

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #1

DATE: 1-Feb-02

COURSE: ECE 2025

NAME: _____
 LAST, FIRST

STUDENT #: _____

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L02:Tues-9:30am (Bordelon) L04:Tues-12:00pm (Yezzi) L05:Thurs-1:30pm (Williams)

L06:Tues-1:30pm (Bordelon) L07:Thur-3:00pm (Williams) L08:Tues-3:00pm (Smith)

L11:Mon-3:00pm (Glytsis) L14:Mon-4:00pm (McClellan) RPK: (Abler)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes (front and back) and a calculator are permitted.
- **JUSTIFY** your reasoning clearly to receive partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	25	
2	25	
3	25	
4	25	

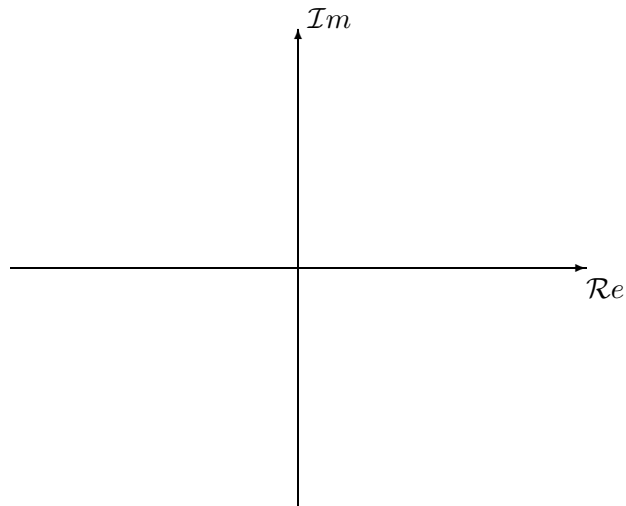
Problem SPRING-02-Q.1.1:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form.

Let $Y = \sqrt{3} - j$ and $Z = e^{j\pi/3}$.

- (a) If $A = Y + Z$, what is its numerical value expressed in rectangular form? **Plot the vectors Y , Z , and A in the complex plane.**

$A =$ -----



- (b) If $B = ZY^*$, what are the numerical values of the magnitude and phase associated with the polar form representation?

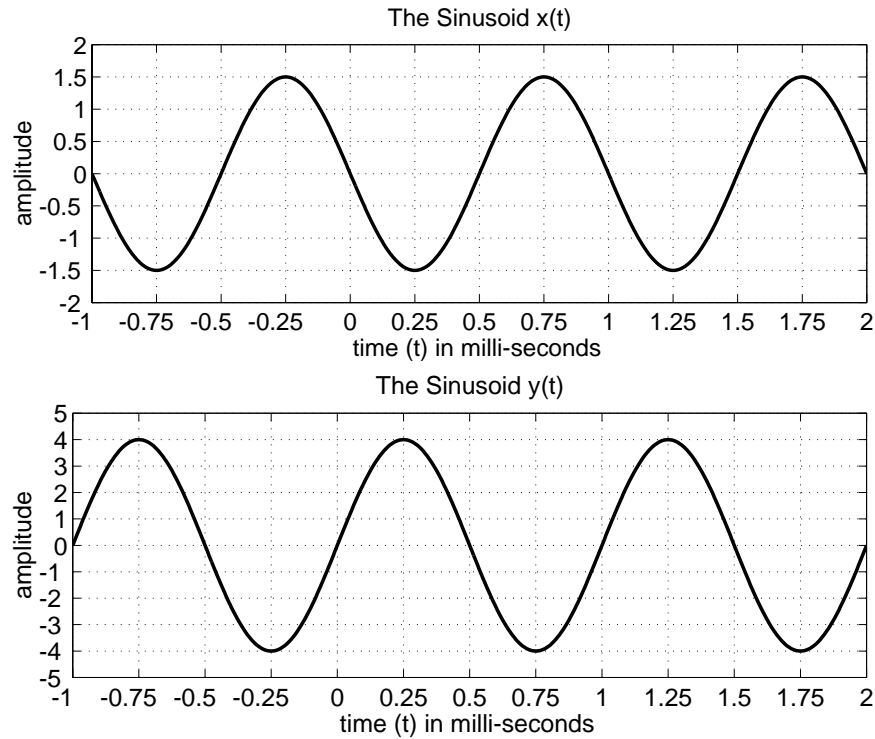
$|B| =$ -----, $\angle B =$ -----

- (c) If $C = (jZ)^{33}$, what is its numerical value expressed in rectangular form?

