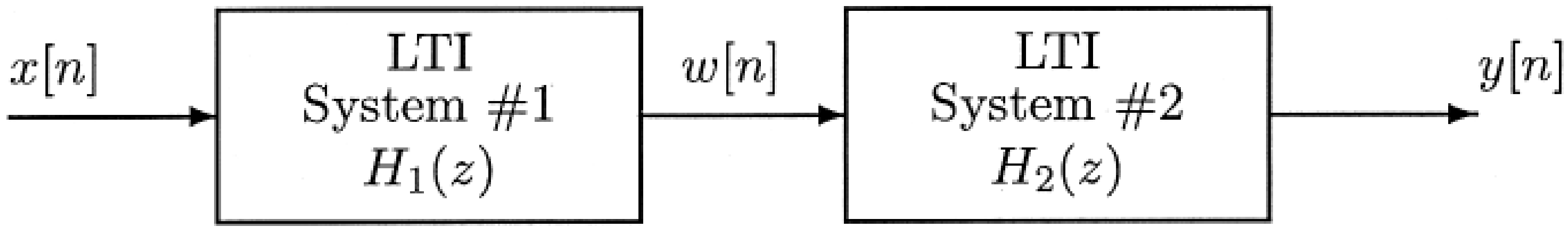
(b) Assuming that the input signal is the same as in part (a), find values for A and Δ that will create the output signal $y(t) = u(t) - 2u(t-2) + u(t-4)$.



$A =$ −1	$\Delta =$ 2

Problem Fall-01-F.2:

A cascade of two FIR discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 - z^{-2}) \quad \text{and} \quad h_2[n] = (0.9)^{n-1}u[n-1].$$

(a) If the input to the first system is

$$x[n] = u[n], \longleftarrow \; X(z) = \frac{1}{1-z^{-1}}$$

determine the output, $w[n]$, of the **first** system.

$$W(z) = H_1(z)X(z) = (1 - z^{-2})\frac{1}{1-z^{-1}} = 1 + z^{-1}$$

$$w[n] = \delta[n] + \delta[n-1]$$

$$\text{or, } w[n] = u[n] - u[n-2]$$

$w[n] = \delta[n] + \delta[n-1]$

(b) Determine the system function $H(z)$ of the overall system.

$$H_2(z) = z^{-1}\frac{1}{1-0.9z^{-1}}$$

$$H(z) = H_1(z)\,H_2(z)$$

$H(z) = (z^{-1} - z^{-3})/(1 - 0.9z^{-1})$

(c) Determine the impulse response of the the overall system.

$$\frac{z^{-1}}{1-0.9z^{-1}} \xrightarrow[Z]{\text{inverse}} (0.9)^{n-1}u[n-1]$$

$h[n] = (0.9)^{n-1}u[n-1] - (0.9)^{n-3}u[n-3]$

Problem Fall-01-F.3:

For each short question, pick a correct frequency[1] and enter its letter in the answer box[2]:

Frequency

(a) 4000 Hz

(b) 2500 Hz

(c) 2000 Hz

(d) 800 Hz

(e) 600 Hz

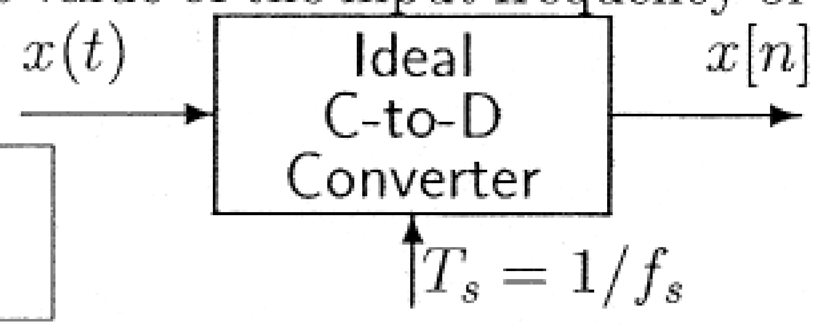
(f) 500 Hz

(g) 400 Hz

(h) 250 Hz

(i) 200 Hz

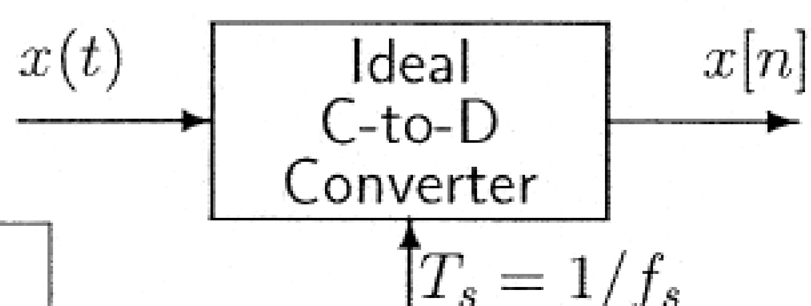
(a) If the output from an ideal C/D converter is $x[n] = A\cos(\pi n)$, and the sampling rate is 5000 samples/sec, then determine one possible value of the input frequency of $x(t)$:



ANS = b

$$\frac{2\pi f}{f_s} = \pi \pm 2\pi \ell \qquad f = 2500 \text{ or } 7500$$

(b) If the output from an ideal C/D converter is $x[n] = A\cos(\pi n)$, and the input signal $x(t)$ defined by: $x(t) = A\cos(2500\pi t)$ then determine one possible value of the sampling frequency of the C-to-D converter:



ANS = f

$$f_s(\pi \pm 2\pi\ell) = 2500\pi \Rightarrow f_s = \frac{2500}{2\ell+1} = 500 \text{ FOR } \ell = 2$$

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by: $x(t) = \Re e\{e^{j2000\pi t} + e^{j1500\pi t}\}$.

ANS = c

$f_{max} = 1000$ Hz.

[1]Some questions have more than one answer, but you only need to pick one correct answer from the list.
[2]It is possible to use an answer more than once.

Problem Fall-01-F.4:

For each of the following problems, **SIMPLIFY** your answer as much as possible.

(a) Evaluate $\Re e\{x[n+1]x^*[n-1]\}$ when $x[n] = -je^{-j(0.1)\pi n}$.

$$= \Re e\left\{-je^{-j(0.1)\pi(n+1)}(j)e^{j(0.1)\pi(n-1)}\right\}$$

$$= \Re e\left\{e^{-j(0.2)\pi}\right\} = \cos 0.2\pi$$

(b) Evaluate the following expression, $|e^{j\pi/6} + e^{-j\pi/6}|^2 =$ 3

$$\left|e^{j\frac{\pi}{6}} + e^{-j\frac{\pi}{6}}\right|^2 = \left|2\cos\frac{\pi}{6}\right|^2 = 4 \cdot \frac{3}{4} = 3$$

(c) Evaluate the following integral, $\int_{-\infty}^{\infty} \delta(t+\frac{1}{2})\sin(2\pi t)e^{j\pi t/2}dt$

$$= \sin\left(2\pi\left(-\tfrac{1}{2}\right)\right)e^{-j\frac{\pi}{4}}$$

$$= \sin(-\pi)\,e^{-j\frac{\pi}{4}} = 0$$

(d) Evaluate the following integral, $\int_{-\infty}^{\infty} e^{-\pi t/2}u(t-3)e^{-j\omega t/2}dt$.

THIS IS A FOURIER TRANSFORM INTEGRAL.

$$\int_{3}^{\infty} e^{-\left(\frac{\pi}{2}+j\frac{\omega}{2}\right)t}\,dt$$

$$= \frac{-1}{\frac{\pi}{2}+j\frac{\omega}{2}}\; e^{-\left(\frac{\pi}{2}+j\frac{\omega}{2}\right)t}\Bigg|_{3}^{\infty}$$

$$= \frac{1}{\frac{\pi}{2}+j\frac{\omega}{2}}\; e^{-\left(\frac{\pi}{2}+j\frac{\omega}{2}\right)3}$$

$$= \frac{2}{\pi+j\omega}\; e^{-3\left(\frac{\pi}{2}+j\frac{\omega}{2}\right)}$$

Problem Fall-01-F.5:

In each of the following problems, find the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or plot. (The symbol $*$ denotes covolution.)

