

Problem Fall-01-Q.1.1:

The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = 4 - 2 \cos(5t - \pi/3)$$

and that

$$x(t) = v(t) \cos(40t).$$

- (a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.

- (b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be clearly indicated.

Problem Fall-01-Q.1.2:

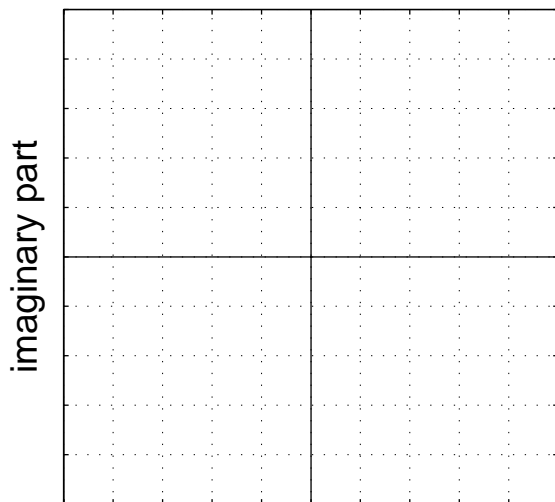
Define $x(t)$ as

$$x(t) = 3\sqrt{3} \cos(10\pi t - \pi/3) + 3 \cos(10\pi(t + 0.05))$$

- (a) Use phasor addition to express $x(t)$ in the form $x(t) = A \cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ , as well as ω_0 .

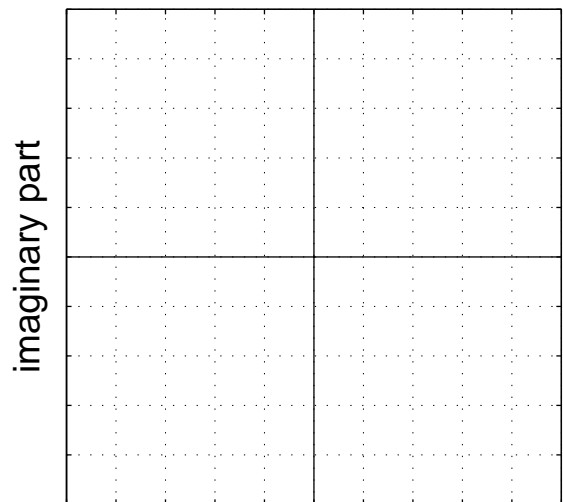
- (b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two vectors here.



real part

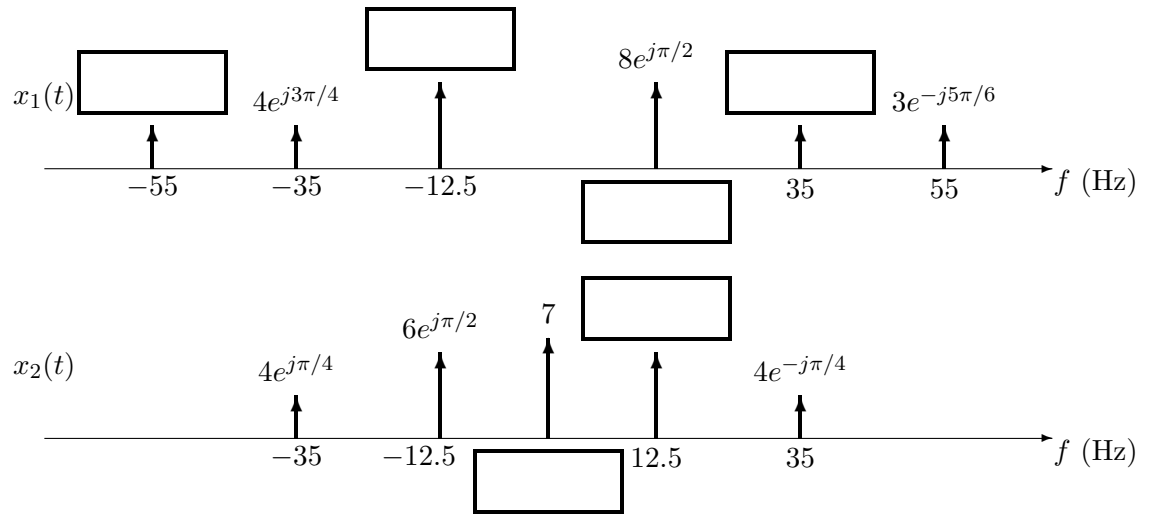
Head-to-tail plot here.



real part

Problem Fall-01-Q.1.3:

- (a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.



