GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 17-Sept-01 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

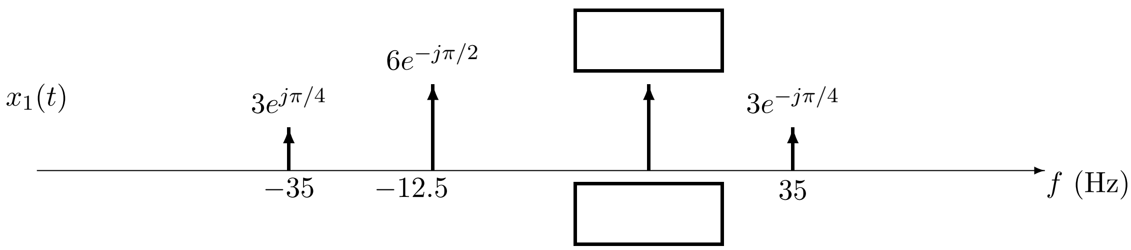
L01:Tues-9:30am (Bordelon)	L02:Thur-9:30am (Casinovi)	L03:Tues-12:00pm (Casinovi)
L04:Thur-12:00pm (Ji)	L05:Tues-1:30pm (Bordelon)	L06:Thur-1:30pm (Ji)
L07:Tues-3:00pm (Casinovi)	L08:Thur-3:00pm (Bordelon)	L09:Tues-4:30pm (Lanterman)
L10:Thur-4:30pm (Bordelon)	L11:Tues-6:00pm (Lanterman)	L13:Mon -3:00pm (Yezzi)
L14:Weds-3:00pm (Taylor)	L15:Mon -4:30pm (Verriest)	L16:Weds-4:30pm (Taylor)
L17:Mon -6:00pm (Verriest)	L20:Mon -1:30pm (McClellan)	RPK: (Abler)

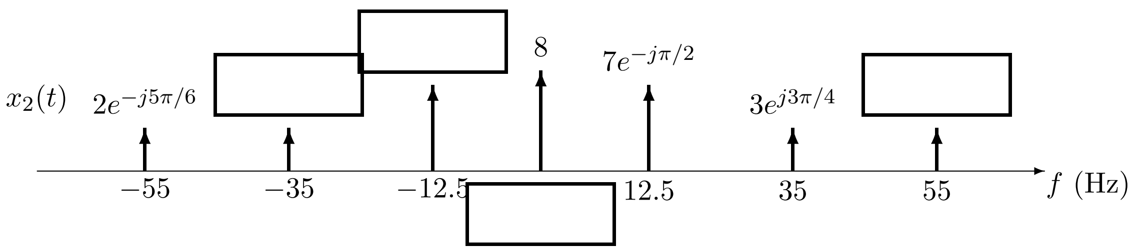
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning clearly to receive any partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem F01-Q.1.1:

(a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.





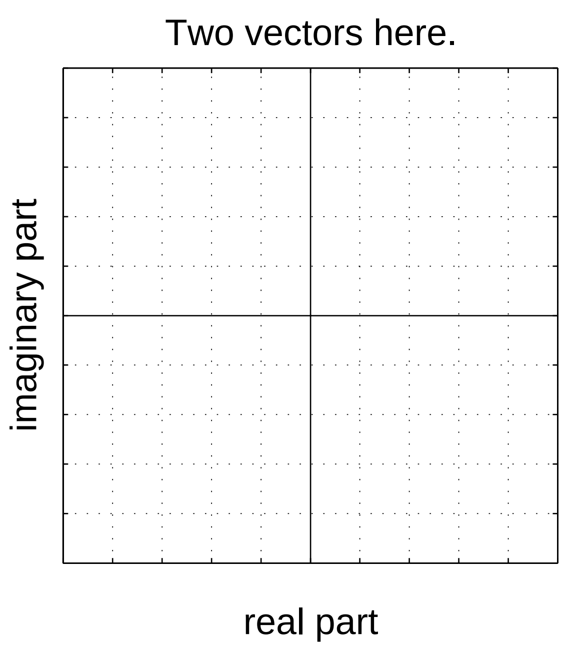
(b) Write an equation for $x_2(t)$ in terms of cosine functions.

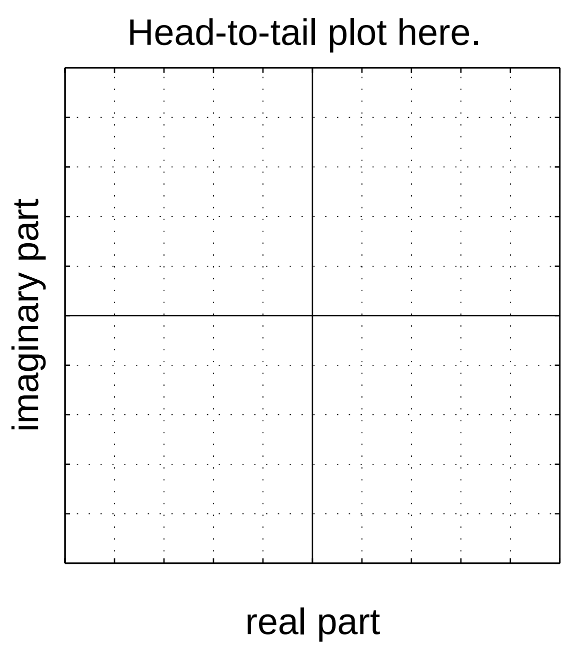
Problem F01-Q.1.2:
Define $x(t)$ as

$$x(t) = \sqrt{2}\cos(10\pi(t + .05)) + \cos(10\pi t - 3\pi/4)$$

(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).



