GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3

DATE: 19-Nov-01 COURSE: ECE 2025

NAME: ______________________________ gtxxx (login) # : ______________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-9:30am (Bordelon)	L02:Thur-9:30am (Casinovi)	L03:Tues-12:00pm (Casinovi)
L04:Thur-12:00pm (Ji)	L05:Tues-1:30pm (Bordelon)	L06:Thur-1:30pm (Ji)
L07:Tues-3:00pm (Casinovi)	L08:Thur-3:00pm (Bordelon)	L09:Tues-4:30pm (Lanterman)
L10:Thur-4:30pm (Bordelon)	L11:Tues-6:00pm (Lanterman)	L13:Mon -3:00pm (Yezzi)
L14:Weds-3:00pm (Taylor)	L15:Mon -4:30pm (Verriest)	L16:Weds-4:30pm (Taylor)
L17:Mon -6:00pm (Verriest)	L20:Mon -1:30pm (McClellan)	RPK: (Abler)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HANDWRITTEN notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive any partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

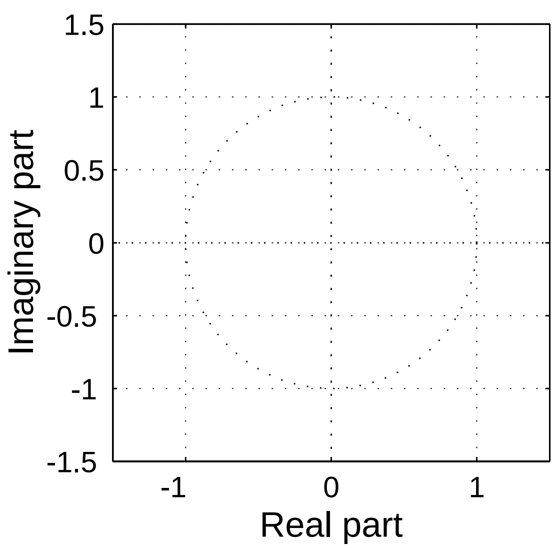
Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem Fall-01-Q.3.1:
A discrete-time system (FIR filter) is defined by the following z-transform system function:

$$H(z) = (1 + 0.9z^{-1})(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.



(c) If the input is of the form $x[n] = A\sin(\omega_0 n + \phi)$, for what value of frequency ω_0 (in the range $0 < \omega_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

Problem Fall-01-Q.3.2:

In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal. Provide some **explanation** or intermediate steps for each answer.

(a) $\displaystyle\int_{-\infty}^{t-7} \delta(\tau+1)\cos(\tau)\,d\tau =$

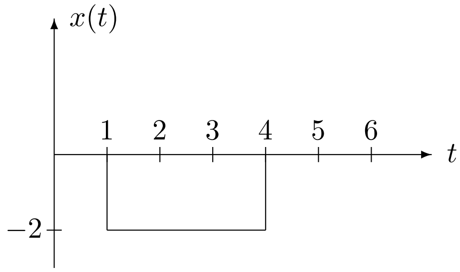
(b) $\displaystyle\frac{d}{dt}\{\sin(2t)u(t-2)\} =$

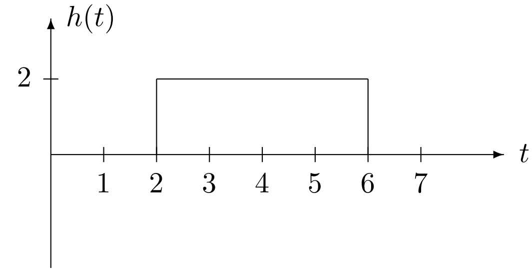
Problem Fall-01-Q.3.3:
In each of the following cases, determine the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or a plot. **Explain** each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = e^{-3(t-2)}u(t-2)$.

(b) Find $s(t)$ when $S(j\omega) = e^{-j\omega/2}[u(\omega + 10\pi) - u(\omega - 10\pi)]$.

Problem Fall-01-Q.3.4:
The following figure shows the signal $x(t) = -2u(t-1)+2u(t-4)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 2u(t-2) - 2u(t-6)$.





(a) Sketch $h(9-\tau)$ as a function of τ in the space below.

(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 9$; that is, determine $y(9)$. It is not necessary to evaluate $y(t)$ for all t, only for $t = 9$. Note: This problem may be answered without performing any integration.

(c) $y(t)$ reaches its minimum value for $T_1 \leq t \leq T_2$. Find the minimum value, y_{min} and also the values for T_1 and T_2.