- (b) Write an equation for $x_2(t)$ in terms of cosine functions.

Problem Fall-01-Q.1.4:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -\sqrt{3} - j3.$$

(a) Express jV in polar form. In addition plot jV as a vector.

(b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.

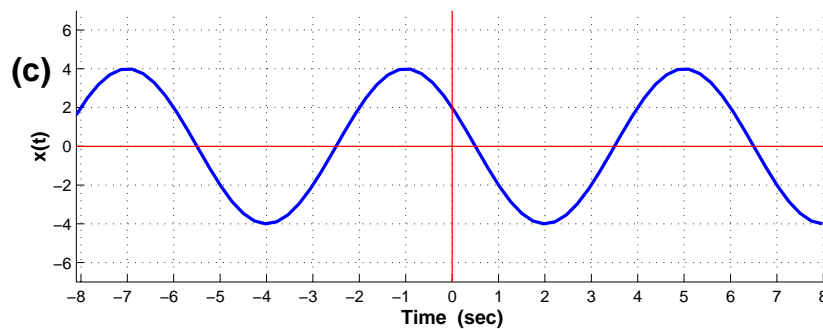
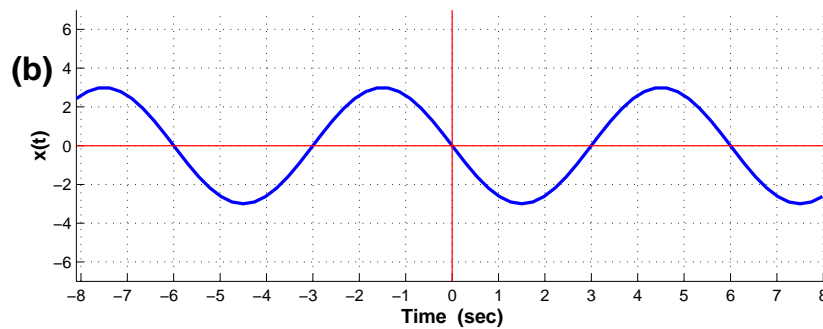
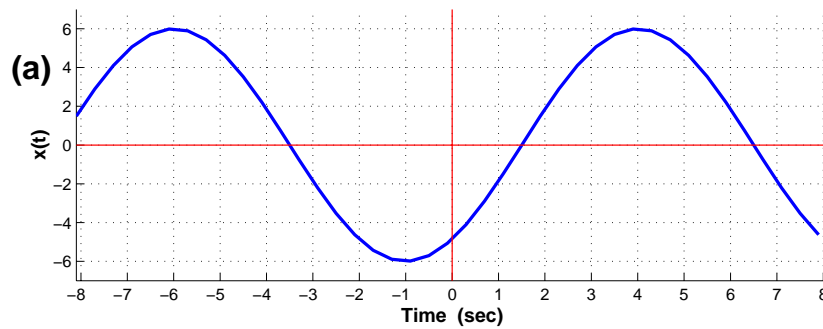
(c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.

(d) Express $\Re\{j^3 V e^{j15t}\}$ in the standard “cosine” form.

Problem Fall-01-Q.1.5:

Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

PLOT	(a)	(b)	(c)
AMPLITUDE			
PHASE (in radians)			
FREQUENCY (in Hz)			



Problem Fall-01-Q.1.1:

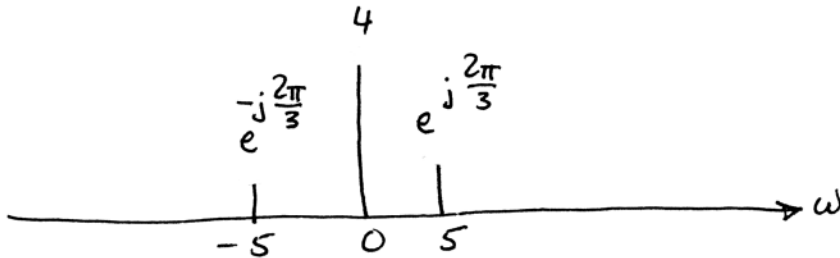
The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = 4 - 2 \cos(5t - \pi/3) = 4 + e^{j\pi} \cdot e^{-j\frac{\pi}{3}} \cdot e^{j5t} + e^{-j\pi} \cdot e^{j\frac{\pi}{3}} \cdot e^{-j5t}$$

and that

$$x(t) = v(t) \cos(40t).$$

- (a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.



