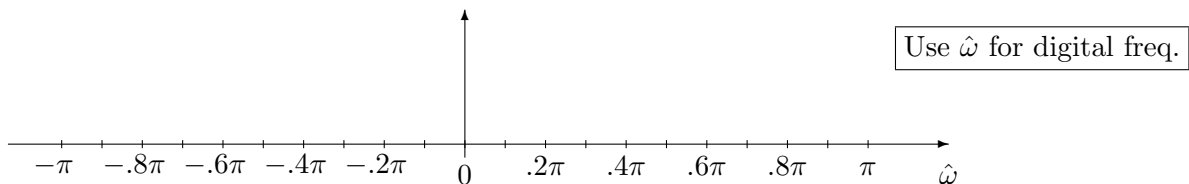


Suppose that the continuous-time input $x(t)$ to the above system is given as

$$x(t) = \cos(14000\pi t) + \cos(2000\pi t) + \cos(1000\pi t).$$

- (a) What is required such that no aliasing occurs for $x(t)$?
- (b) Given that $f_s = 12,000$ samples/second, the frequency spectrum for $x[n]$.



- (c) Given that $x(t) = \cos(25000\pi t)$ and $f_s = 10000$ samples/second, write a simplified expression for the in terms of cosine functions.

Problem Fall-01-Q.2.2:

A periodic signal, $x(t)$, is given by

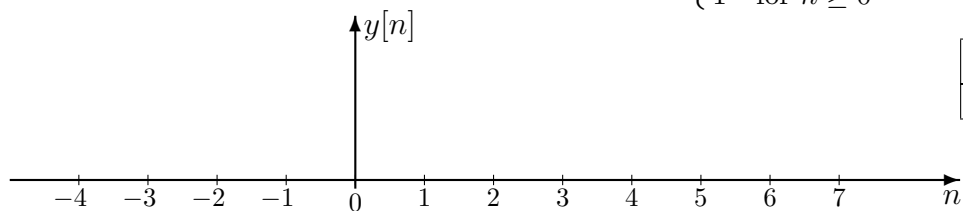
$$x(t) = 2 + \sin(300\pi t) + 3 \cos(600\pi t + \pi/3)$$

(a) What is the period of $x(t)$?

(b) Find the Fourier series coefficients of $x(t)$ for $-6 \leq k \leq 6$.

Problem Fall-01-Q.2.3:

- (a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, 0, -2\}$, make a plot of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$

**Label Carefully**Plot zero values also

- (b) Suppose that the frequency response of a different FIR filter is

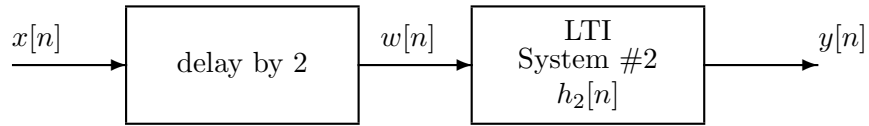
$$\mathcal{H}(\hat{\omega}) = (2 - 2 \cos(\hat{\omega}))e^{-j3\hat{\omega}}$$

If the input signal is $x[n] = 5 + 1 \cos(\pi n/4 + 0.3\pi)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$y[n] =$

Problem Fall-01-Q.2.4:

Consider the following cascade system:



(a) Find and plot the magnitude of the frequency response of the first filter $|\mathcal{H}_1(\hat{\omega})|$.

(b) If the overall impulse response of the cascade is

$$h_{eq}[n] = \delta[n - 3] + \delta[n - 5]$$

determine the impulse response of the second filter $h_2[n]$.

Problem Fall-01-Q.2.5:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response:

Impulse response:

Difference equation: $y[n] = x[n] - x[n - 2]$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega}))$

Impulse response:

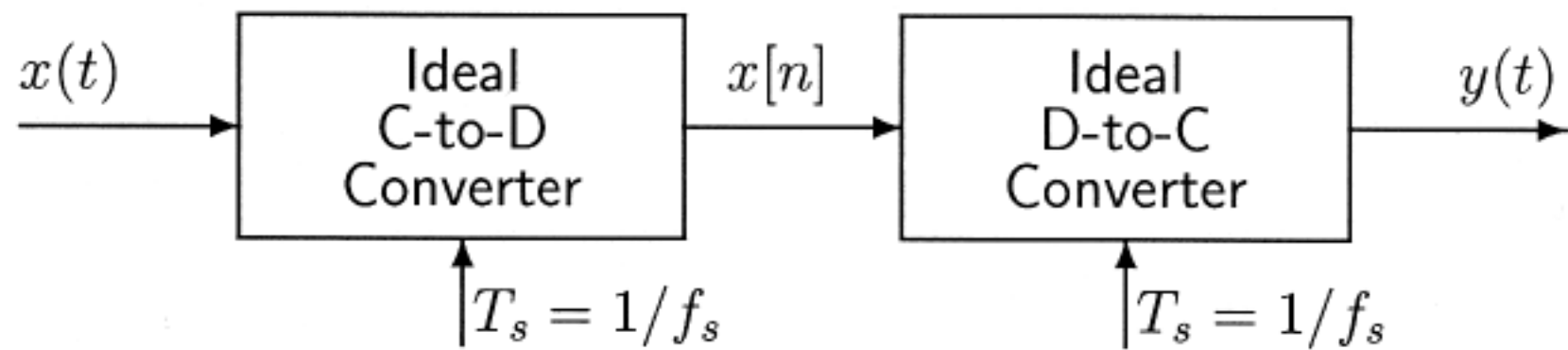
Difference equation:

