GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #3

DATE: 5-April-02 COURSE: ECE 2025

NAME: ______________________________ GT #: gt________________
LAST, First

Recitation Section: **CIRCLE THE DAY & TIME** when your Recitation Section meets:

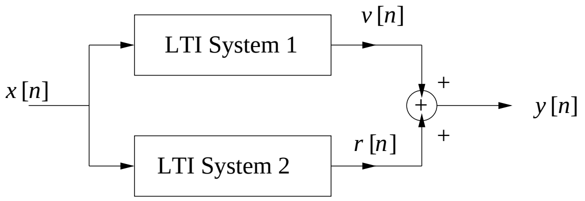
L02:Tues-9:30am (Bordelon)	L04:Tues-12:00pm (Yezzi)	L05:Thurs-1:30pm (Williams)
L06:Tues-1:30pm (Bordelon)	L07:Thur-3:00pm (Williams)	L08:Tues-3:00pm (Smith
L11:Mon-3:00pm (Glytsis)	L14:Mon-4:00pm (McClellan)	RPK: (Abler) Vald: (Fares)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2} \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your ANSWERS, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem 1: (20%)

Consider the parallel form LTI system depicted below.

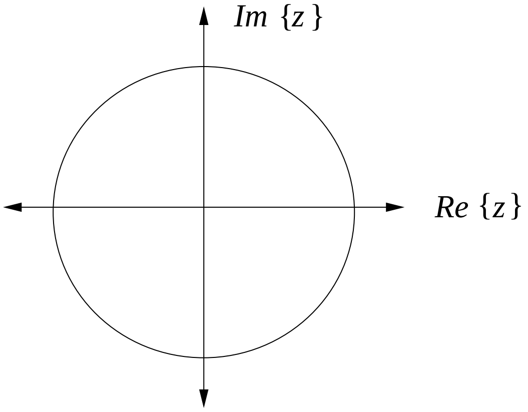


System 1 is defined by the difference equation $v[n] = x[n] - x[n-7]$.

System 2 is defined by the system function $H_2(z) = -1 + \frac{1}{2}z^{-3} + z^{-4}$.

(a) Determine the system function $H_1(z)$ associated with System 1 and plot the zeros of $H_1(z)$.

$H_1(z) =$



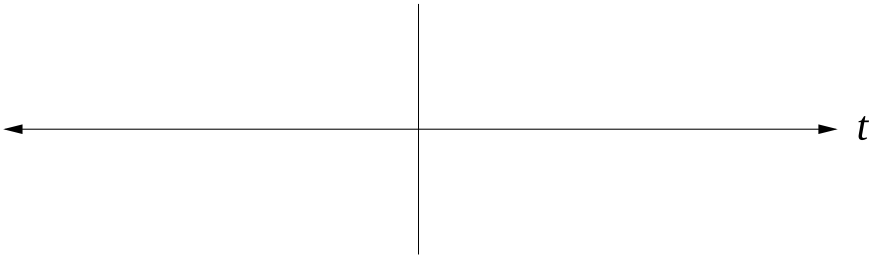
(b) Determine the impulse response of the overall parallel form system. That is, find $h[n]$ such that $y[n] = x[n] * h[n]$.

$h[n] =$

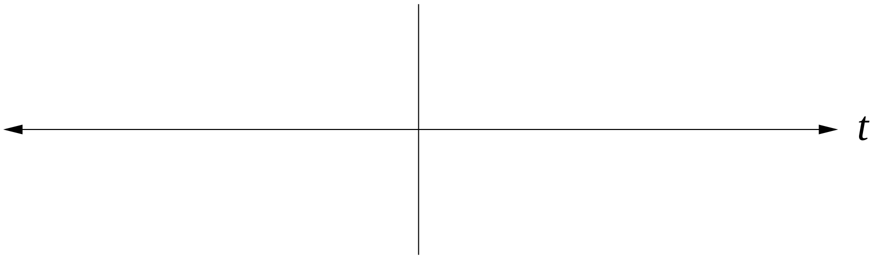
Problem 2: (20%)

Let $x(t) = \frac{1}{2}u(t) - \frac{1}{2}u(t-5)$.