$C =$ -----

Problem SPRING-02-Q.1.2:

Consider the sinusoidal signals $x(t)$ and $y(t)$ plotted below.



- (a) Determine A , f_0 , and ϕ in the representation of $x(t)$ as $x(t) = A \cos(2\pi f_0 t + \phi)$.

$A =$ _____ $\phi =$ _____ $f_0 =$ _____ (in Hz)

- (b) Now suppose that $B \cos(\omega_0 t + \psi) = x(t) + 3 \cos(\omega_0 t - 5\pi/6)$. Determine B , ω_0 , and ψ .

$B =$ _____

$\omega_0 =$ _____

$\psi =$ _____

- (c) The signal $y(t)$ can be expressed in terms of $x(t)$. That is, we can write $x(t) = C y(t - t_1)$. Determine the numerical values of the scale factor C and the time shift t_1 , where $t_1 \geq 0$.

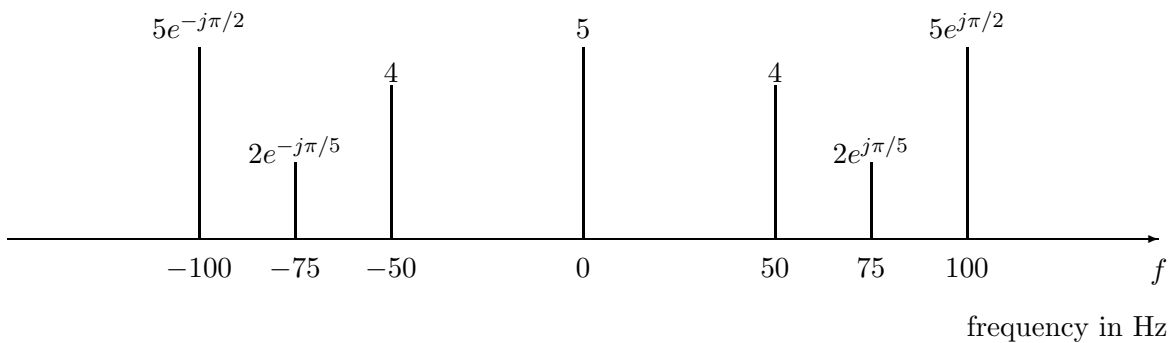
$C =$ _____ $t_1 =$ _____

Problem SPRING-02-Q.1.3:

A real signal

$$x(t) = A \cos(200\pi t + \phi) + B \cos(\omega_1(t - \tau)) + C \cos(\omega_2 t) + D$$

has the following two-sided spectrum:



- (a) Determine A , B , C , D , ω_1 , ω_2 , ϕ , and τ the signal $x(t)$ with the above spectrum.

$$A = \text{-----}$$

$$B = \text{-----}$$

$$C = \text{-----}$$

$$D = \text{-----}$$

$$\phi = \text{-----}$$

$$\omega_1 = \text{-----}$$

$$\omega_2 = \text{-----}$$

$$\tau = \text{-----}$$

- (b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0 , of the signal $x(t)$.

$$f_0 = \text{-----}$$

Problem SPRING-02-Q.1.4:

A signal $x(t)$ is given by the equation

$$x(t) = 4[A + \cos(300\pi t)] \cos(1000\pi t + \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^N D_k \cos(\omega_k t + \phi_k), \tag{1}$$

where the ω_k 's are different frequencies.

- (a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).