$y_{min} =$ ____________
$T_1 =$ ____________ sec
$T_2 =$ ____________ sec

Problem Fall-01-Q.3.5:

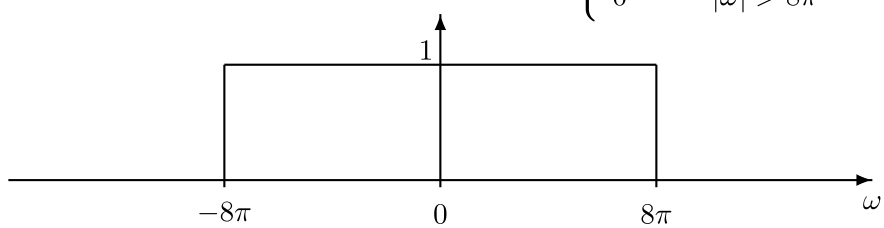
The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 1/3}$ seconds. The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} 0.5 & k = 0 \\ \dfrac{3\sin(\pi k/2)}{\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency ω_0 of the input signal $x(t)$? $\boxed{\omega_0 = \underline{\qquad\qquad} \text{ rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 8\pi \\ 0 & |\omega| > 8\pi \end{cases}$$



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). **<u>Justify your answer</u>** by sketching the spectrum of $y(t)$ (or its Fourier transform).

(c) Draw the spectrum of the output signal superimposed on the plot of $H(j\omega)$.

Problem Fall-01-Q.3.1:

A discrete-time system (FIR filter) is defined by the following z-transform system function:

$$H(z) = (1 + 0.9z^{-1})(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})$$

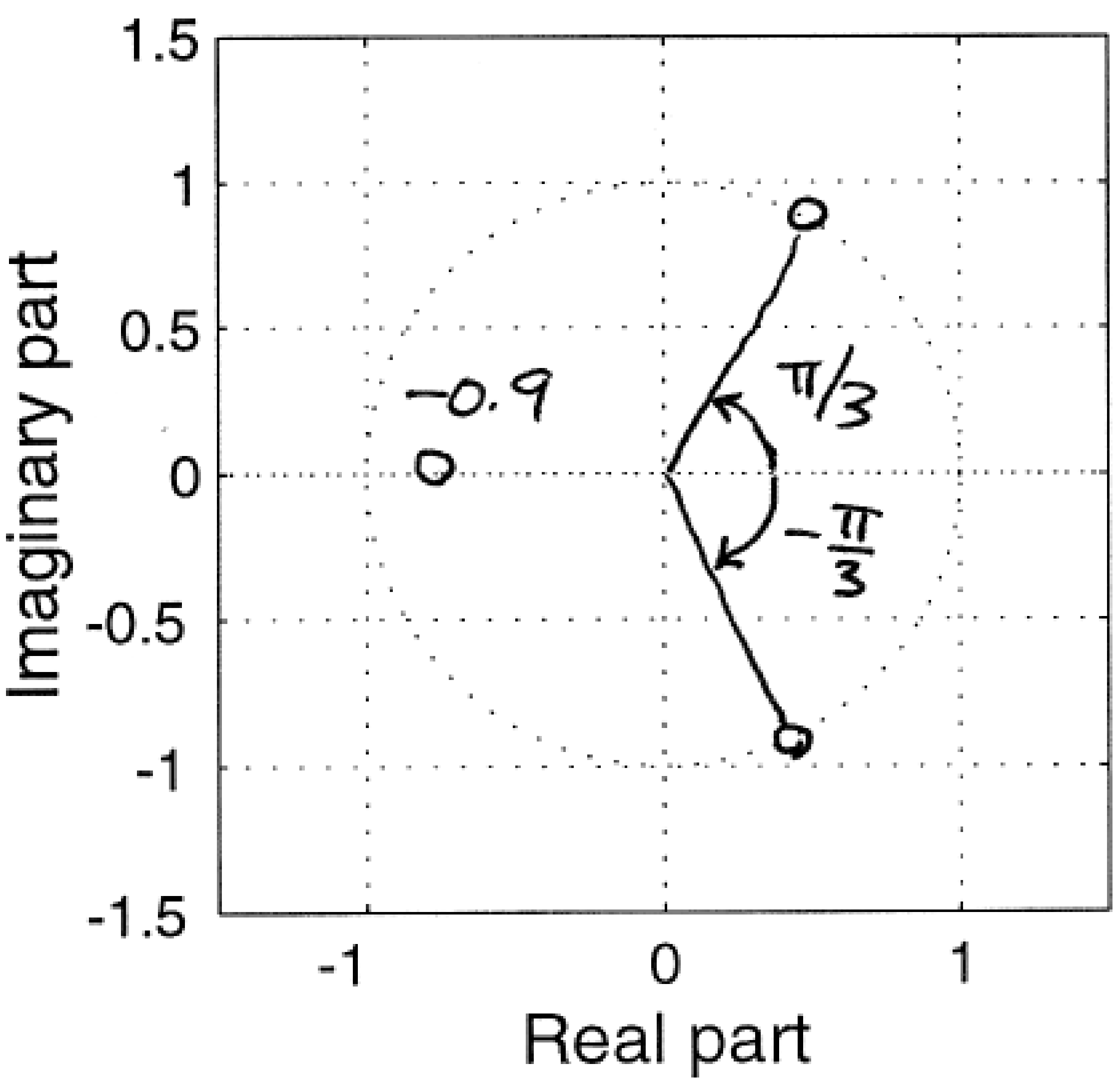
(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

