GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 1-Feb-02 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ______________

LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L02:Tues-9:30am (Bordelon)	L04:Tues-12:00pm (Yezzi)	L05:Thurs-1:30pm (Williams)
L06:Tues-1:30pm (Bordelon)	L07:Thur-3:00pm (Williams)	L08:Tues-3:00pm (Smith)
L11:Mon-3:00pm (Glytsis)	L14:Mon-4:00pm (McClellan)	RPK: (Abler)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning CLEARLY to receive partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

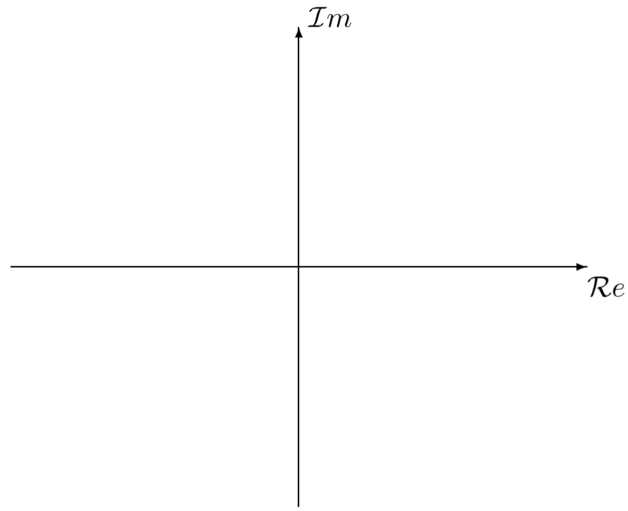
Problem Spring-02-Q.1.1:

Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form.

Let $\quad Y = 1 + j\sqrt{3} \quad$ and $\quad Z = e^{-j\pi/6}$.

(a) If $A = Y + Z$, what is its numerical value expressed in rectangular form? **Plot the vectors Y, Z, and A in the complex plane.**

$A =$ ______________________________



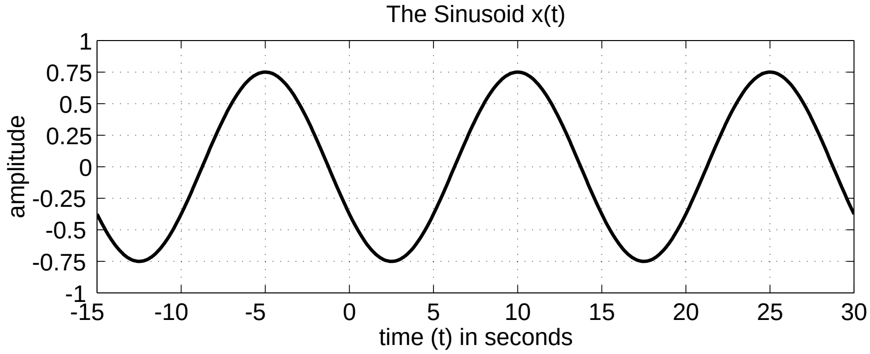
(b) If $B = ZY^*$, what are the numerical values of the magnitude and phase associated with the polar form representation?

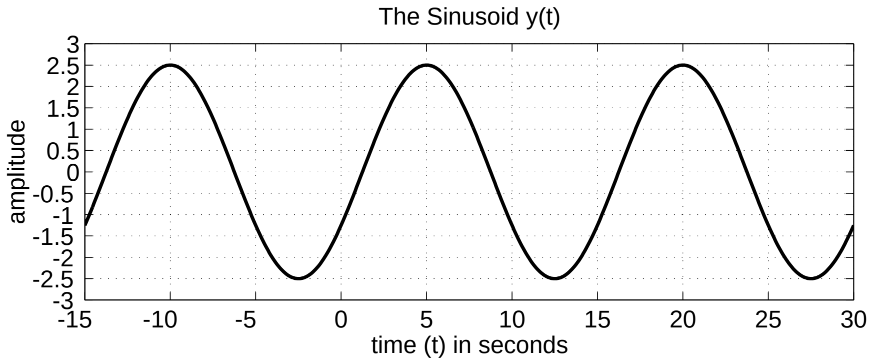
$|B| =$ ______________________________, $\quad \angle B =$ ______________________________

(c) If $C = (jZ)^{66}$, what is its numerical value expressed in rectangular form?

$C =$ ______________________________

Problem Spring-02-Q.1.2:
Consider the sinusoidal signals $x(t)$ and $y(t)$ plotted below.