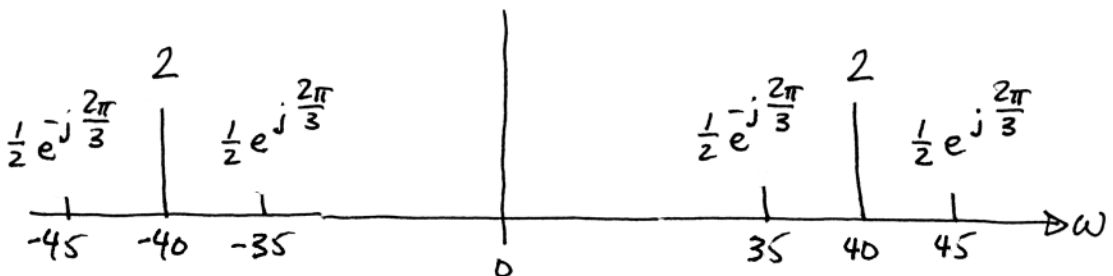
- (b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.

$$x(t) = v(t) \cos(40t)$$

$$= \left[4 + e^{j\frac{2\pi}{3}} e^{j5t} + e^{-j\frac{2\pi}{3}} e^{-j5t} \right] \frac{1}{2} \left[e^{j40t} + e^{-j40t} \right]$$

$$= 2e^{j40t} + \frac{1}{2} e^{j\frac{2\pi}{3}} e^{j45t} + \frac{1}{2} e^{-j\frac{2\pi}{3}} e^{j35t} + 2e^{-j40t}$$

$$+ \frac{1}{2} e^{j\frac{2\pi}{3}} e^{-j35t} + \frac{1}{2} e^{-j\frac{2\pi}{3}} e^{-j45t}$$



Problem Fall-01-Q.1.2:Define $x(t)$ as

$$x(t) = \overbrace{3\sqrt{3}\cos(10\pi t - \pi/3)}^{x_1} + \overbrace{3\cos(10\pi(t+0.05))}^{x_2}$$

- (a) Use phasor addition to express $x(t)$ in the form $x(t) = A \cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ , as well as ω_0 .

$$X_1 = 3\sqrt{3} e^{-j\pi/3} = \frac{3\sqrt{3}}{2} - j\frac{9}{2}$$

$$x_2(t) = 3\cos(10\pi(t+0.05)) = 3\cos(10\pi t + \frac{\pi}{2})$$

$$X_2 = 3e^{j\pi/2} = j3$$

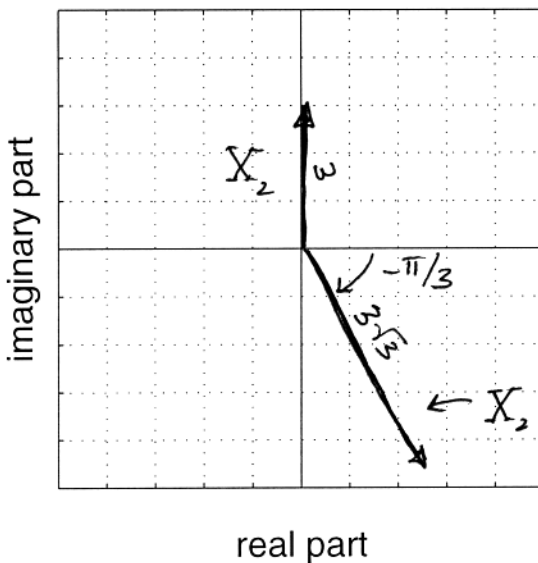
$$X = X_1 + X_2 = \frac{3\sqrt{3}}{2} - j\frac{3}{2} = 3e^{-j\pi/6}$$