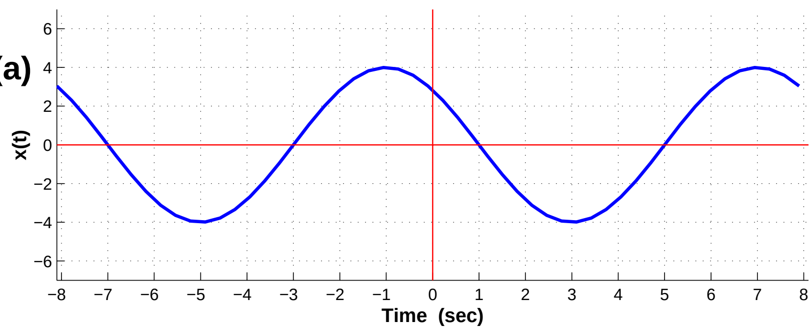


Problem F01-Q.1.3:

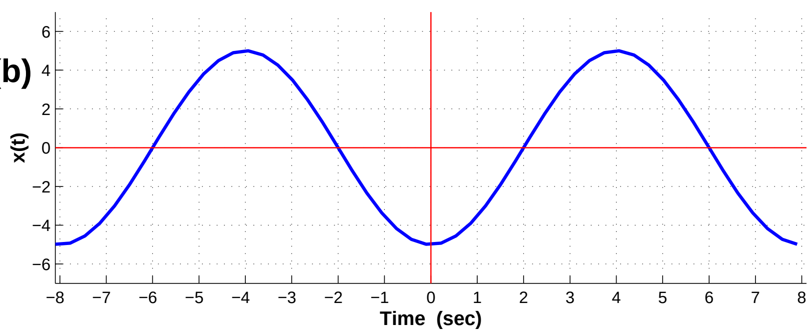
Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

Plot	(a)	(b)	(c)
Amplitude			
Phase (in radians)			
Frequency (in Hz)			

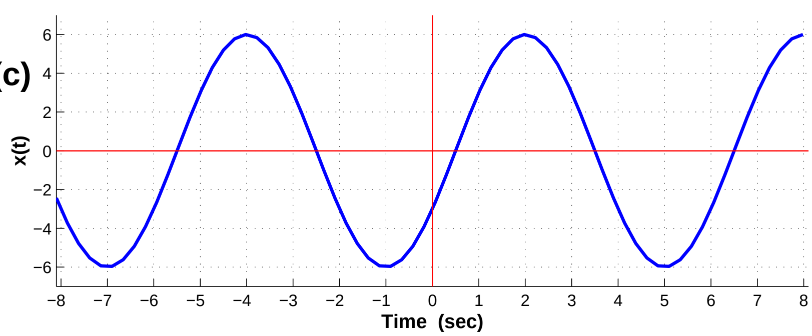
(a)



(b)



(c)



Problem F01-Q.1.4:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -\sqrt{3} + j3.$$

(a) Express jV in polar form. In addition plot jV as a vector.

(b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.

(c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.

(d) Express $\Re e\{j^3 V e^{j15t}\}$ in the standard "cosine" form.

Problem F01-Q.1.5:
The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = 4 + 4\cos(6t + \pi/2)$$