$$H(z) = (1 + 0.9z^{-1})(1 - z^{-1} + z^{-2})$$

$$= 1 - 0.1z^{-1} + 0.1z^{-2} + 0.9z^{-3}$$

$$\boxed{y[n] = x[n] - 0.1x[n-1] + 0.1x[n-2] + 0.9x[n-3]}$$

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.



(c) If the input is of the form $x[n] = A\sin(\hat{\omega}_0 n + \phi)$, for what value of frequency $\hat{\omega}_0$ (in the range $0 < \hat{\omega}_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

$$y[n] = A|H(e^{j\hat{\omega}_0})|\sin(\hat{\omega}_0 n + \phi + \angle H(e^{j\hat{\omega}_0}))$$

FOR $y[n]$ TO BE ZERO, $|H(e^{j\omega_0})|$ SHOULD BE ZERO, WHICH REQUIRES A ZERO OF THE SYSTEM FN. ON THE UNIT CIRCLE AT THE APPROPRIATE FREQUENCY.

$$\boxed{\hat{\omega}_0 = \frac{\pi}{3} \text{ RAD.}}$$

Problem Fall-01-Q.3.2:
In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal. Provide some **explanation** or intermediate steps for each answer.

(a) $\displaystyle\int_{-\infty}^{t-7} \delta(\tau+1)\cos(\tau)\,d\tau = \cos(-1)\int_{-\infty}^{t-7}\delta(\tau+1)\,d\tau$: SIFTING PROPERTY

$= \cos(1)\, u(\tau+1)\Big|_{-\infty}^{t-7}$: $\delta = \dfrac{du}{dt}$

$= \cos(1)\, u(t-6)$: EVALUATION AT LIMITS

(b) $\dfrac{d}{dt}\{\sin(2t)u(t-2)\} = \sin 2t\, \dfrac{du(t-2)}{dt} + 2\cos 2t\, u(t-2)$: PRODUCT RULE

$= \sin 2t\, \delta(t-2) + 2\cos(2t)\, u(t-2)$: $\delta = \dfrac{du}{dt}$

$= \sin(4)\,\delta(t-2) + 2\cos(2t)\, u(t-2)$: SIFTING PROPERTY

Problem Fall-01-Q.3.3:
In each of the following cases, determine the Fourier transform, or inverse Fourier transform. Give your answer as a simple formula or a plot. **Explain** each answer by stating which property and transform pair you used.

(a) Find $X(j\omega)$ when $x(t) = e^{-3(t-2)}u(t-2)$.

$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega} \quad : \text{ BASIC XFORM}$$

$$e^{-3t}u(t) \longleftrightarrow \frac{1}{3+j\omega} \quad : \; a=3$$

$$e^{-3(t-2)}u(t-2) \longleftrightarrow \frac{e^{-j2\omega}}{3+j\omega} \quad : \text{ DELAY PROPERTY}$$