$$x(t) = 3\cos(10\pi t - \frac{\pi}{6})$$

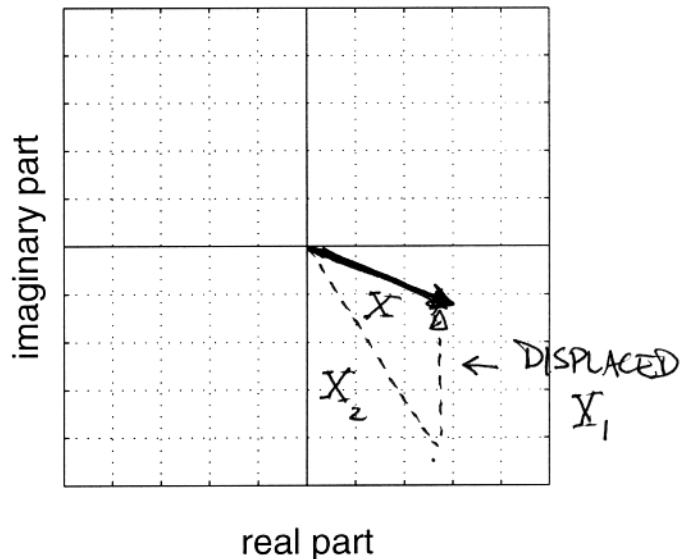
$$\begin{aligned} A &= 3 \\ \omega_0 &= 10\pi \\ \phi &= -\frac{\pi}{6} \end{aligned}$$

- (b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two vectors here.

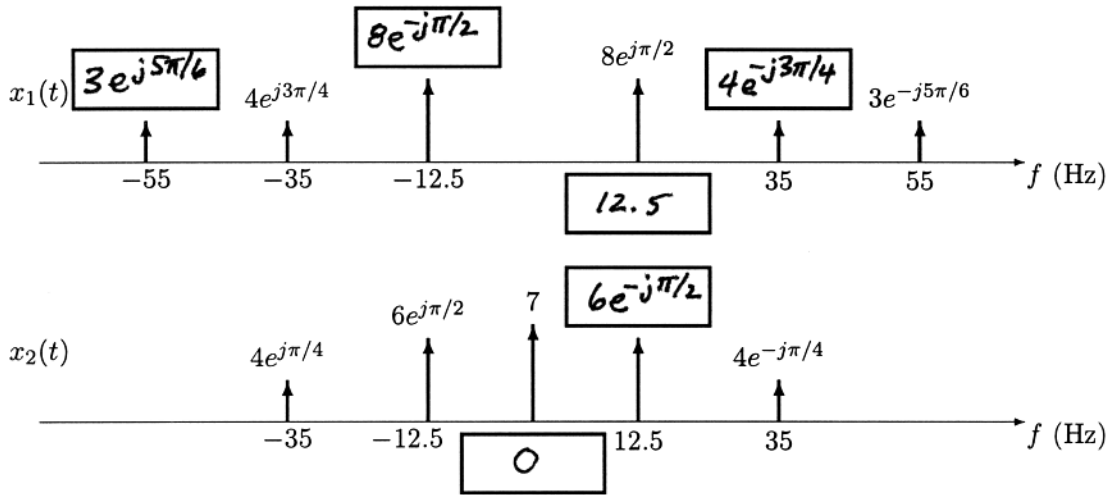


Head-to-tail plot here.



Problem Fall-01-Q.1.3:

- (a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.



- (b) Write an equation for $x_2(t)$ in terms of cosine functions.

$$x_2(t) = 7 + 12 \cos\left(2\pi(12.5)t - \frac{\pi}{2}\right) + 8 \cos\left(2\pi(35)t - \frac{\pi}{4}\right)$$