and that

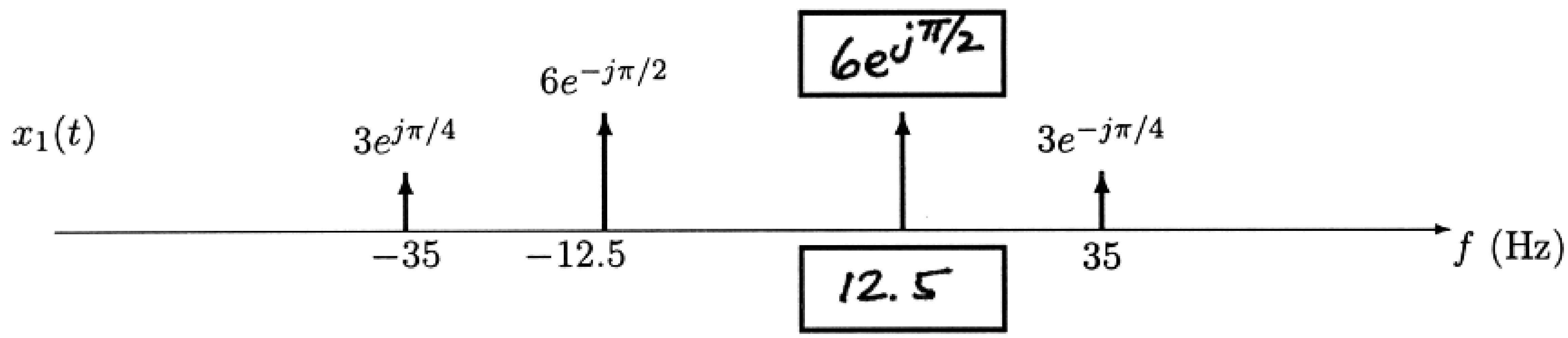
$$x(t) = v(t)\cos(20t).$$

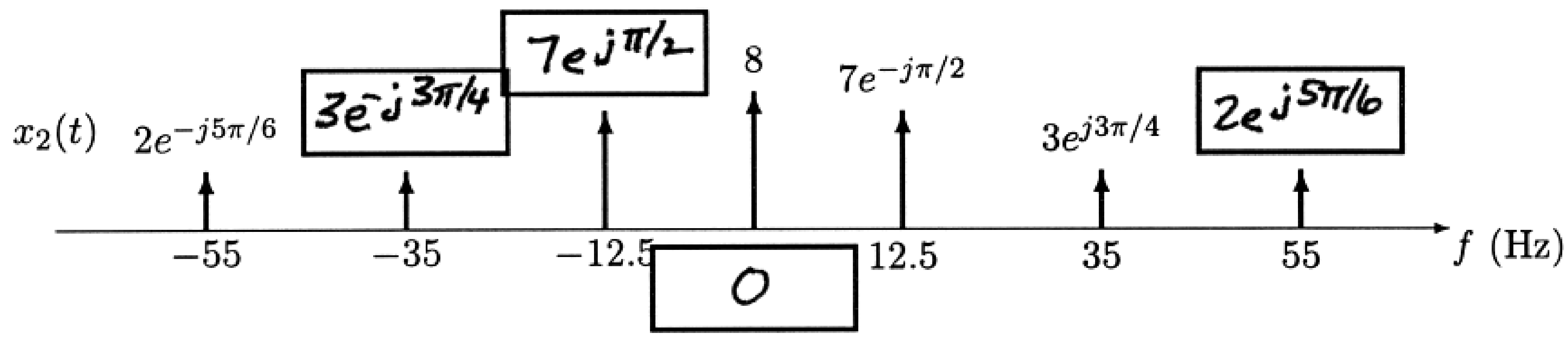
(a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.

(b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be clearly indicated.

Problem F01-Q.1.1:

(a) The incomplete spectra for two *real* signals $x_1(t)$ and $x_2(t)$ are shown in the following figures. Fill in the empty boxes for the missing components.





(b) Write an equation for $x_2(t)$ in terms of cosine functions.

$$x_2(t) = 8 + 14\cos\left(2\pi(12.5)t - \frac{\pi}{2}\right) + 6\cos\left(2\pi(35)t + \frac{3\pi}{4}\right) + 4\cos\left(2\pi(55)t + \frac{5\pi}{6}\right)$$

Problem F01-Q.1.2:
Define $x(t)$ as

$$x(t) = \underbrace{\sqrt{2}\cos(10\pi(t+.05))}_{x_1} + \underbrace{\cos(10\pi t - 3\pi/4)}_{x_2}$$

(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

$x_1(t) = \sqrt{2}\cos(10\pi t + \frac{\pi}{2}) \quad X_1 = \sqrt{2}\, e^{j\frac{\pi}{2}} = j\sqrt{2}$

$X_2 = e^{-j\frac{3\pi}{4}} = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$X = X_1 + X_2 = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} = e^{j\frac{3\pi}{4}}$