$N =$ _____

- (b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

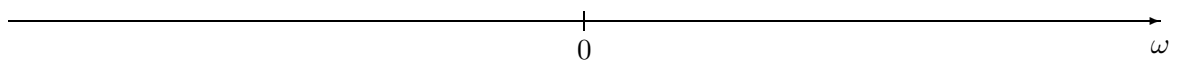
lowest $\omega_k =$ _____

highest $\omega_k =$ _____

- (c) The spectrum of $x(t)$ contains a component at frequency 1000π rad/sec with complex amplitude $4j$. What is the numerical value of A ?

$A =$ _____

- (d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.



frequency in rad/sec

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QUIZ #1

DATE: 1-Feb-02

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NAME: Answer Key
LAST, FIRST

STUDENT #: B

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Problem SPRING-02-Q.1.1:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form.

Let $Y = \sqrt{3} - j$ and $Z = e^{j\pi/3}$.

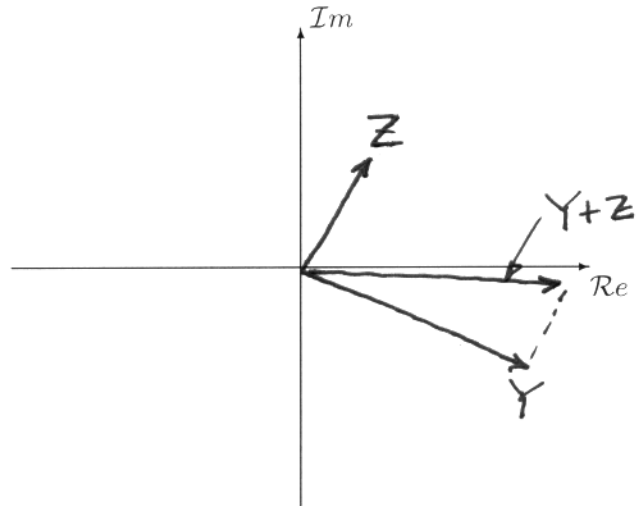
- (a) If $A = Y + Z$, what is its numerical value expressed in rectangular form? **Plot the vectors Y , Z , and A in the complex plane.**

$$A = \underline{2.232 - j0.134}$$

$$A = 2.236 e^{-j0.06}$$

-3.43°

$$Y = 2 e^{-j\pi/6}$$



- (b) If $B = ZY^*$, what are the numerical values of the magnitude and phase associated with the polar form representation?

$$|B| = \underline{2}, \quad \angle B = \underline{\pi/2} \text{ rads}$$

$$\begin{aligned} ZY^* &= (e^{j\pi/3})(2e^{-j\pi/6})^* = 2e^{j(\pi/3 + \pi/6)} \\ &= 2e^{j\pi/2} \end{aligned}$$

90°

- (c) If $C = (jZ)^{33}$, what is its numerical value expressed in rectangular form?