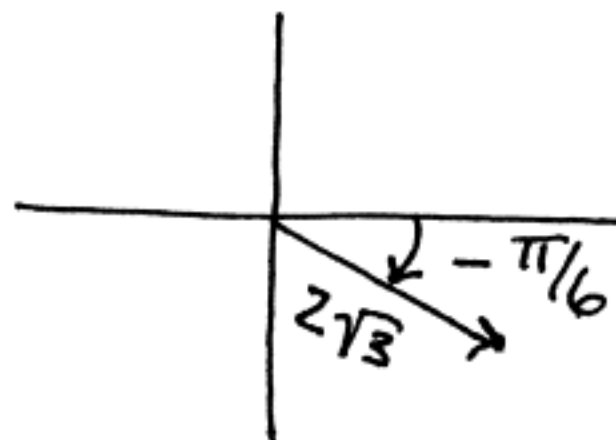
Problem Fall-01-Q.1.4:

Simplify the following complex-valued expressions. In each case reduce the answers to a simple numerical form. Let

$$V = -\sqrt{3} - j3.$$

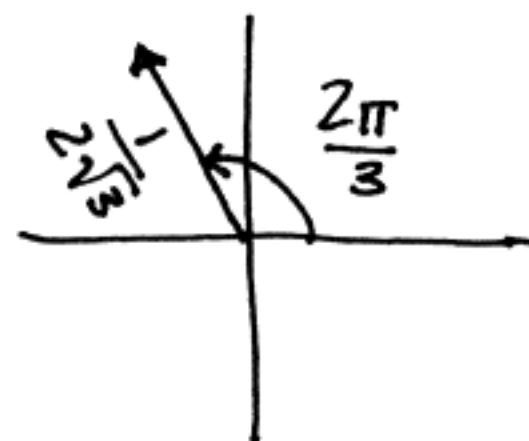
- (a) Express jV in polar form. In addition plot jV as a vector.

$$\begin{aligned} jV &= 3 - j\sqrt{3} \\ &= 2\sqrt{3} e^{-j\frac{\pi}{6}} \end{aligned}$$



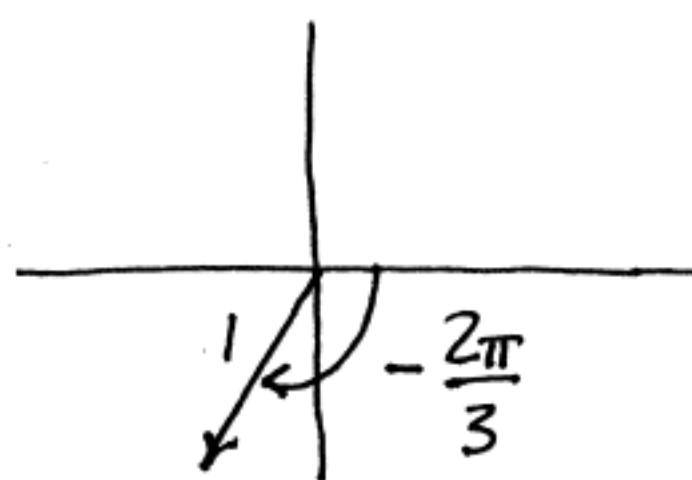
- (b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.

$$\begin{aligned} V &= 2\sqrt{3} e^{-j\frac{2\pi}{3}} \\ \frac{1}{V} &= \frac{1}{2\sqrt{3}} e^{j\frac{2\pi}{3}} = -\frac{1}{4\sqrt{3}} + j\frac{1}{4} \end{aligned}$$



- (c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.

$$\frac{1}{Z} = \frac{2\sqrt{3}}{2\sqrt{3} e^{j\frac{2\pi}{3}}} = e^{-j\frac{2\pi}{3}}$$



- (d) Express $\Re\{j^3 V e^{j15t}\}$ in the standard "cosine" form.

$$\begin{aligned} \Re\{j^3 V e^{j15t}\} &= \Re\left\{e^{-j\frac{\pi}{2}} \cdot 2\sqrt{3} \cdot e^{-j\frac{2\pi}{3}} \cdot e^{j15t}\right\} \\ &= 2\sqrt{3} \cos\left(15t + \frac{5\pi}{6}\right) \end{aligned}$$

Problem Fall-01-Q.1.5:

Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

PLOT	(a)	(b)	(c)
AMPLITUDE	6	3	4
PHASE (in radians)	$-\frac{4\pi}{5}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$
FREQUENCY (in Hz)	$\frac{1}{10}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\phi = -\omega t_m = -2\pi f t_m$$

← ALLOW FOR EQUIVALENT PHASE