(a) Sketch $\frac{d}{dt}x(t)$. Carefully label your plot.



(b) Sketch $x(3-t)$. Carefully label your plot.



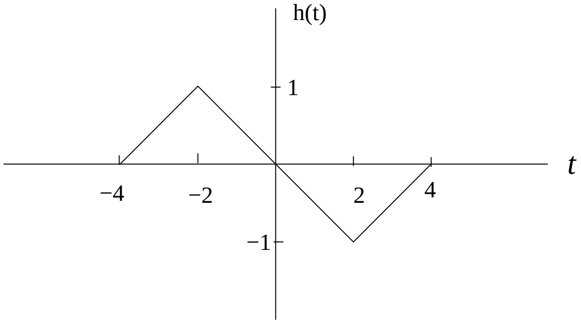
(c) If $x(t) * x(t-4) * \delta(t-T) = x(t-6) * x(t-3)$, determine the numerical value of T.

$T =$

Problem 3: (20%)

Consider the LTI System whose output is $y(t) = x(t) * h(t)$, where $x(t) = u(t)$

and $h(t)$ is given by



(a) Determine $y(-2)$, the value of $y(t)$ at $t = -2$.

$y(-2) =$

(b) You should be able to see that $y(t) = 0$ in two regions: $T_1 \leq t \leq T_2$ and $T_3 \leq t \leq T_4$. Determine T_1, T_2, T_3, and T_4. **Explain carefully to receive full credit.**

$T_1 =$ ____________, $T_2 =$ ____________, $T_3 =$ ____________, $T_4 =$ ____________

(c) Is this system stable? yes or no (Circle one)

Problem 4: (20%)

Let $h(t) = -\delta(t+2) + 4\delta(t) - \delta(t-2)$.

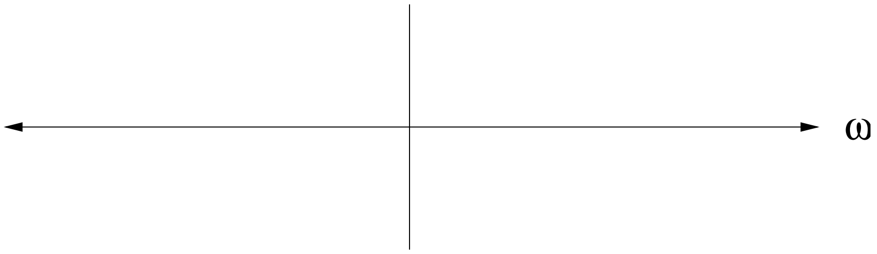
(a) Find $H(j\omega)$.

$H(j\omega) =$

(b) Let $y(t) = h(t-7)$. Find the phase of $Y(j\omega)$, the Fourier transform of $y(t)$.

$\angle Y(j\omega) =$

(c) If $x(t) = \dfrac{\sin 60\pi t}{\pi t} - \dfrac{2\sin 20\pi t}{\pi t}$, plot $X(j\omega)$. Carefully label your plot.



Problem 5: (20%)

Assume that $x(t)$ is the periodic function given by

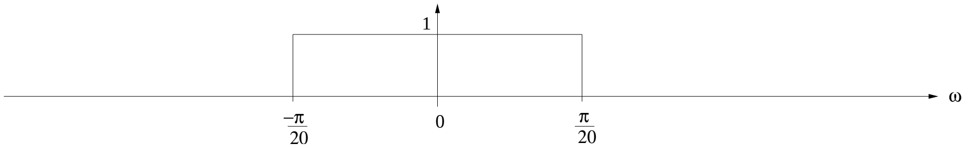
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-55k) = \sum_{k=-\infty}^{\infty} \frac{1}{55} e^{j\omega_0 kt}.$$

(a) Determine the value of the fundamental frequency ω_0.