(a) Determine A, f_0, and ϕ in the representation of $x(t)$ as $x(t) = A\cos(2\pi f_0 t + \phi)$.

$A =$______________. $\phi =$______________. $f_0 =$______________. (in Hz)

(b) Now suppose that $B\cos(\omega_0 t + \psi) = x(t) + 0.375\cos(\omega_0 t)$. Determine B, ω_0, and ψ.

$B =$______________

$\omega_0 =$______________

$\psi =$______________

(c) The signal $y(t)$ can be expressed in terms of $x(t)$. That is, we can write $x(t) = Cy(t - t_1)$. Determine the numerical values of the scale factor C and the time shift t_1, where $t_1 \geq 0$.

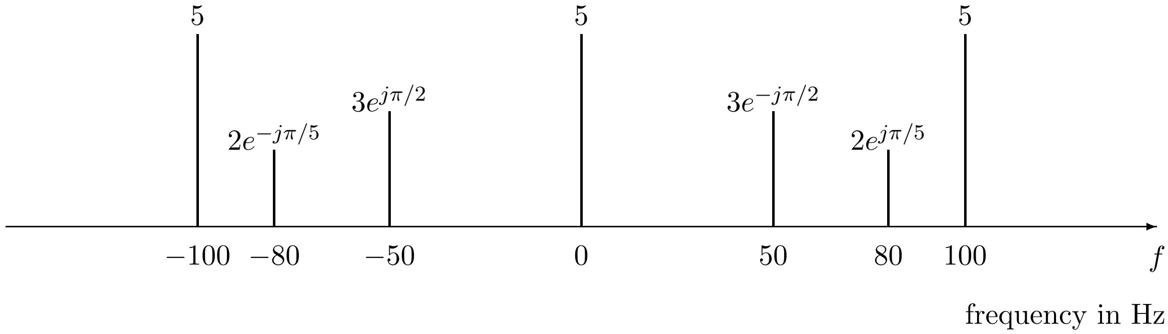
$C =$______________ $t_1 =$______________

Problem Spring-02-Q.1.3:

A real signal

$$x(t) = A\cos(160\pi t + \phi) + B\cos(\omega_1(t - \tau)) + C\cos(\omega_2 t) + D$$

has the following two-sided spectrum:



(a) Determine A, B, C, D, ω_1, ω_2, ϕ, and τ the signal $x(t)$ with the above spectrum.

$A =$____________

$B =$____________

$C =$____________

$D =$____________

$\phi =$____________

$\omega_1 =$____________

$\omega_2 =$____________

$\tau =$____________

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0, of the signal $x(t)$.

$f_0 =$____________

Problem Spring-02-Q.1.4:

A signal $x(t)$ is given by the equation

$$x(t) = 2[A + \cos(200\pi t)]\cos(2000\pi t + \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k), \qquad (1)$$

where the ω_k's are different freqeuncies.

(a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).

$N =$ ________________

(b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

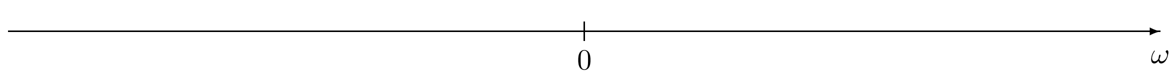
lowest $\omega_k =$ ________________

highest $\omega_k =$ ________________

(c) The spectrum of $x(t)$ contains a component at frequency 2000π rad/sec with complex amplitude $6j$. What is the numerical value of A?

$A =$________________

(d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.



frequency in rad/sec

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Problem Spring-02-Q.1.1:
Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form.

Let $Y = 1 + j\sqrt{3}$ and $Z = e^{-j\pi/6}$.

(a) If $A = Y + Z$, what is its numerical value expressed in rectangular form? **Plot the vectors Y, Z, and A in the complex plane.**