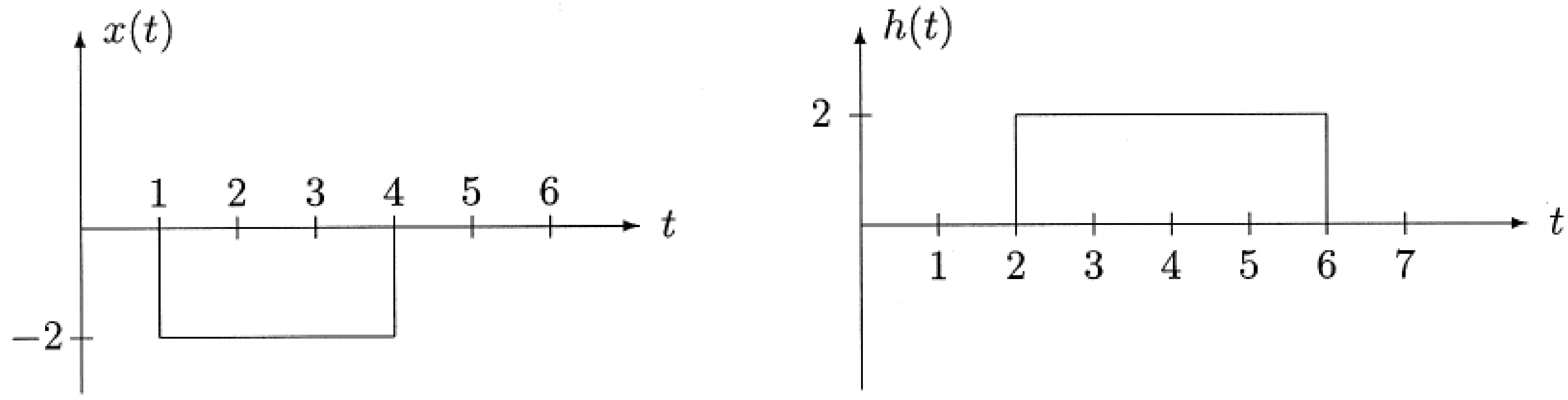
(b) Find $s(t)$ when $S(j\omega) = e^{-j\omega/2}[u(\omega+10\pi) - u(\omega-10\pi)]$.

$$\frac{\sin \omega_0 t}{\pi t} \longleftrightarrow u(\omega+\omega_0) - u(\omega-\omega_0) : \text{ BASIC XFORM}$$

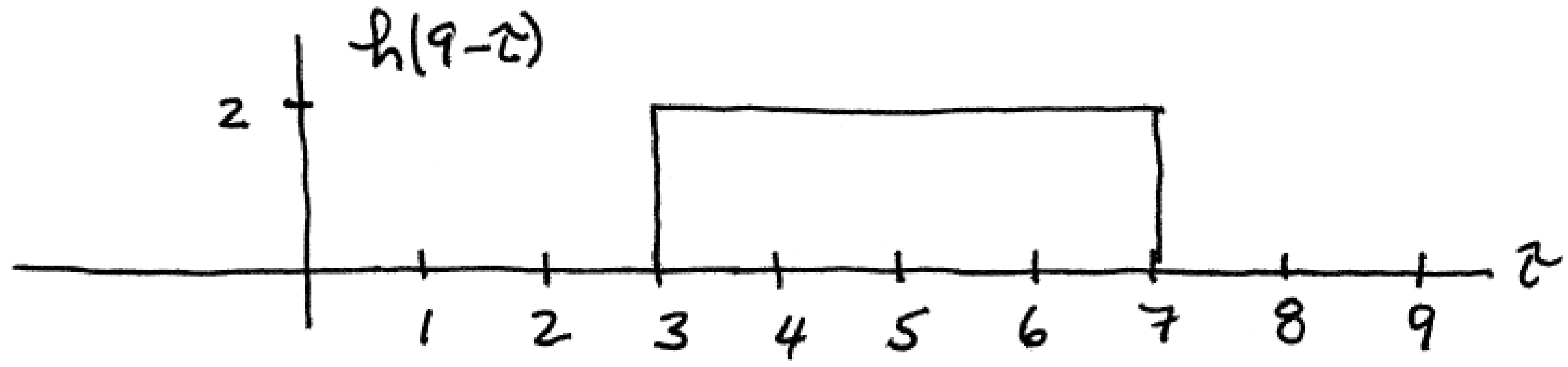
$$\frac{\sin 10\pi t}{\pi t} \longleftrightarrow u(\omega+10\pi) - u(\omega-10\pi) : \; \omega_0 = 10\pi$$

$$\boxed{\frac{\sin 10\pi(t-\frac{1}{2})}{\pi(t-\frac{1}{2})} \longleftrightarrow e^{-j\frac{\omega}{2}}\left[u(\omega+10\pi) - u(\omega-10\pi)\right] : \text{ SHIFT PROPERTY}}$$