$x(t) = \cos(10\pi t + \frac{3\pi}{4})$

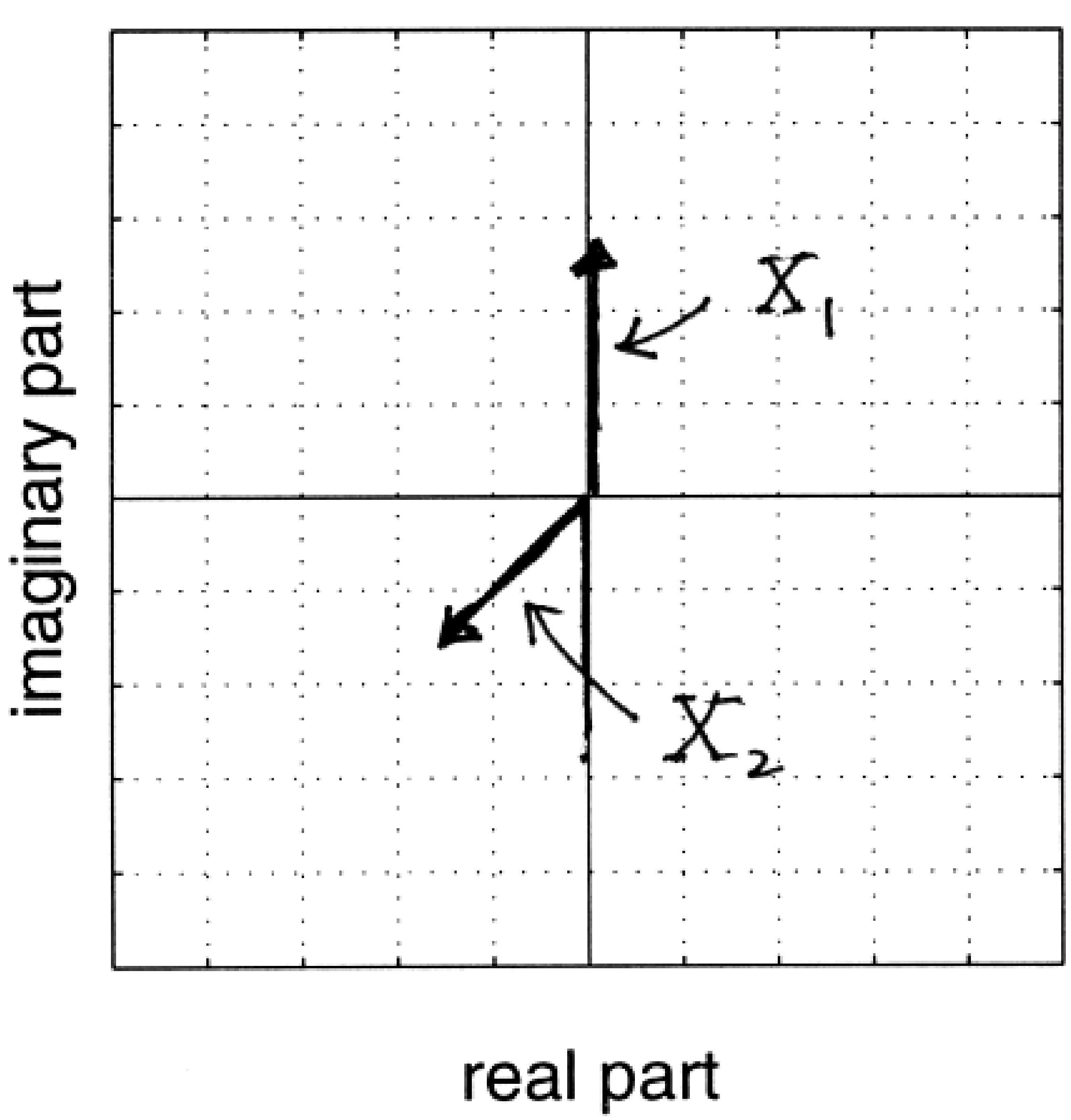
$A = 1$

$\omega_0 = 10\pi$

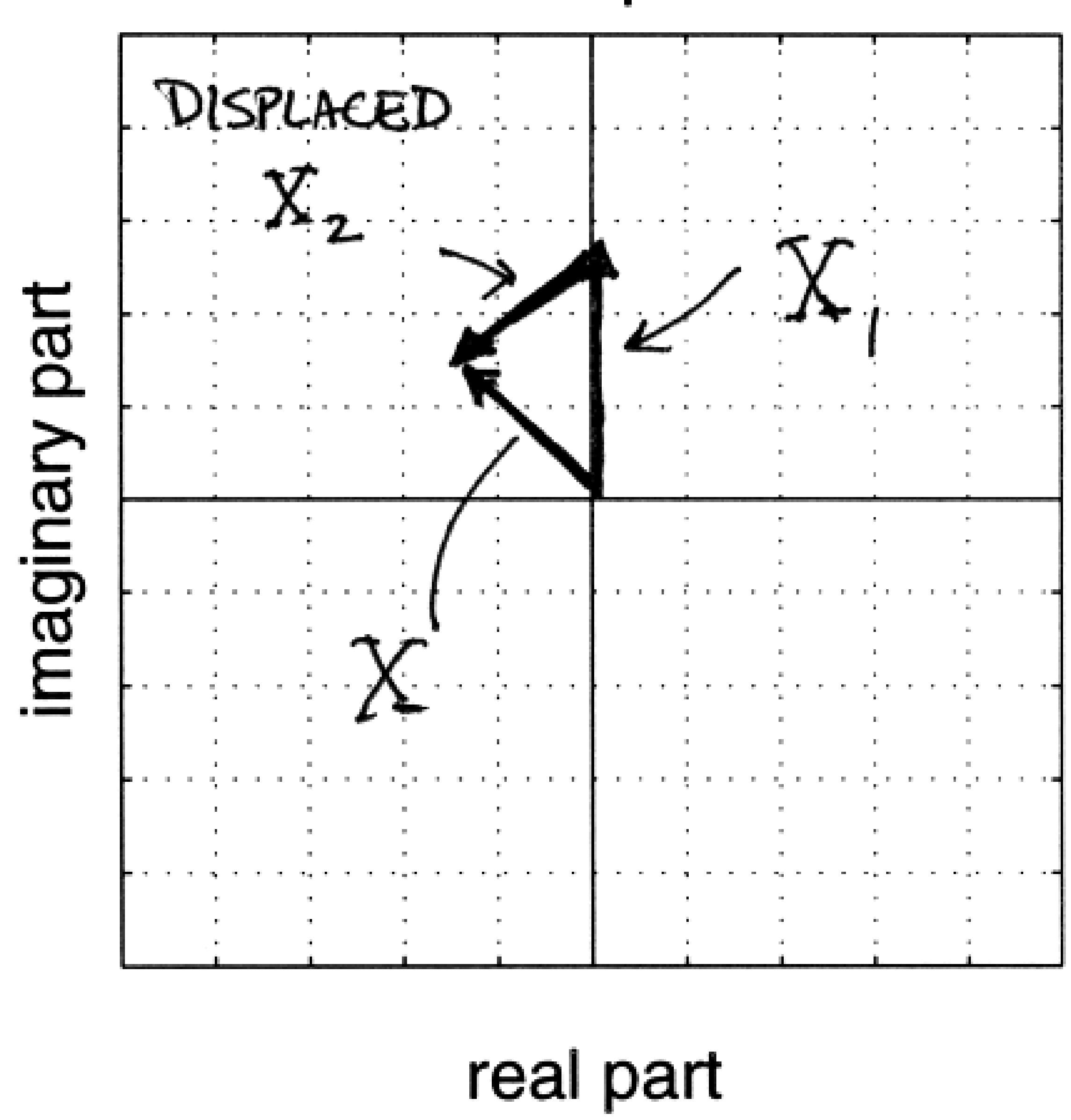
$\phi = \frac{3\pi}{4}$