(a) Find $Y(j\omega)$ when $y(t) = h(t) * x(t) = \cos(\pi t) * (\delta(t - 0.5) - \delta(t + 0.5))$.

$$y(t) = \cos(\pi(t-0.5)) - \cos(\pi(t+0.5))$$
$$= 2\cos(\pi t - \pi/2)$$

$$Y(jw) = 2\pi e^{-j\pi/2}\delta(w-\pi) + 2\pi e^{j\pi/2}\delta(w+\pi)$$

(b) Find $h(t)$ when $H(j\omega) = e^{j3\omega}\left(\frac{j\omega}{2 + j\omega}\right)$.

$$H(jw) = e^{j3w} \cdot jw \cdot \frac{1}{2+jw} \qquad \frac{1}{2+jw} \rightarrow e^{-2t}u(t)$$

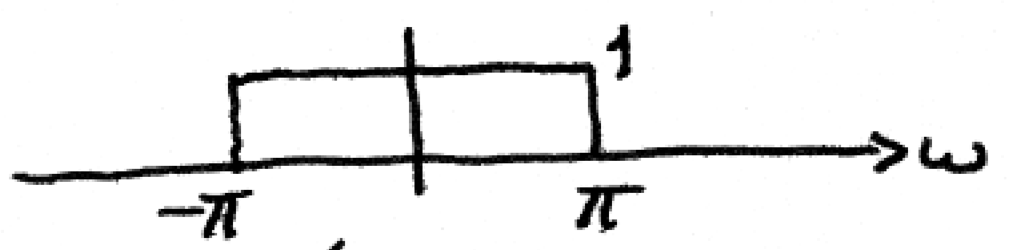
$jw \rightarrow$ derivative

$e^{-jw} \rightarrow$ delay

$e^{j3w} \rightarrow$ delay by -3

$$\frac{d}{dt}\{e^{-2t}u(t)\} = -2e^{-2t}u(t) + \delta(t)$$
$$\Rightarrow h(t) = -2e^{-2(t+3)}u(t+3) + \delta(t+3)$$

(c) Find $v(t)$ when $V(j\omega) = \begin{cases} \cos(2\omega), & |\omega| < \pi \\ 0, & |\omega| > \pi \end{cases}$.



$$\cdot \left(\frac{1}{2}e^{jw2} + \frac{1}{2}e^{-j2w}\right)$$

Time delay

$$\frac{\sin(\pi t)}{\pi t}$$

$$v(t) = \frac{1}{2}\frac{\sin(\pi(t+2))}{\pi(t+2)} + \frac{1}{2}\frac{\sin(\pi(t-2))}{\pi(t-2)}$$

(d) Find $H(j\omega)$ when $h(t) = \frac{3\sin(2\pi(t-1))}{\pi(t-1)}$. ← delay by 1

$$3e^{-jw}\{u(w+2\pi) - u(w-2\pi)\} = \begin{cases} 3e^{-jw} & |w| \le 2\pi \\ 0 & |w| > 2\pi \end{cases}$$

Problem Fall-01-F.6:

(a) The Fourier coefficients for the Fourier Series of a periodic signal $x(t)$ are defined using the following MATLAB code:

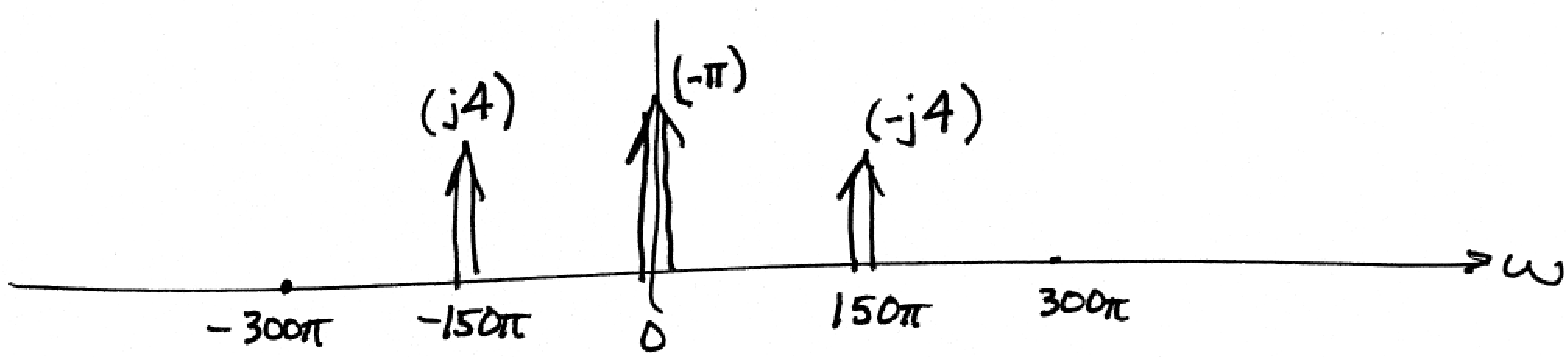
```
N = 2;
for k = -N:N
    if k == 0
        ak(k+N+1) = -0.5; % DC term
    else
        ak(k+N+1) = (-j/(pi*k))*(1 - exp(j*k*pi));
    end
end
```

If the fundamental frequency is 75 Hz, sketch the Fourier transform of the signal, $X(j\omega)$.