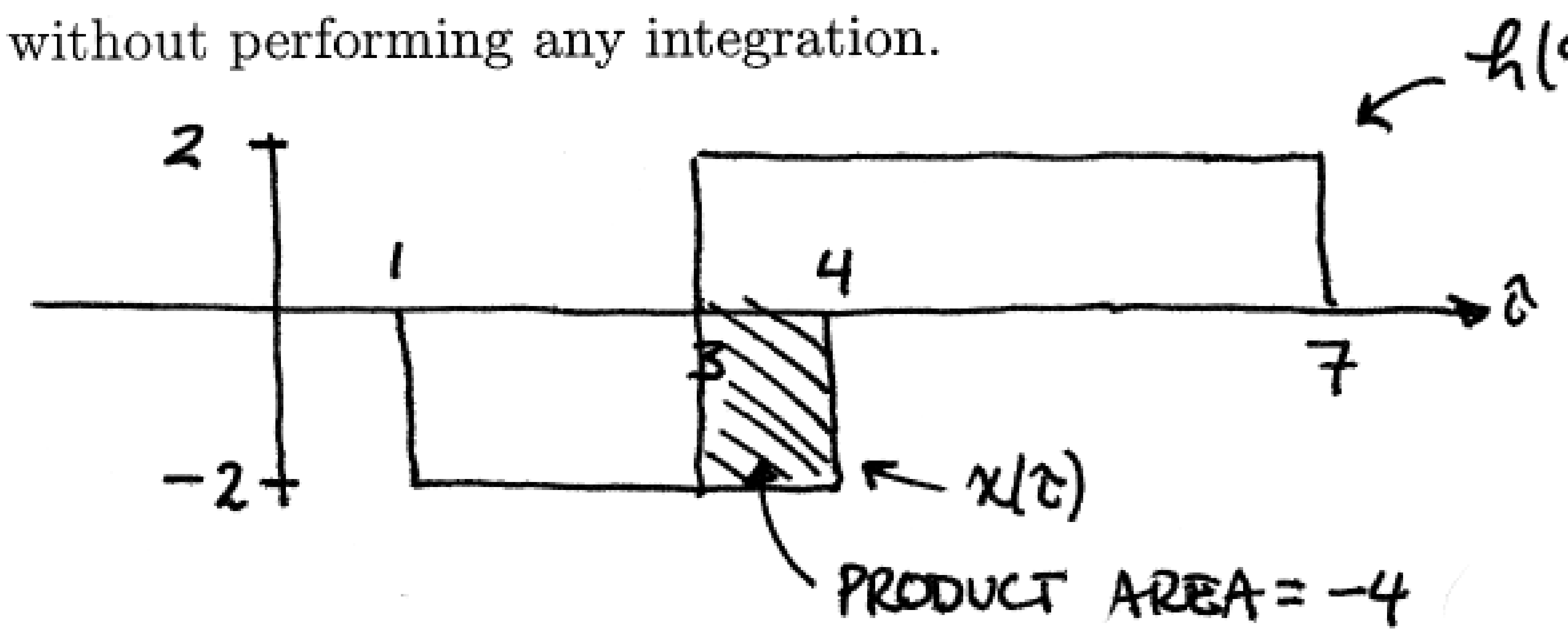
Problem Fall-01-Q.3.4:
The following figure shows the signal $x(t) = -2u(t-1)+2u(t-4)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 2u(t-2) - 2u(t-6)$.



(a) Sketch $h(9-\tau)$ as a function of τ in the space below.