$A =$ $1.866 + j1.232$

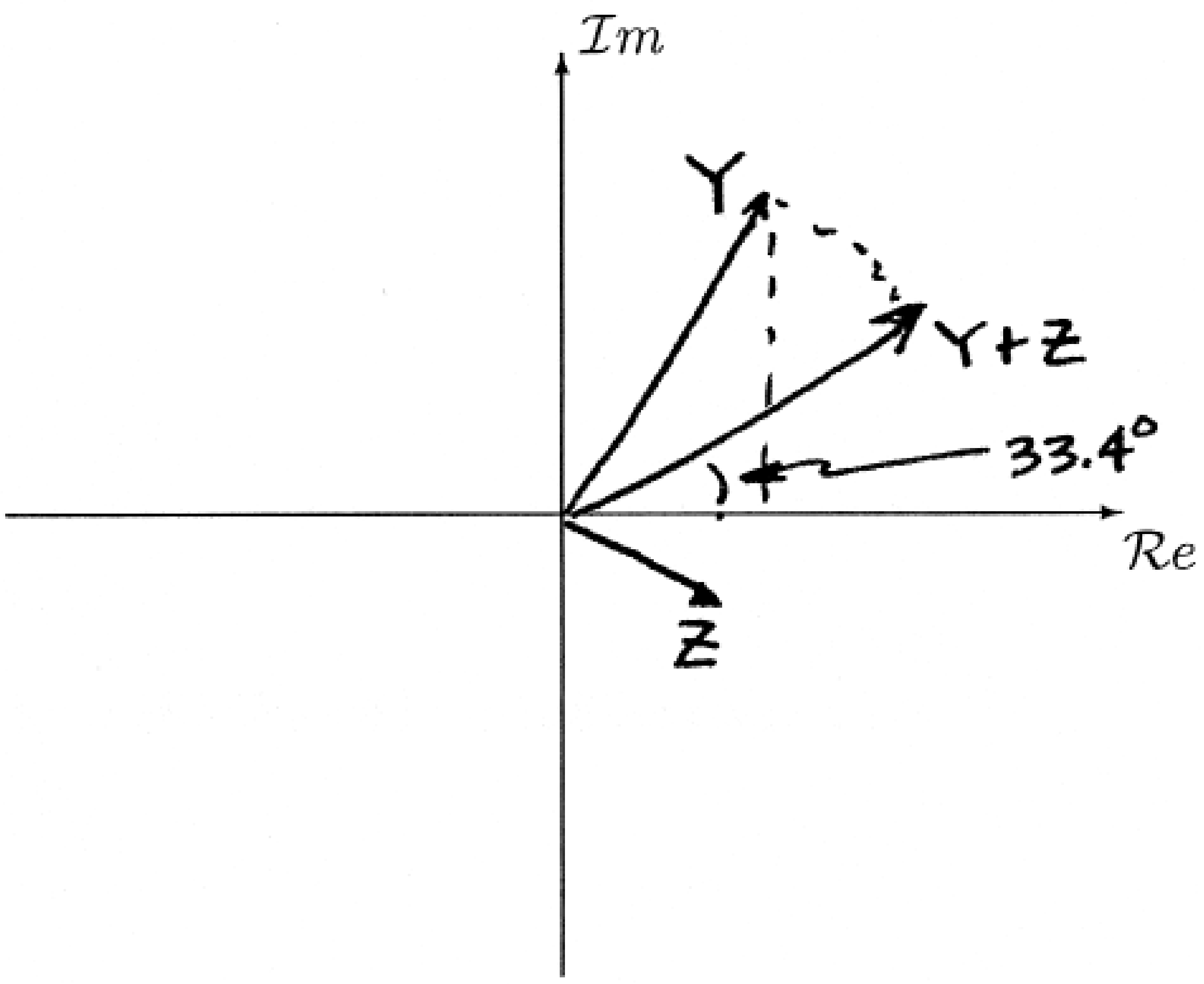
$Z = \frac{\sqrt{3}}{2} - j\frac{1}{2}$

$Y + Z = \left(1 + \frac{\sqrt{3}}{2}\right) + j\left(\sqrt{3} - \frac{1}{2}\right)$

$= 1.866 + j1.232$

$= 2.236\, e^{j0.584}$

0.584 rads $= 33.43°$



(b) If $B = ZY^*$, what are the numerical values of the magnitude and phase associated with the polar form representation?

$|B| =$ 2, $\angle B =$ $-\pi/2$ radians

$Y = 2e^{j\pi/3}$ $B = e^{-j\pi/6}\left(2e^{-j\pi/3}\right) = 2e^{-j\pi/2}$

(c) If $C = (jZ)^{66}$, what is its numerical value expressed in rectangular form?