$\omega_0 =$

(b) Suppose that $x(t)$ is the input to an LTI system with the frequency response illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| \leq \dfrac{\pi}{20} \\ 0 & |\omega| > \dfrac{\pi}{20} \end{cases}$$

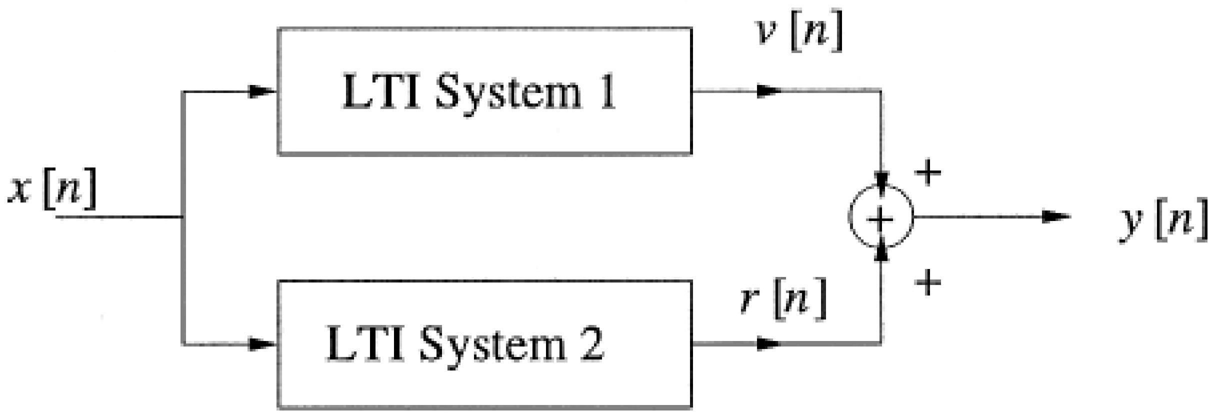


Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. Your answer should be expressed in terms of only real quantities. (Hint: Plot the spectrum of $x(t)$ on the plot of the frequency response.)

$y(t) =$

Problem 1: (20%)

Consider the parallel form LTI system depicted below.



System 1 is defined by the difference equation $v[n] = x[n] - x[n-7]$.

System 2 is defined by the system function $H_2(z) = -1 + \frac{1}{2}z^{-3} + z^{-4}$.

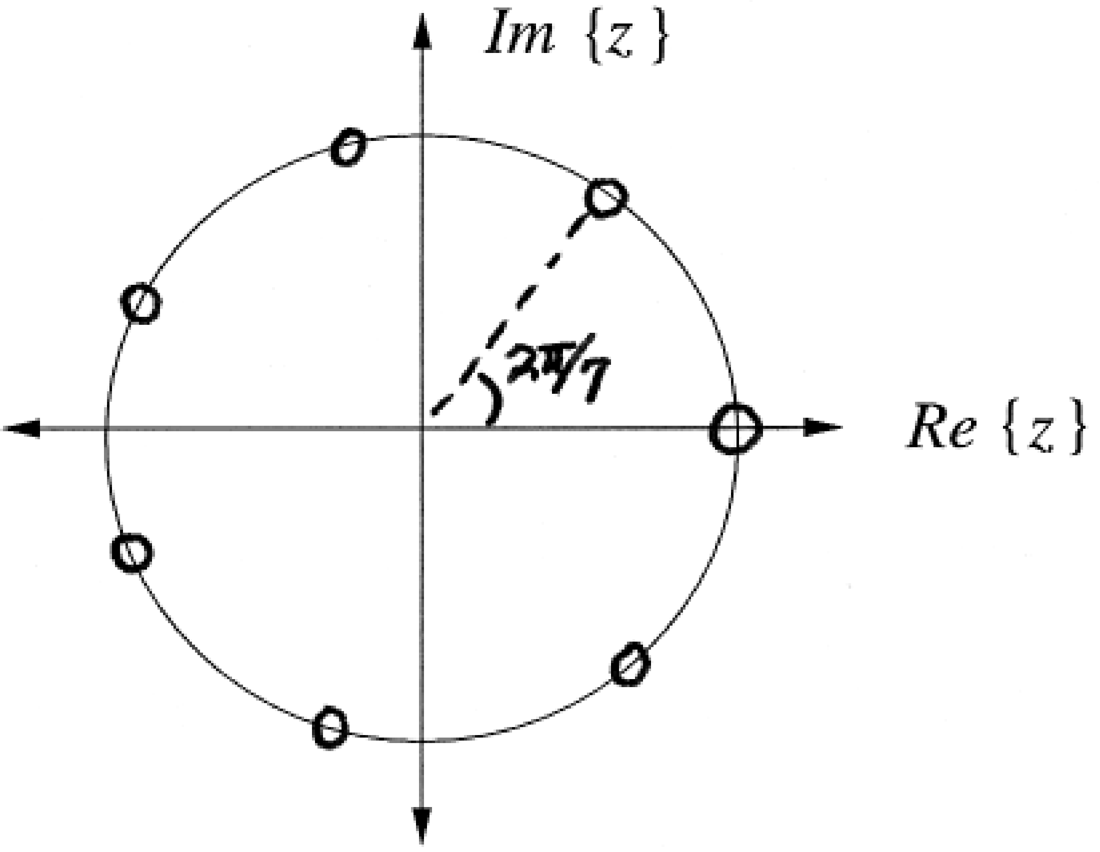
(a) Determine the system function $H_1(z)$ associated with System 1 and plot the zeros of $H_1(z)$.