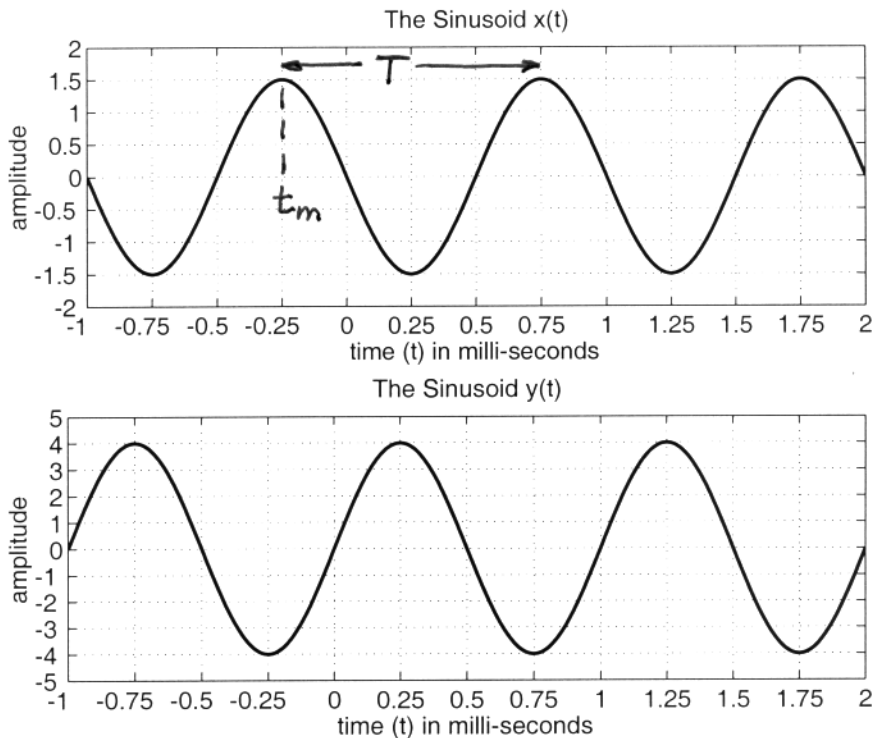
$$C = \underline{-j = 0 - j}$$

$$\begin{aligned} C &= (e^{j\pi/2} e^{j\pi/3})^{33} = (e^{j5\pi/6})^{33} = e^{j165\pi/6} \\ &\quad \swarrow e^{-j\pi/2} = 0 - j \end{aligned}$$

$$\frac{165\pi}{6} = 28\pi - \frac{3\pi}{6}$$

Problem SPRING-02-Q.1.2:

Consider the sinusoidal signals $x(t)$ and $y(t)$ plotted below.



- (a) Determine A , f_0 , and ϕ in the representation of $x(t)$ as $x(t) = A \cos(2\pi f_0 t + \phi)$.

$$A = \underline{1.5} \quad \phi = \underline{\pi/2} \quad f_0 = \underline{1000} \text{ (in Hz)}$$

$$t_m = -1/4 \text{ ms} \quad T = 1 \text{ ms} = 10^{-3}$$

$$\varphi = -\omega t_m = -2\pi(1000)(-1/4) \times 10^{-3} \quad f = 1/T = 1000 \text{ Hz}$$

- (b) Now suppose that $B \cos(\omega_0 t + \psi) = x(t) + 3 \cos(\omega_0 t - 5\pi/6)$. Determine B , ω_0 , and ψ .

$$B = \underline{2.598} \quad B = 1.5 e^{j\pi/2} + 3 e^{-j5\pi/6}$$

$$\omega_0 = \underline{2000\pi \text{ rad/s}} \quad = 2.598 e^{j\pi}$$

$$\psi = \underline{\pi \text{ rads}} \quad \omega_0 \text{ has to be the same for all}$$

↙ $\alpha - \pi$

sinusoids in the sum

$$\underline{2.598 \cos(2000\pi t + \pi)}$$

- (c) The signal $y(t)$ can be expressed in terms of $x(t)$. That is, we can write $x(t) = C y(t - t_1)$. Determine the numerical values of the scale factor C and the time shift t_1 , where $t_1 \geq 0$.

$$C = \underline{0.375} \quad t_1 = \underline{0.5 \text{ msec}}$$

$$C = 1.5/4 = 3/8 \quad t_1 = 0.75 - 0.25 = 0.5 \text{ msec}$$

$$\text{If } y(t) = D x(t - t_2), \text{ then } D = 4/1.5 = 8/3$$

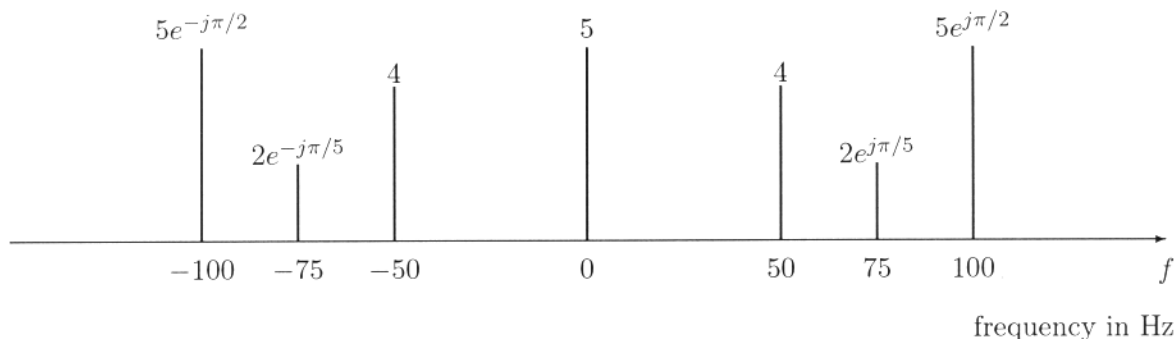
$$t_2 = 0.25 - (-0.25) = 0.5 \text{ msec}$$