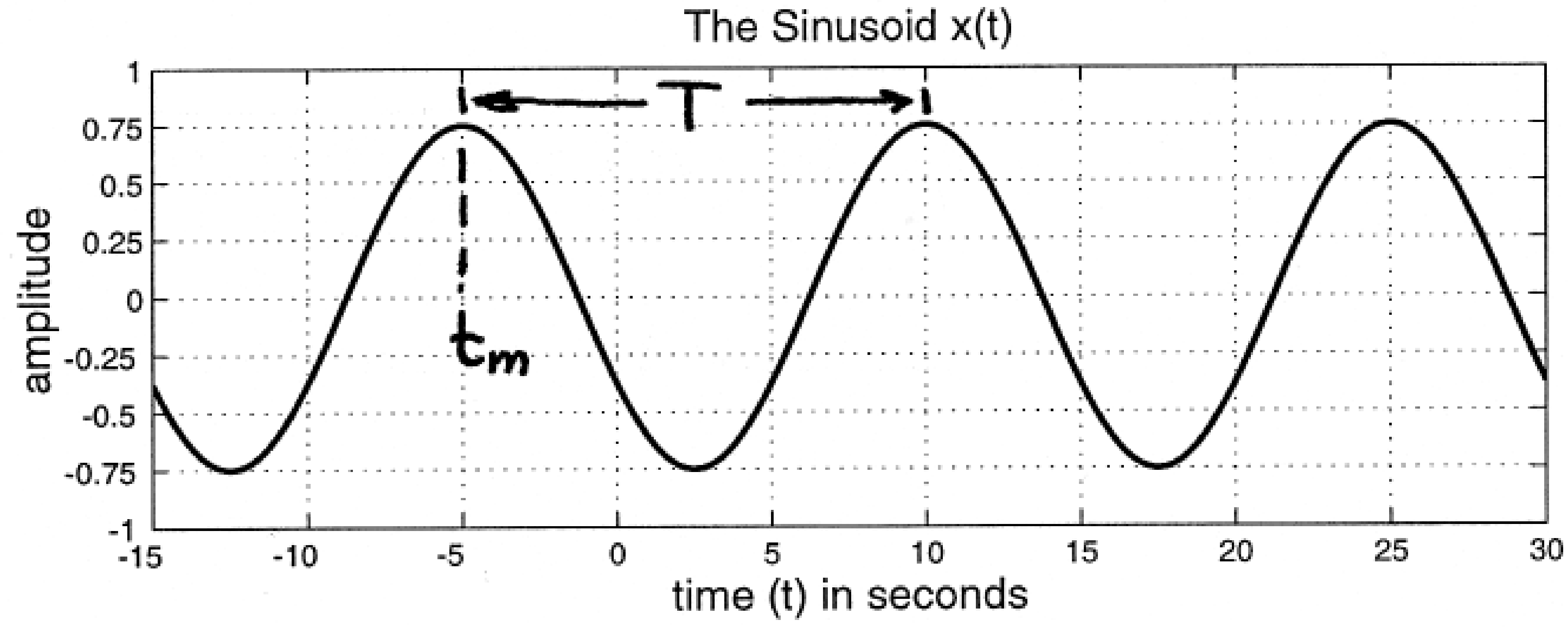
$C =$ $1 + j0$

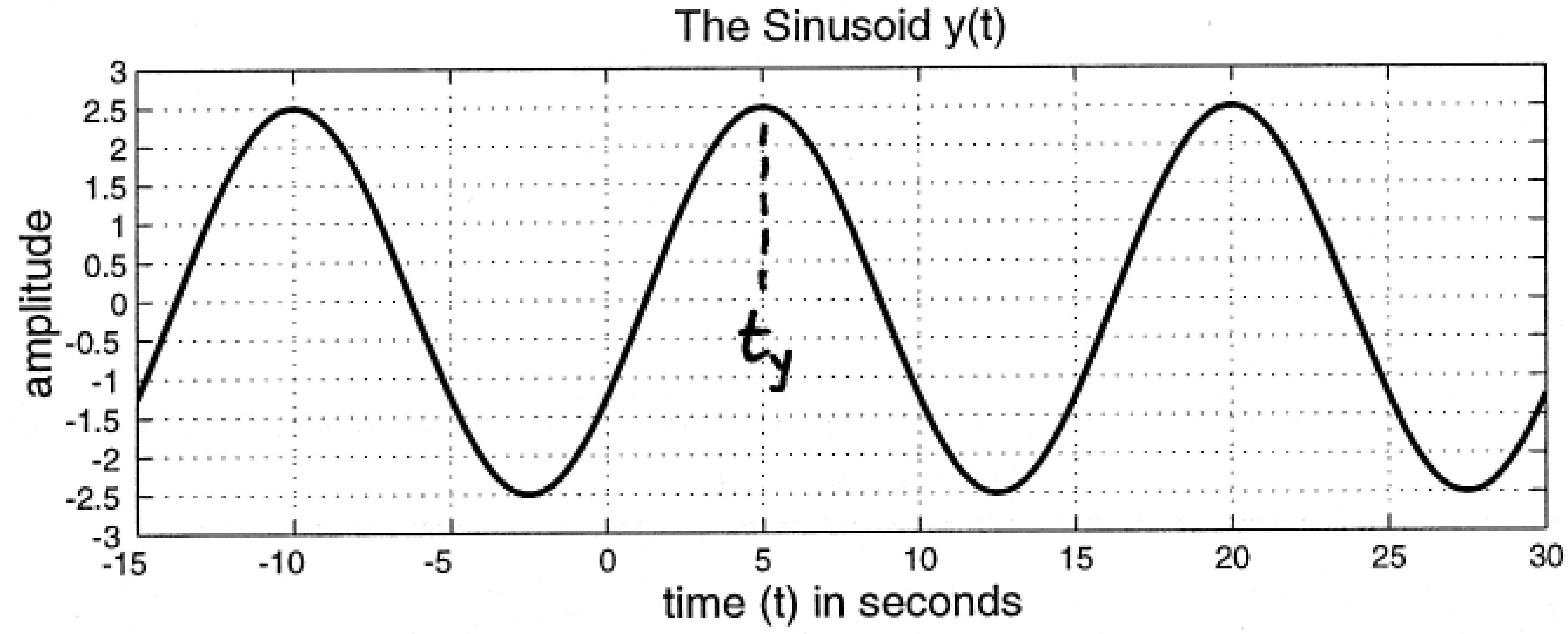
$je^{-j\pi/6} = e^{j\pi/2}e^{-j\pi/6} = e^{j\pi/3}$

$C = \left(e^{j\pi/3}\right)^{66} = e^{j66\pi/3} = e^{j22\pi} = 1$

Problem Spring-02-Q.1.2:

Consider the sinusoidal signals plotted below.





(a) Determine A, f_0, and ϕ in the representation of $x(t)$ as $x(t) = A\cos(2\pi f_0 t + \phi)$.

$A =$ 0.75 $\qquad \phi =$ $2\pi/3$ rads $\qquad f_0 =$ $1/15$ (in Hz)

Use max value $\qquad t_m = -5$ sec $\qquad T = 15s \Rightarrow f_0 = \frac{1}{T} = \frac{1}{15}$ Hz

$\varphi = -\omega t_m = -\frac{2\pi}{15}(-5)$

(b) Now suppose that $A\cos(\omega_0 t + \phi) = x(t) + 0.375\cos(\omega_0 t)$. Determine A, ω_0, and ϕ.

$A =$ 0.6495 $\qquad$ Add complex amps:

$\omega_0 =$ $2\pi/15$ rad/s $\qquad 0.75e^{j2\pi/3} + 0.375e^{j0}$

$\phi =$ $\pi/2$ rads $\qquad = 0.6495e^{j\pi/2}$

$\pi/2$ rads $= 1.571$ rads

Frequencies of sinusoids must be the same

(c) The signal $y(t)$ can be expressed in terms of $x(t)$. That is, we can write $x(t) = Cy(t - t_1)$. Determine the numerical values of the scale factor C and the time shift t_1, where $t_1 \geq 0$.