$H_1(z) = 1 - z^{-7}$

Zeros of $H_1(z)$:

$$1 - z^{-7} = 0$$
$$z^7 - 1 = 0$$
$$z^7 = 1 = e^{j2\pi k}$$
$$z = e^{j2\pi k/7}$$
$$k = 0, 1, 2, 3, 4, 5, 6$$

Equally spaced around the unit circle.



(b) Determine the impulse response of the overall parallel form system. That is, find $h[n]$ such that $y[n] = x[n] * h[n]$.

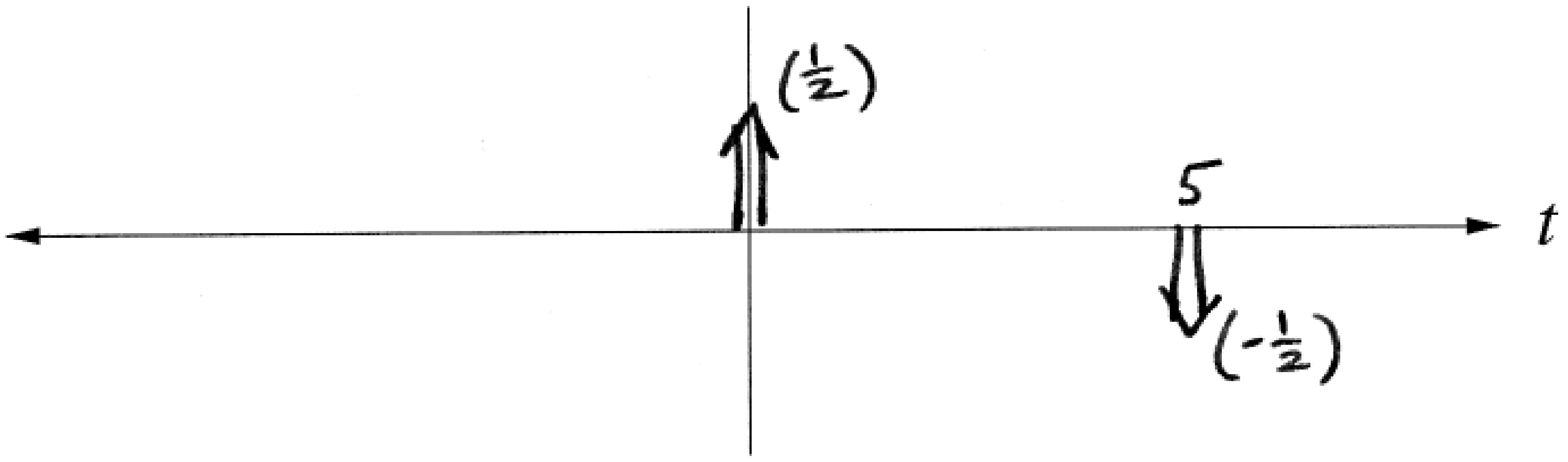
$h[n] = \frac{1}{2}\delta[n-3] + \delta[n-4] - \delta[n-7]$