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two vectors here.



Head-to-tail plot here.



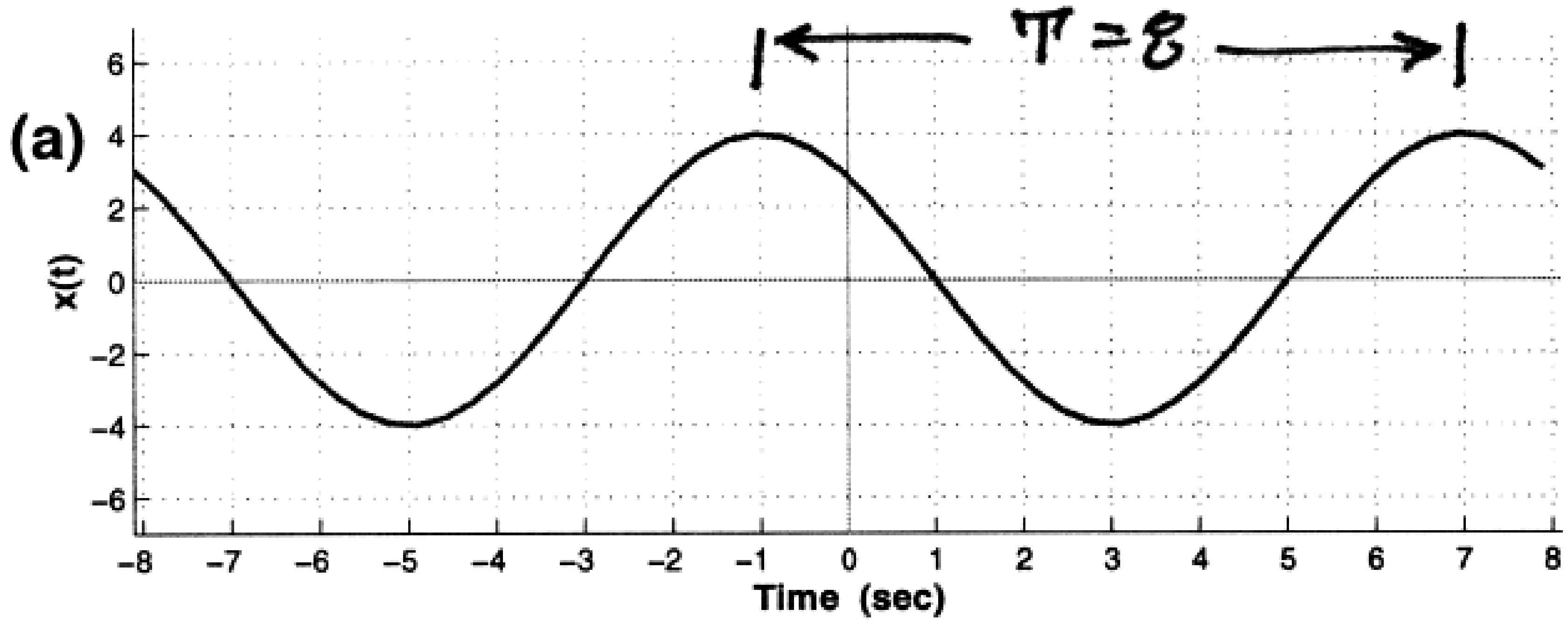
Problem F01-Q.1.3:

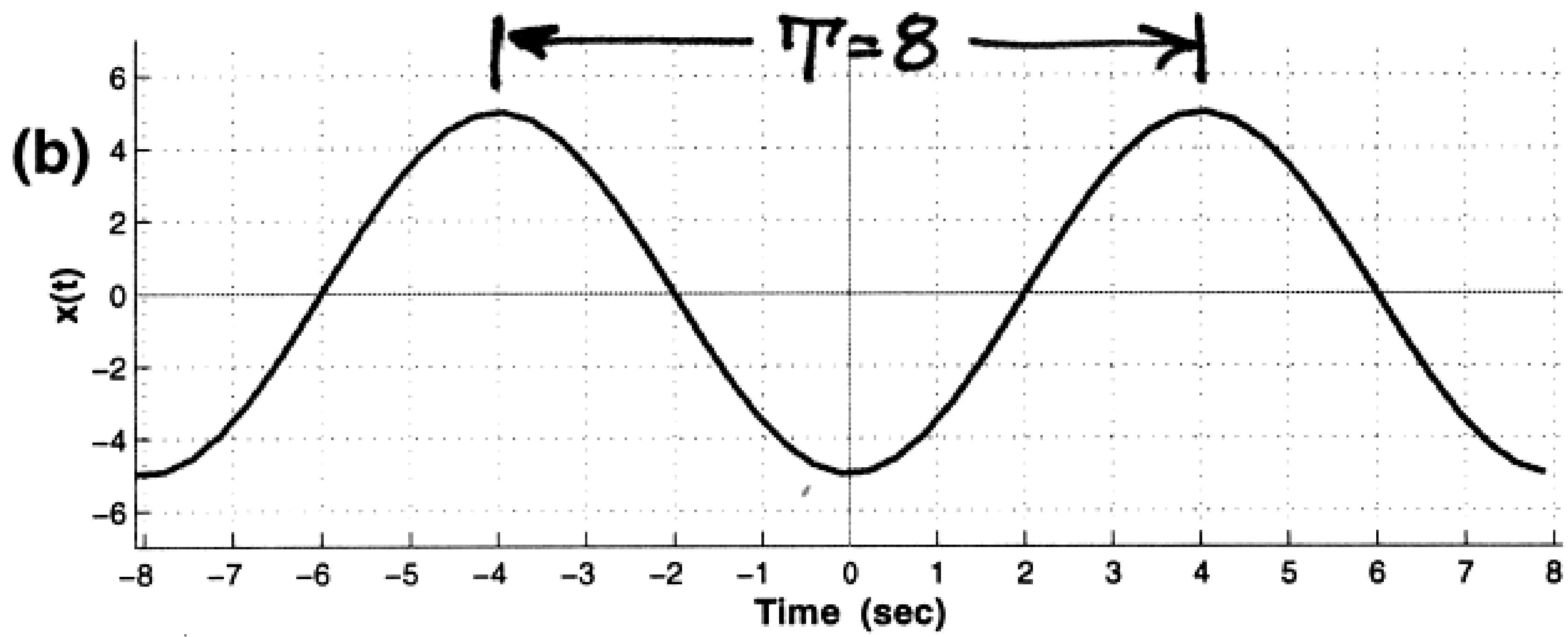
Several sinusoidal signals are plotted below. For each plot (a)–(c), determine the amplitude, phase (in radians) and frequency (in Hz). Write your answers in the following table:

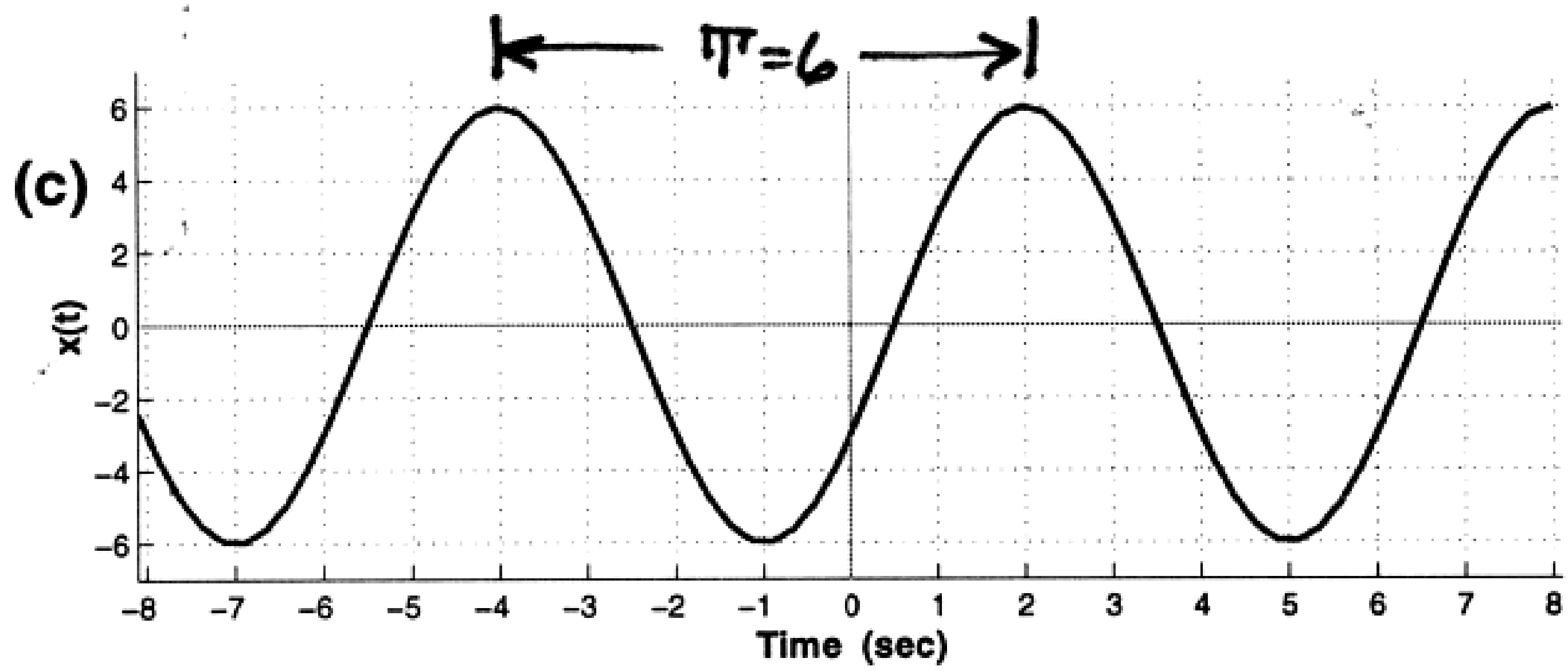
PLOT	(a)	(b)	(c)
AMPLITUDE	4	5	6
PHASE (in radians)	$\frac{\pi}{4}$	π	$-\frac{2\pi}{3}$
FREQUENCY (in Hz)	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{6}$

$\phi = -\omega t_m = -2\pi f t_m$

← ALLOW FOR EQUIVALENT PHASE







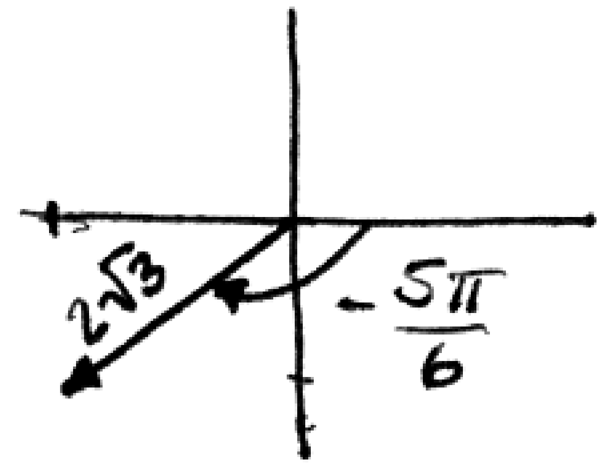
Problem F01-Q.1.4:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -\sqrt{3} + j3.$$

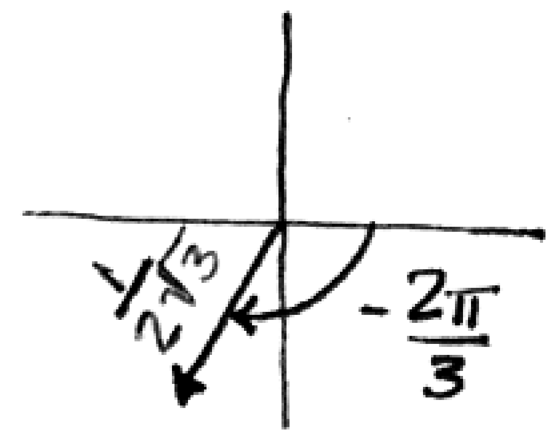
(a) Express jV in polar form. In addition plot jV as a vector.