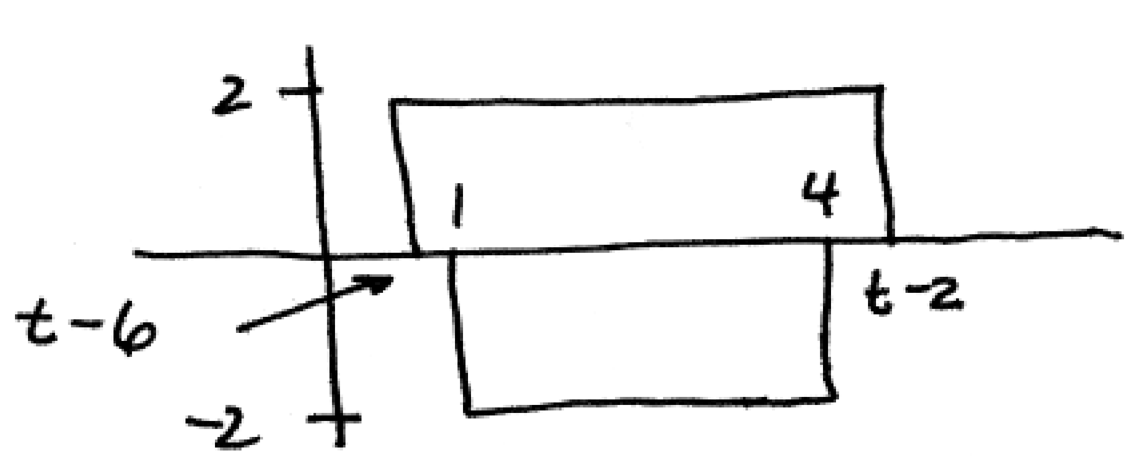


(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 9$; that is, determine $y(9)$. It is not necessary to evaluate $y(t)$ for all t, only for $t = 9$. Note: This problem may be answered without performing any integration.



$\therefore y(9) = -4$

(c) $y(t)$ reaches its minimum value for $T_1 \le t \le T_2$. Find the minimum value, y_{min} and also the values for T_1 and T_2.



$y_{min} =$ -12

$T_1 =$ 6 sec

$T_2 =$ 7 sec

MIN. VALUE ACHIEVED WITH COMPLETE OVERLAP

$\Rightarrow t-6 \le 1$ AND $t-2 \ge 4$

$\Rightarrow t \le 7$ AND $t \ge 6$

Problem Fall-01-Q.3.5:

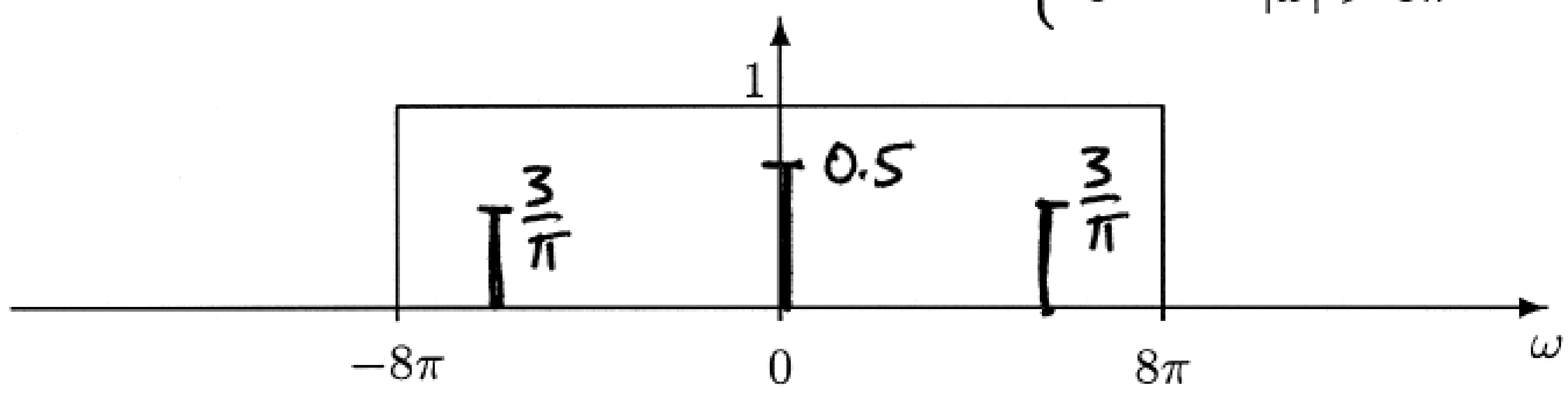
The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 1/3}$ seconds. The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} 0.5 & k = 0 \\ \dfrac{3\sin(\pi k/2)}{\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency ω_0 of the input signal $x(t)$? $\boxed{\omega_0 = \frac{2\pi}{1/3} = 6\pi \quad \text{rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 8\pi \\ 0 & |\omega| > 8\pi \end{cases}$$



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). **Justify your answer** by sketching the spectrum of $y(t)$ (or its Fourier transform).

$$y(t) = 0.5 + \frac{3}{\pi} e^{j6\pi t} + \frac{3}{\pi} e^{-j6\pi t}$$

$$\boxed{y(t) = 0.5 + \frac{6}{\pi}\cos(6\pi t)}$$

(c) Draw the spectrum of the output signal superimposed on the plot of $H(j\omega)$.