$$H(z) = H_1(z) + H_2(z)$$
$$= (1 - z^{-7}) + (-1 + \tfrac{1}{2}z^{-3} + z^{-4})$$
$$= \tfrac{1}{2}z^{-3} + z^{-4} - z^{-7}$$

Problem 2: (20%)

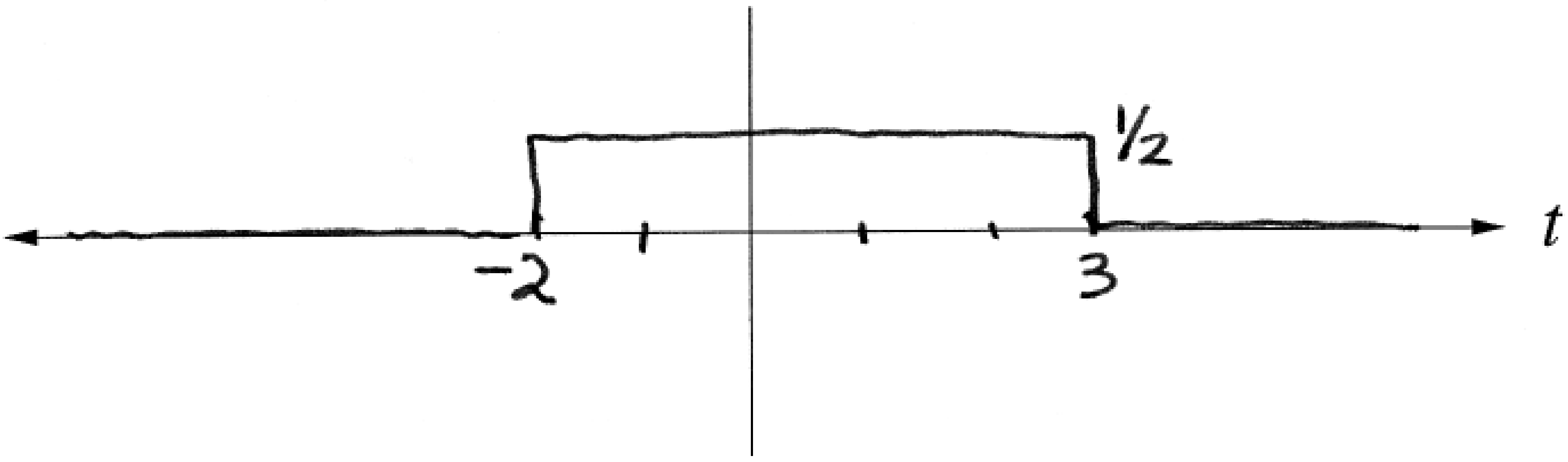
Let $x(t) = \frac{1}{2}u(t) - \frac{1}{2}u(t-5)$.

(a) Sketch $\frac{d}{dt}x(t)$. Carefully label your plot.

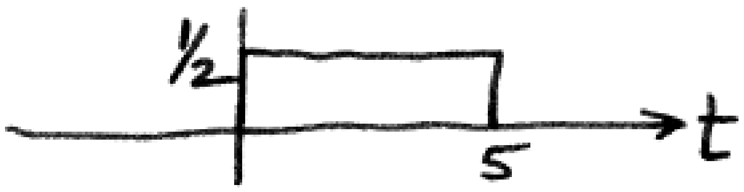


$$\frac{d}{dt}\left\{\frac{1}{2}u(t)-\frac{1}{2}u(t-5)\right\}$$
$$=\frac{1}{2}\delta(t)-\frac{1}{2}\delta(t-5)$$

(b) Sketch $x(3-t)$. Carefully label your plot.