$C =$ $3/10$ $\qquad t_1 =$ 5 sec

$C = 0.75/2.5 \qquad t_1 = 10 - 5 = 5$ sec

If $y(t) = Dx(t - t_2)$, then $D = 2.5/0.75 = 10/3$

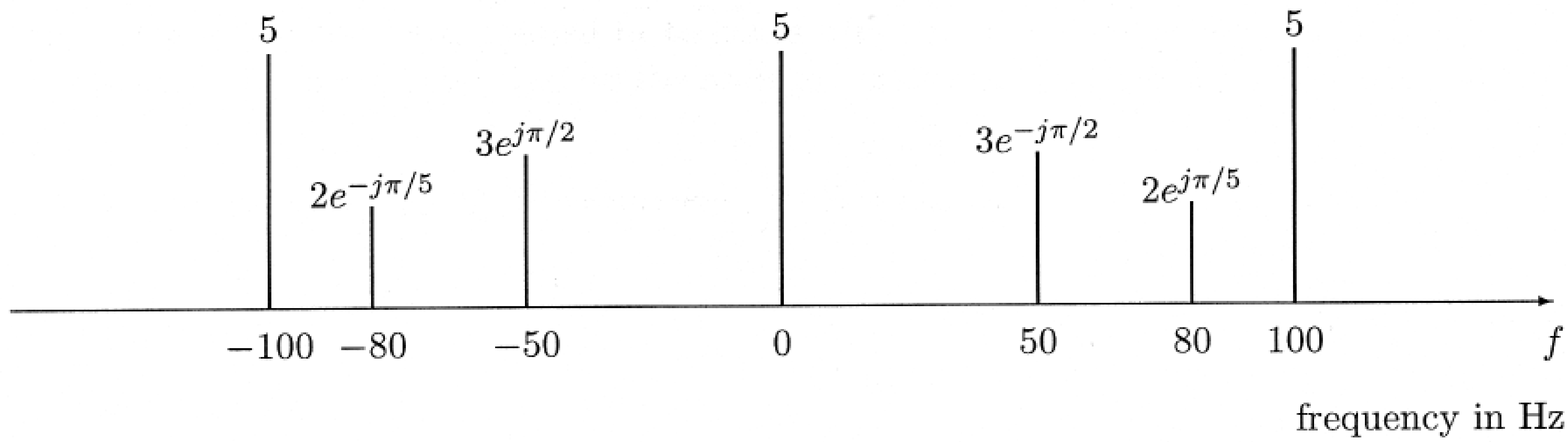
$t_2 = 5 - (-5) = 10$ sec

Problem Spring-02-Q.1.3:

A real signal

$$x(t) = A\cos(160\pi t + \phi) + B\cos(\omega_1(t - \tau)) + C\cos(\omega_2 t) + D$$

has the following two-sided spectrum:



(a) Determine A, B, C, D, ω_1, ω_2, ϕ, and τ the signal $x(t)$ with the above spectrum.

$A =$ 4

$B =$ 6

$C =$ 10

$D =$ 5

$\phi =$ $\pi/5$ rads

$\omega_1 =$ 100π rad/s

$\omega_2 =$ 200π rad/s

$\tau =$ 1/200 sec

- The DC is 5 $\Rightarrow$ $D = 5$
- 160π rad/s $\rightarrow$ 80 Hz
  Complex amp $= 2e^{j\pi/5}$ at $f = 80$Hz
  $\Rightarrow A = 4 \quad \varphi = \pi/5$
- 100 Hz component has zero phase
  $\Rightarrow C = 10 \qquad \omega_2 = 2\pi(100)$
- Remaing lines at $\pm$ 50Hz form
  $B\cos(\omega_1(t-\tau)) \qquad \Rightarrow \omega_1 = 2\pi(50) = 100\pi$
  Complex amp $= 3e^{-j\pi/2}$
  $\Rightarrow B = 6 \qquad$ phase $= -\pi/2$
  convert to time delay
  $-\frac{\pi}{2} = -\omega_1\tau = -2\pi(50)\tau$
  $\tau = \frac{1}{200}$ sec.

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0, of the signal $x(t)$.

$f_0 =$ 10 Hz $\qquad$ $GCD\{50, 80, 100\} = 10$

Problem Spring-02-Q.1.4:

A signal $x(t)$ is given by the equation

$$x(t) = 2[A + \cos(200\pi t)]\cos(2000\pi t + \pi/2).$$

The signal $x(t)$, which is given as a *product*, can be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k), \tag{1}$$

where the ω_ks are different freqeuncies.

(a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).

$N =$ 3

(b) What are the lowest and highest frequencies of all the sinusoids in the sum form of $x(t)$?

lowest $\omega_k =$ 1800π rad/s

highest $\omega_k =$ 2200π rad/s

As a sum of sinusoids:

$$x(t) = \cos(1800\pi t + \pi/2) + 2A\cos(2000\pi t + \pi/2) + \cos(2200\pi t + \pi/2)$$

(c) The spectrum of $x(t)$ contains a component at frequency 2000π rad/sec with complex amplitude $6j$. What is the numerical value of A?

$A =$ 6

$A e^{j\pi/2} = 6j \Rightarrow A = 6$

NOTE: $j = e^{j\pi/2}$

(d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.

$$x(t) = [A + \tfrac{1}{2}e^{j200\pi t} + \tfrac{1}{2}e^{-j200\pi t}] \cdot [e^{j2000\pi t}e^{j\pi/2} + e^{-j2000\pi t}e^{-j\pi/2}]$$

When you multiply out all the terms, you get 6 complex exps.