$a_0 = -\frac{1}{2}$ $\quad a_k = \frac{-j}{\pi k}(1 - e^{j\pi k})$ $\quad k = -2, -1, 1, 2$

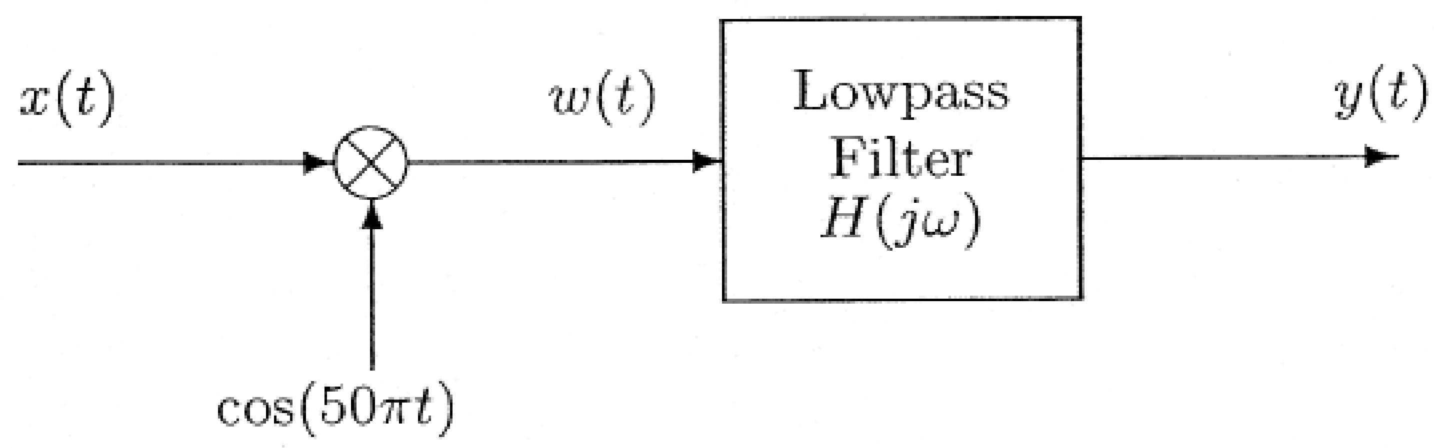
$a_1 = \frac{-j}{\pi}(1 - e^{j\pi}) = -j\frac{2}{\pi}$ $\quad a_2 = \frac{-j}{2\pi}(1 - e^{j2\pi}) = 0$

$a_{-1} = \frac{-j}{-\pi}(1 - e^{-j\pi}) = j\frac{2}{\pi}$ $\quad a_{-2} = 0$



$\omega_0 = 2\pi(75) = 150\pi$

(b)



In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform, $X(j\omega)$, as depicted in Figure A below.