Sketch $x(t)$



Then flip and shift by 3

(c) If $x(t) * x(t-4) * \delta(t-T) = x(t-6) * x(t-3)$, determine the numerical value of T.

$T =$ 5 secs

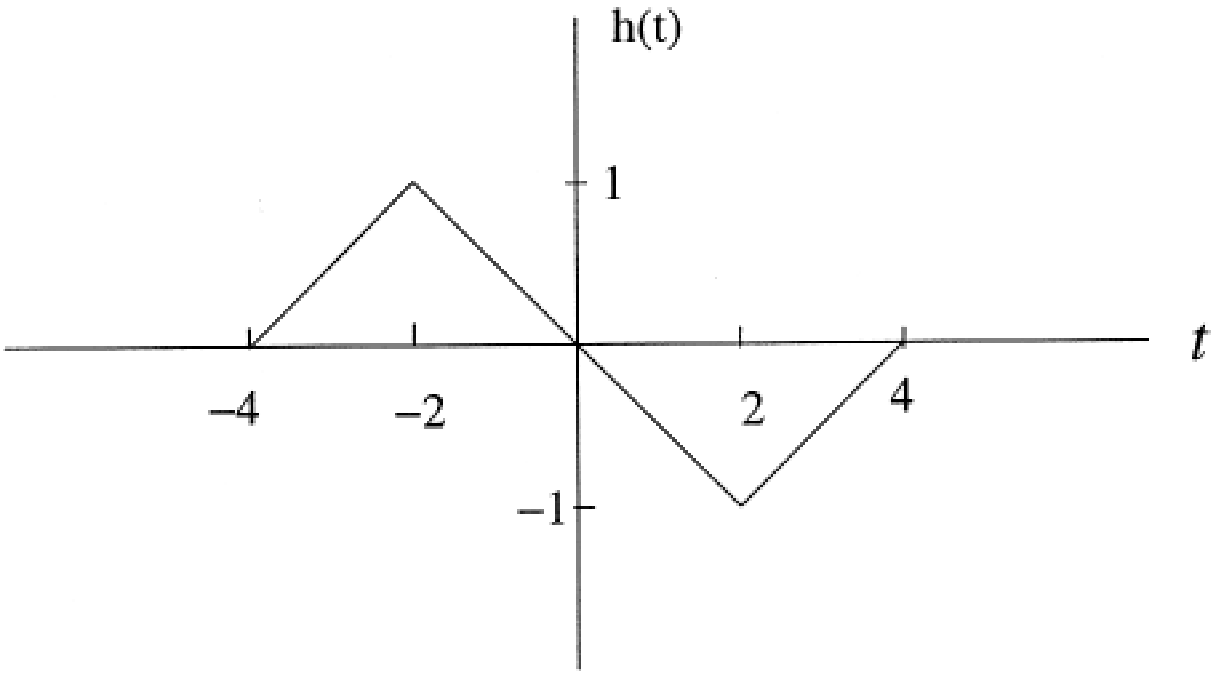
If $x(t) * x(t) = s(t)$, then we get

$$\underbrace{s(t-4) * \delta(t-T)}_{\text{This convolution is time-shifting by } T} = s(t-9)$$

Problem 3: (20%)