(c) Frequency response:

Impulse response:

Difference equation: $y = \text{conv}([0, 2, 0, 2], x)$

Problem Fall-01-Q.2.1:

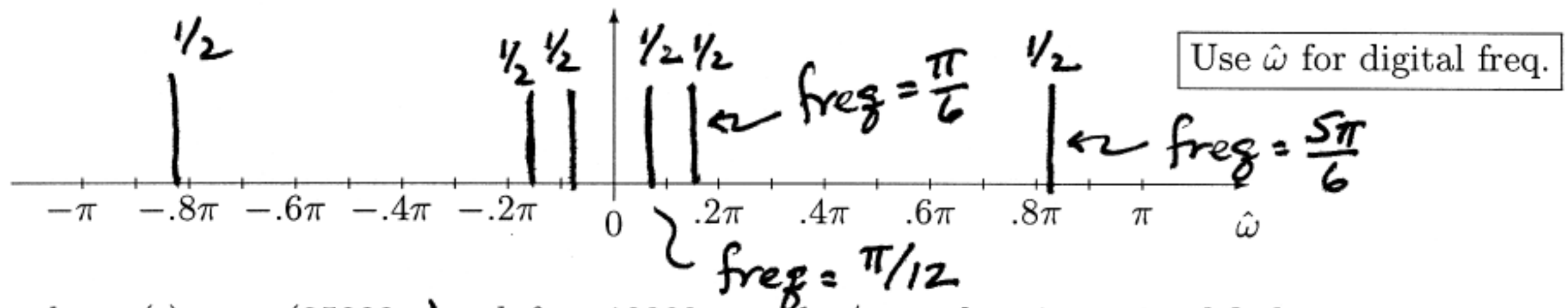


Suppose that the continuous-time input $x(t)$ to the above system is given as

$$x(t) = \cos(14000\pi t) + \cos(2000\pi t) + \cos(1000\pi t).$$

$$f_{\max} = 7,000 \text{ Hz}$$

- (a) What sampling rate is required such that no aliasing occurs for $x(t)$? $f_s = 14,000 \text{ Hz}$
- (b) Given that $f_s = 12,000$ samples/second, plot the frequency spectrum for $x[n]$.



- (c) Given that $x(t) = \cos(25000\pi t)$ and $f_s = 10000$ samples/second, write a simplified expression for the output $y(t)$ in terms of cosine functions.

$$y(t) = \cos(5000\pi t)$$

NOTE THAT ALIASING OCCURS.

Problem Fall-01-Q.2.2:

A periodic signal, $x(t)$, is given by

$$x(t) = 2 + \sin(300\pi t) + 3 \cos(600\pi t + \pi/3)$$

(a) What is the period of $x(t)$?

FUNDAMENTAL FREQ.: $\omega_0 = 300\pi = \frac{2\pi}{T}$

$$\Rightarrow T = \frac{1}{150} \text{ SEC.}$$

(b) Find the Fourier series coefficients of $x(t)$ for $-6 \leq k \leq 6$.

USING EULER'S RELATION

$$\begin{aligned} x(t) = & 2 + \frac{1}{2} e^{-j\pi/2} \cdot e^{j300\pi t} + \frac{1}{2} e^{j\pi/2} e^{-j300\pi t} \\ & + \frac{3}{2} e^{j\pi/3} \cdot e^{j2(300\pi)t} + \frac{3}{2} e^{-j\pi/3} e^{-j2(300\pi)t} \end{aligned}$$