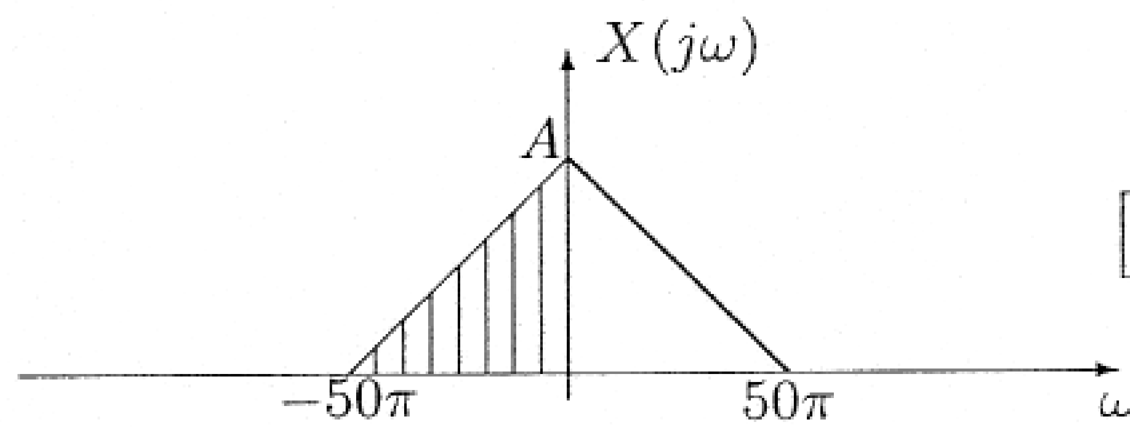


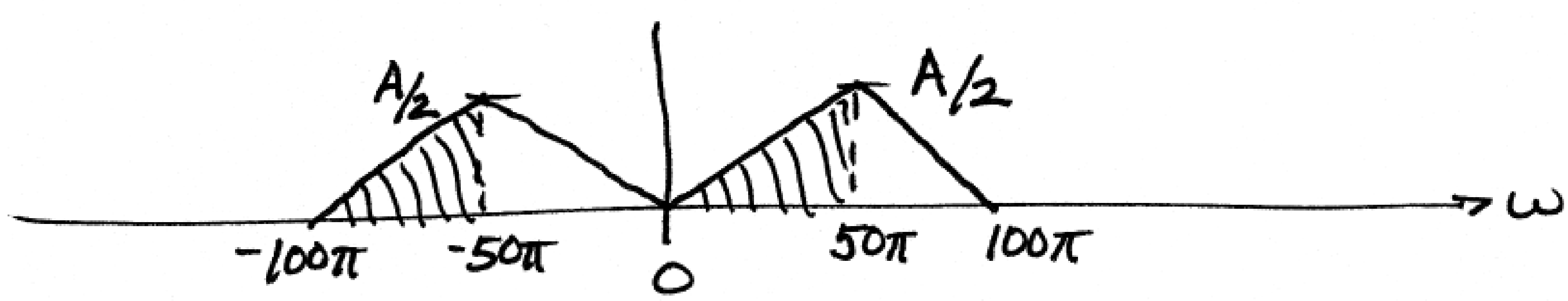
Figure A

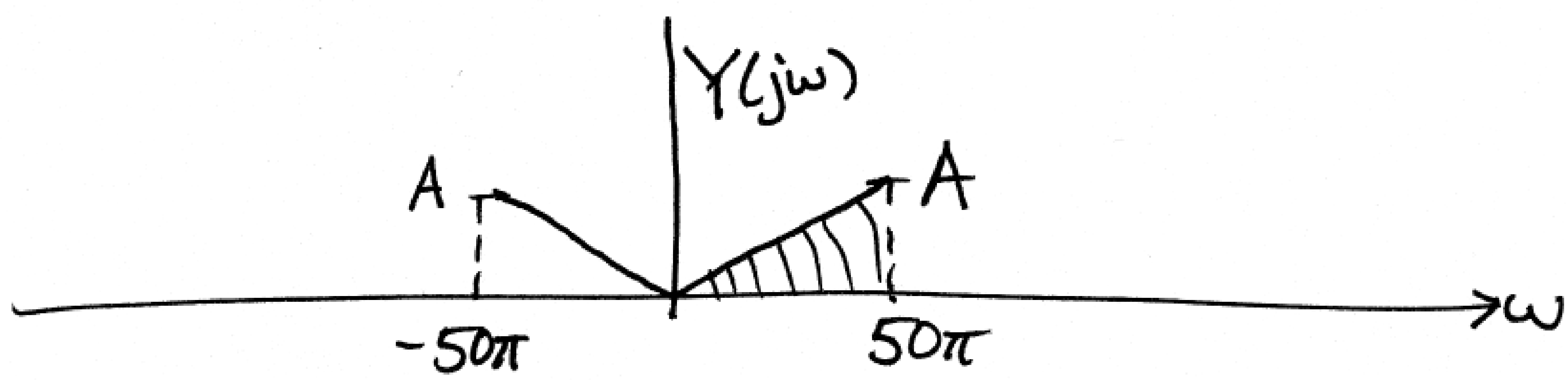
The frequency response of the lowpass filter is

$$H(j\omega) = \begin{cases} 2 & |\omega| \leq 50\pi \\ 0 & |\omega| > 50\pi \end{cases}$$