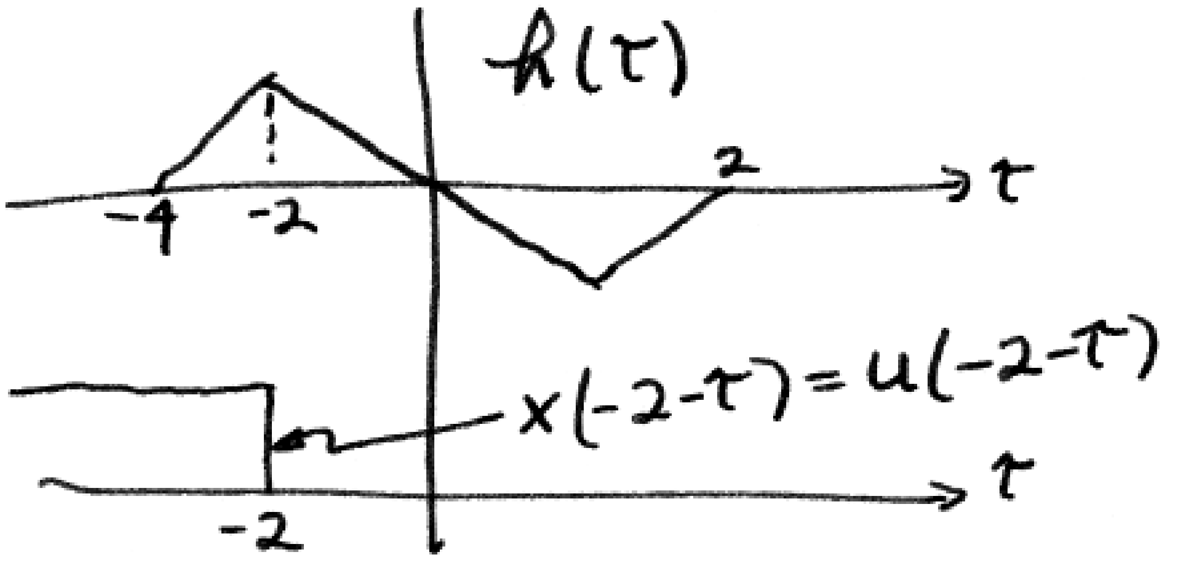
Consider the LTI System whose output is $y(t) = x(t) * h(t)$, where $x(t) = u(t)$

and $h(t)$ is given by



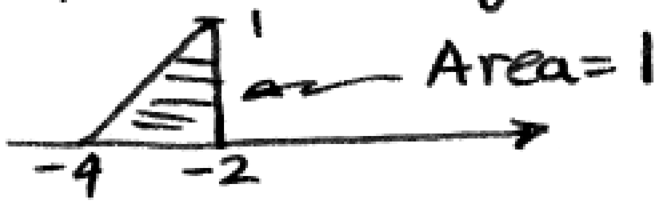
(a) Determine $y(-2)$, the value of $y(t)$ at $t = -2$.

$y(-2) =$ 1



For $t=-2$, the overlap is from $\tau=-4$ to $\tau=-2$

Thus, the convolution integral is the area of a triangle



(b) You should be able to see that $y(t) = 0$ in two regions: $T_1 \le t \le T_2$ and $T_3 \le t \le T_4$. Determine T_1, T_2, T_3, and T_4. **Explain carefully to receive full credit.**

$T_1 =$ $-\infty$, $T_2 =$ -4, $T_3 =$ 4, $T_4 =$ ∞

No overlap for $t < -4 \Rightarrow y(t) = 0$ $\therefore T_2 = -4$

Complete overlap for $t \geq 4$.

The convolution integral is therefore the total area in $h(\tau)$. The two triangle-shaped regions cancel giving $y(t)=0$ for $t \geq 4$. $\Rightarrow T_3 = 4$.

(c) Is this system stable? [✓ yes] or no (Circle one)

Stable because $\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-4}^{4} |h(t)|\, dt = 2+2=4 < \infty$

Problem 4: (20%)

Let $h(t) = -\delta(t+2) + 4\delta(t) - \delta(t-2)$.