Problem SPRING-02-Q.1.3:

A real signal

$$x(t) = A \cos(200\pi t + \phi) + B \cos(\omega_1(t - \tau)) + C \cos(\omega_2 t) + D$$

has the following two-sided spectrum:

(a) Determine A , B , C , D , ω_1 , ω_2 , ϕ , and τ the signal $x(t)$ with the above spectrum.

$A = \underline{10}$

At 200π , $f = 100$ Hz

$B = \underline{4}$

$A = 10$ $\phi = \pi/2$

$C = \underline{8}$

Zero phase at 50 Hz

$D = \underline{5}$

$\omega_2 = 2\pi(50) = 100\pi$ rad/s

$C = 8$

$\phi = \underline{\pi/2}$ rads

Last frequency is 75 Hz

$\omega_1 = \underline{150\pi}$ rad/s

$\omega_1 = 2\pi(75) = 150\pi$ rad/s

$\omega_2 = \underline{100\pi}$ rad/s

$B = 4$

$\tau = \underline{-0.001333...}$ sec

$\psi = \pi/5$ $\tau = -\psi/\omega_1$

$= \underline{-1/750}$

$\tau = \frac{-\pi/5}{150\pi} = \underline{-\frac{1}{750}}$

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0 , of the signal $x(t)$.

$f_0 = \underline{25}$ Hz

GCD $\{0, 50, 75, 100\}$

Problem SPRING-02-Q.1.4:

A signal $x(t)$ is given by the equation

$$x(t) = 4[A + \cos(300\pi t)] \cos(1000\pi t + \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^N D_k \cos(\omega_k t + \phi_k), \quad (1)$$

where the ω_k 's are different frequencies.

- (a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).

$$N = \underline{\underline{3}}$$

- (b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

$$\text{lowest } \omega_k = \underline{\underline{700\pi \text{ rad/s}}}$$

$$\text{highest } \omega_k = \underline{\underline{1300\pi \text{ rad/s}}}$$

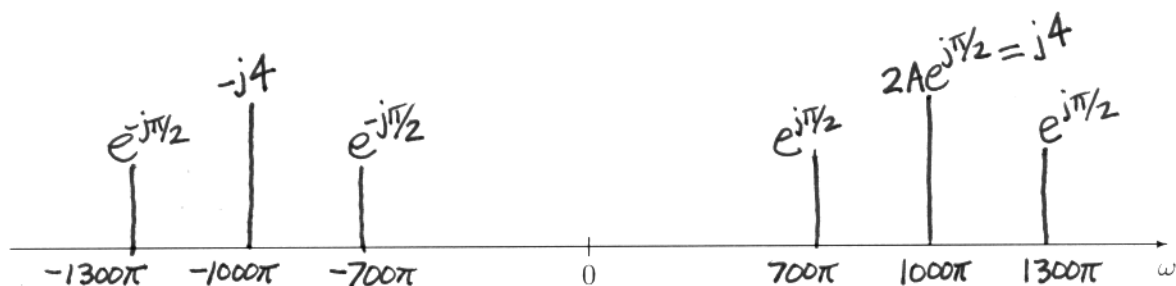
- (c) The spectrum of $x(t)$ contains a component at frequency 1000π rad/sec with complex amplitude $4j$. What is the numerical value of A ?

$$A = \underline{\underline{2}}$$

$$j2A = j4 \Rightarrow A = 2$$

- (d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.

$$x(t) = [2A + e^{j300\pi t} + e^{-j300\pi t}] \cdot [e^{j\pi/2} e^{j1000\pi t} + e^{-j\pi/2} e^{-j1000\pi t}]$$



frequency in rad/sec