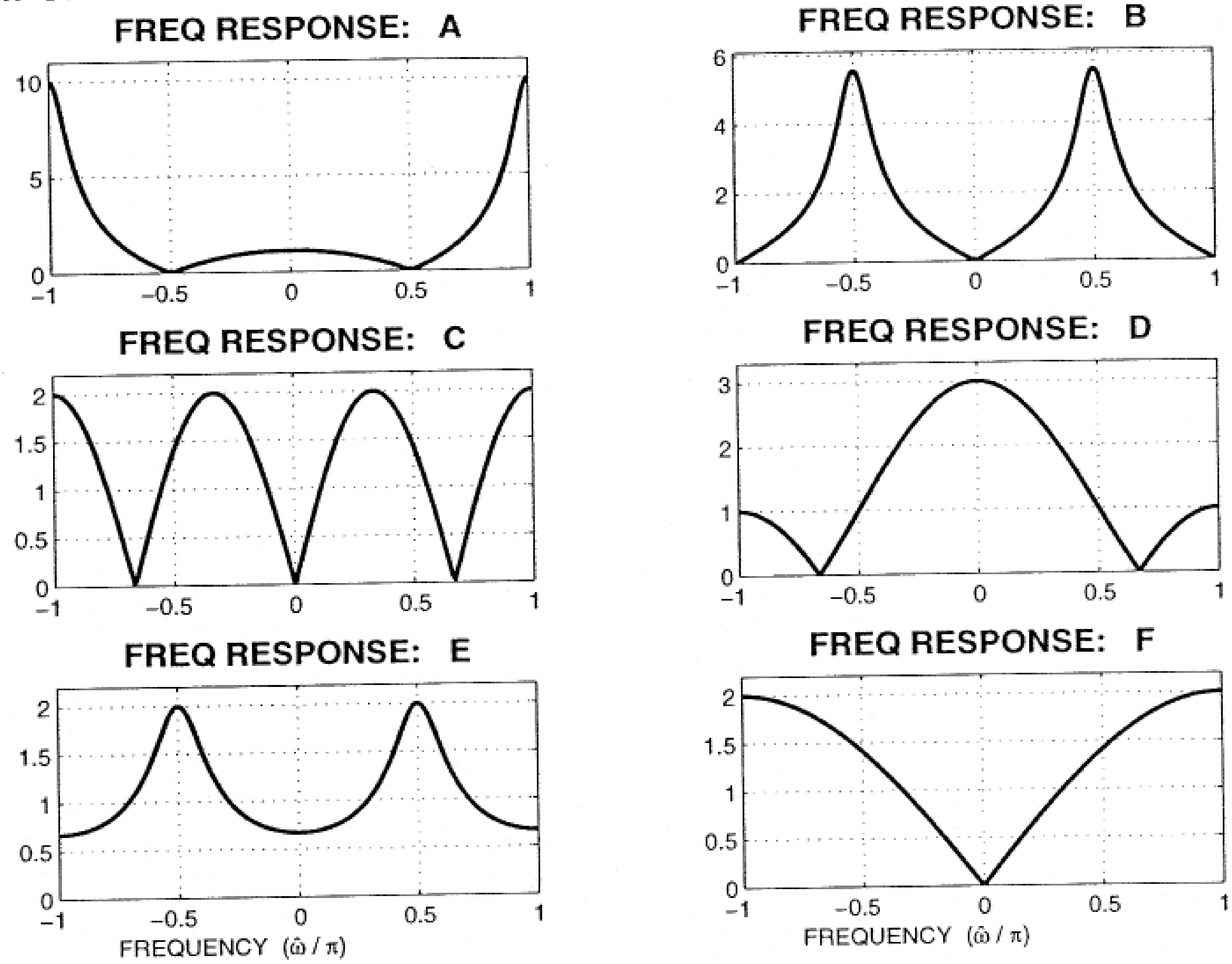
Draw $Y(j\omega)$.

$$W(j\omega) = \frac{1}{2} X(j(\omega - 50\pi)) + \frac{1}{2} X(j(\omega + 50\pi))$$





Problem Fall-01-F.7:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: the frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1$: $y[n] = 0.8y[n-1] + 0.5x[n]$

$\mathcal{S}_2$: $y[n] = -0.5y[n-2] + x[n-1]$

$\mathcal{S}_3$: $y[n] = -0.8y[n-1] + x[n] + x[n-2]$

$\mathcal{S}_4$: $y[n] = x[n] - x[n-1]$

$\mathcal{S}_5$: $H(z) = z^{-1} - z^{-4}$

$\mathcal{S}_6$: $H(z) = \dfrac{1 + z^{-1}}{1 - 0.9z^{-1}}$

$\mathcal{S}_7$: $H(z) = 1 + z^{-1} + z^{-2}$

$\mathcal{S}_8$: $H(z) = \dfrac{1 - z^{-2}}{1 + 0.64z^{-2}}$

Mark your answers in the following table:

FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)
A	3	B	8
C	5	D	7
E	2	F	4