$$jV = -j\sqrt{3} - 3$$
$$= 2\sqrt{3}\, e^{-j\frac{5\pi}{6}}$$



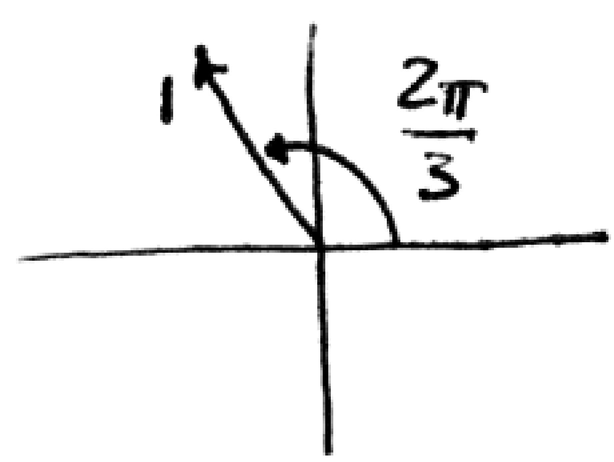
(b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.

$$V = 2\sqrt{3}\, e^{j\frac{2\pi}{3}}$$
$$\frac{1}{V} = \frac{1}{2\sqrt{3}}\, e^{-j\frac{2\pi}{3}}$$
$$= -\frac{1}{4\sqrt{3}} - j\frac{1}{4}$$



(c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.

$$Z = \frac{2\sqrt{3}}{2\sqrt{3}\, e^{-j\frac{2\pi}{3}}} = e^{j\frac{2\pi}{3}}$$



(d) Express $\Re e\{j^3 V e^{j15t}\}$ in the standard "cosine" form.

$$Re\{j^3 V e^{j15t}\} = Re\left\{e^{-j\frac{\pi}{2}} \cdot 2\sqrt{3}\, e^{j\frac{2\pi}{3}} \cdot e^{j15t}\right\}$$
$$= 2\sqrt{3}\cos\left(15t + \frac{\pi}{6}\right)$$

Problem F01-Q.1.5:

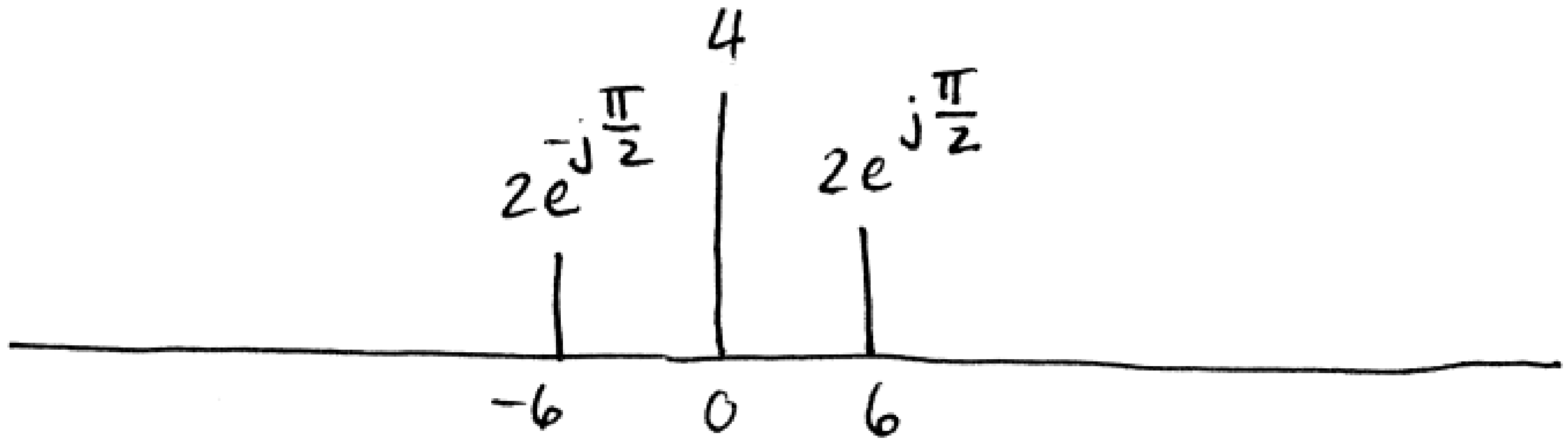
The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation. Assume that

$$v(t) = 4 + 4\cos(6t + \pi/2) = 4 + 2e^{j\pi/2}e^{j6t} + 2e^{-j\frac{\pi}{2}}e^{-j6t}$$

and that

$$x(t) = v(t)\cos(20t).$$

(a) Draw the spectrum for $v(t)$. Your sketch should be clearly labeled and all complex amplitudes should be indicated.



(b) Draw the spectrum for $x(t)$. Your sketch should be clearly labeled and all complex amplitudes should be clearly indicated.

$$x(t) = \left[4 + 2e^{j\frac{\pi}{2}}e^{j6t} + 2e^{-j\frac{\pi}{2}}e^{-j6t}\right]\frac{1}{2}\left[e^{j20t} + e^{-j20t}\right]$$

$$= 2e^{j20t} + e^{j\frac{\pi}{2}}e^{j26t} + e^{-j\frac{\pi}{2}}e^{j14t} + 2e^{-j20t} + e^{j\frac{\pi}{2}}e^{-j14t} + e^{-j\frac{\pi}{2}}e^{-j26t}$$