\therefore

$$a_0 = 2$$

$$a_1 = \frac{1}{2} e^{-j\pi/2}$$

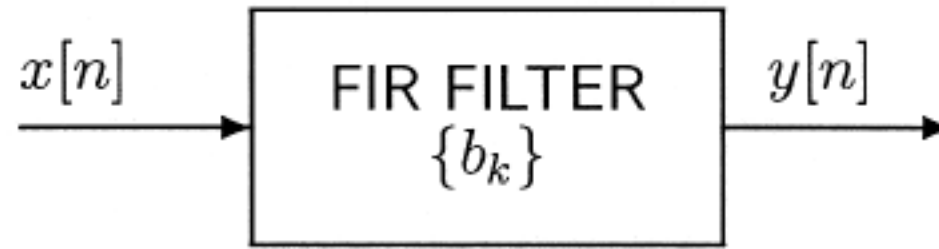
$$a_{-1} = \frac{1}{2} e^{j\pi/2}$$

$$a_2 = \frac{3}{2} e^{j\pi/3}$$

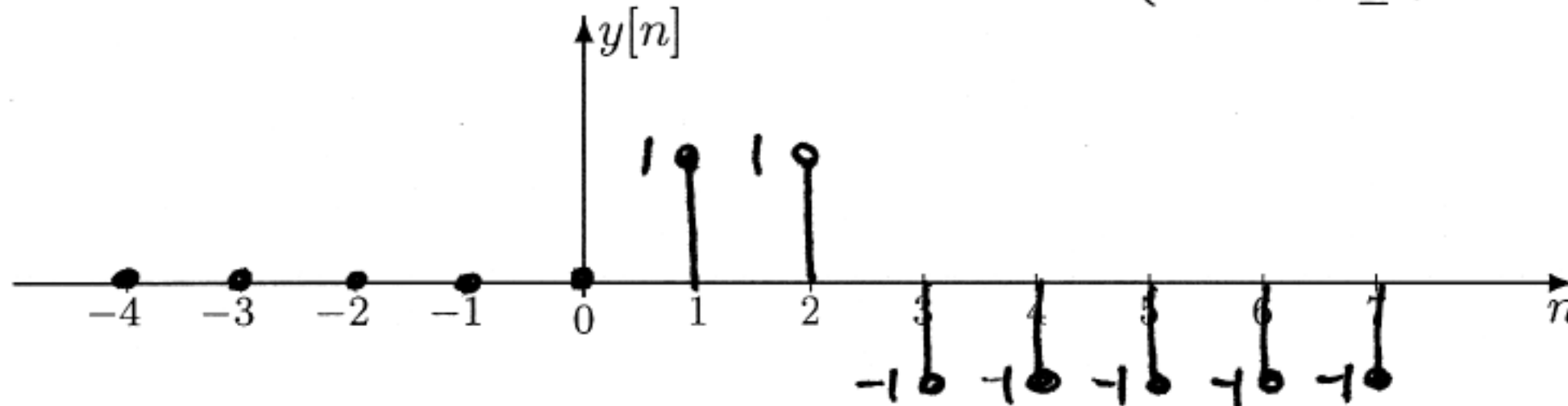
$$a_{-2} = \frac{3}{2} e^{-j\pi/3}$$

$$a_k = 0 \text{ FOR ALL OTHER } k$$

Problem Fall-01-Q.2.3:



- (a) If the filter coefficients of an FIR filter are $\{b_k\} = \{0, 1, 0, -2\}$, make a plot of the output when the input is the unit step signal: $x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$



Label Carefully

Plot zero values also

$$\begin{array}{r}
 x[n-1]: \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad \dots \\
 -2x[n-3]: \quad \quad \quad 0 \quad 0 \quad 0 \quad 0 \quad -2 \quad -2 \quad -2 \quad -2 \quad -2 \\
 \hline
 y[n]: \quad \quad \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1
 \end{array}$$

- (b) Suppose that the frequency response of a different FIR filter is

$$H(\hat{\omega}) = (2 - 2 \cos(\hat{\omega}))e^{-j3\hat{\omega}}$$

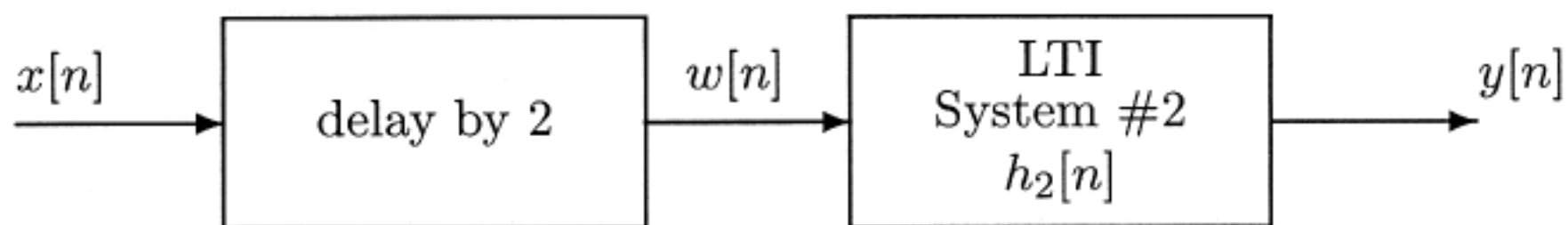
If the input signal is $x[n] = 5 + 1 \cos(\pi n/4 + 0.3\pi)$ for $-\infty < n < \infty$, determine a simple mathematical expression for the output signal $y[n]$.

$$y[n] = (2 - \sqrt{2}) \cos\left(\frac{\pi n}{4} + 0.3\pi - \frac{3\pi}{4}\right) = 0.586 \cos\left(\frac{\pi n}{4} - .45\pi\right)$$

$$H(0) = 0 \quad H\left(\frac{\pi}{4}\right) = (2 - \sqrt{2}) e^{-j3\pi/4}$$

Problem Fall-01-Q.2.4:

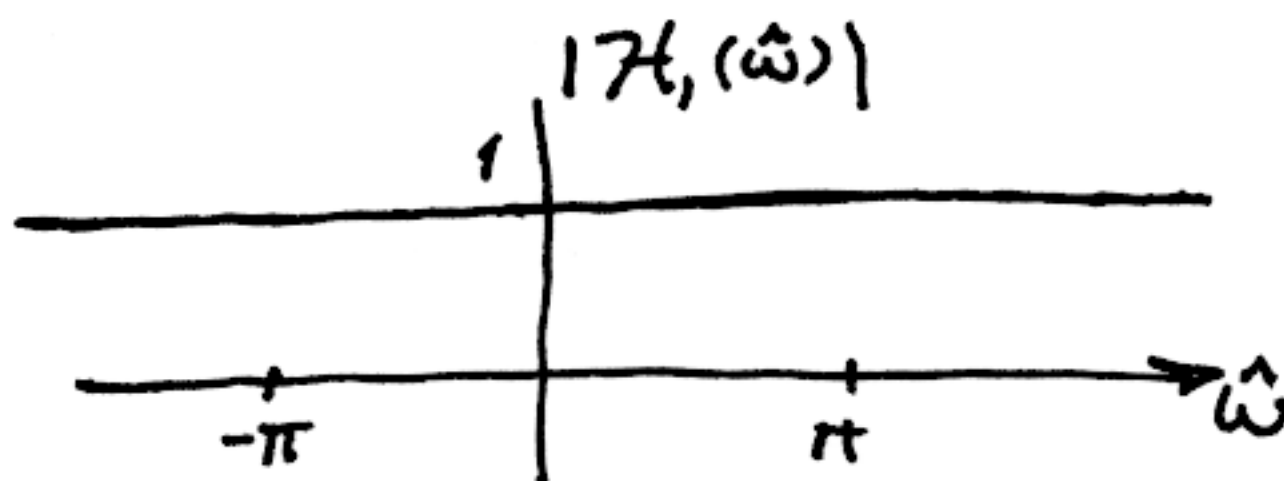
Consider the following cascade system:



- (a) Find and plot the magnitude of the frequency response of the first filter $|\mathcal{H}_1(\hat{\omega})|$.

$$h_1[n] = \delta[n-2]$$
$$\mathcal{H}_1(\hat{\omega}) = e^{-j2\hat{\omega}}$$

$$|\mathcal{H}_1(\hat{\omega})| = 1$$

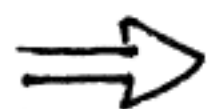


- (b) If the overall impulse response of the cascade is

$$h_{eq}[n] = \delta[n-3] + \delta[n-5]$$

determine the impulse response of the second filter $h_2[n]$.

$$\delta[n-2] * h_2[n] = \delta[n-3] + \delta[n-5]$$



$$h_2[n] = \delta[n-1] + \delta[n-3]$$

Problem Fall-01-Q.2.5:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response: $\mathcal{H}(\hat{\omega}) = 1 - e^{-j2\hat{\omega}}$

Impulse response: $h[n] = \delta[n] - \delta[n-2]$

Difference equation: $y[n] = x[n] - x[n-2]$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{j\hat{\omega}}(2 + 2\cos(\hat{\omega})) = 2e^{j\hat{\omega}} + 1e^{j2\hat{\omega}} + 1$

Impulse response: $h[n] = \delta[n] + 2\delta[n+1] + \delta[n+2]$

Difference equation: $y[n] = x[n] + 2x[n+1] + x[n+2]$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega})) = 2e^{-j\hat{\omega}} + 1 + 1e^{-j2\hat{\omega}}$

Impulse response: $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

Difference equation: $y[n] = x[n] + 2x[n-1] + x[n-2]$

(c) Frequency response: $\mathcal{H}(\hat{\omega}) = 2e^{-j\hat{\omega}} + 2e^{-j3\hat{\omega}}$

Impulse response: $h[n] = 2\delta[n-1] + 2\delta[n-3]$

Difference equation: $y = \text{conv}([0, 2, 0, 2], x)$