(a) Find $H(j\omega)$.

$$\boxed{H(j\omega) = -e^{j2\omega} + 4 - e^{-j2\omega}}$$

Simplify:

$$H(j\omega) = 4 - 2\cos(2\omega)$$

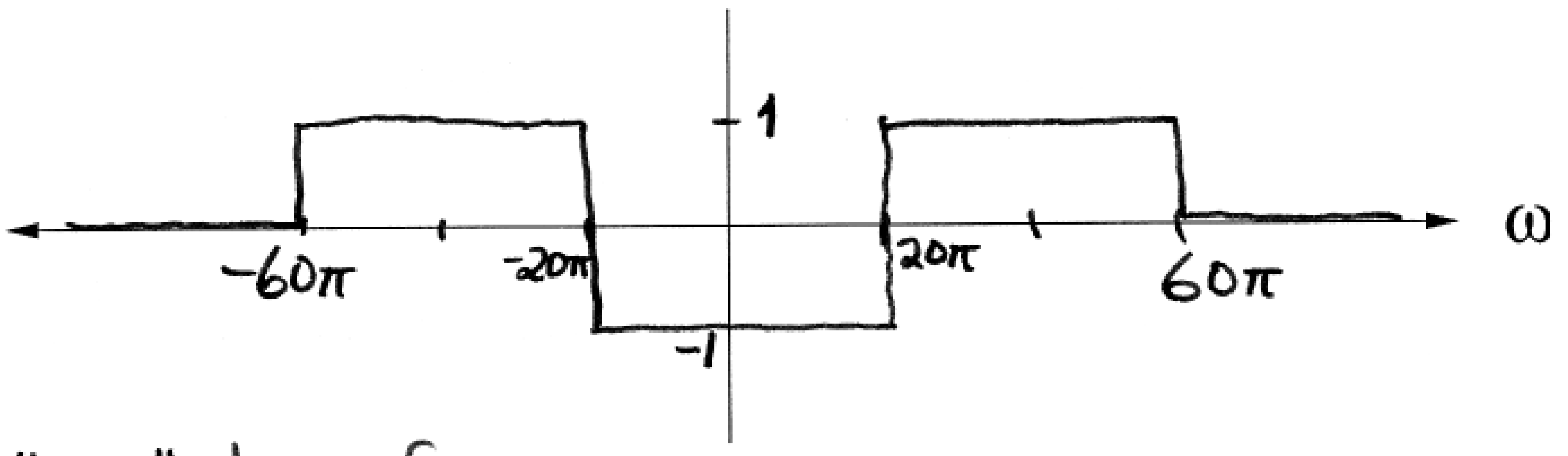
(b) Let $y(t) = h(t-7)$. Find the phase of $Y(j\omega)$, the Fourier transform of $y(t)$.

$$\boxed{\angle Y(j\omega) = -7\omega}$$

$$Y(j\omega) = e^{-j7\omega} H(j\omega)$$

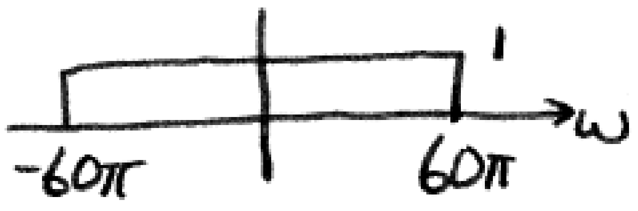
$$= \underset{\substack{\uparrow \\ \text{phase}}}{e^{-j7\omega}} \underbrace{\left(4 - 2\cos(2\omega)\right)}_{\text{Magnitude}}$$

(c) If $x(t) = \dfrac{\sin 60\pi t}{\pi t} - \dfrac{2\sin 20\pi t}{\pi t}$, plot $X(j\omega)$. Carefully label your plot.

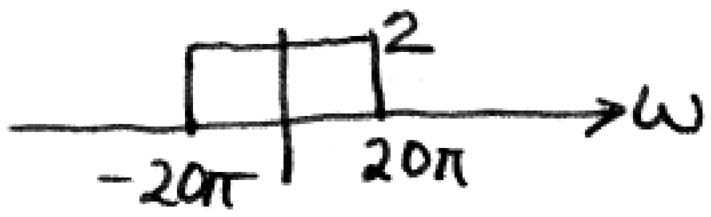


Each "sinc" transforms to a rectangle, then subtract the rectangles

$$\frac{\sin 60\pi t}{\pi t} \xrightarrow{F.T.}$$



$$\frac{2\sin 20\pi t}{\pi t} \rightarrow$$



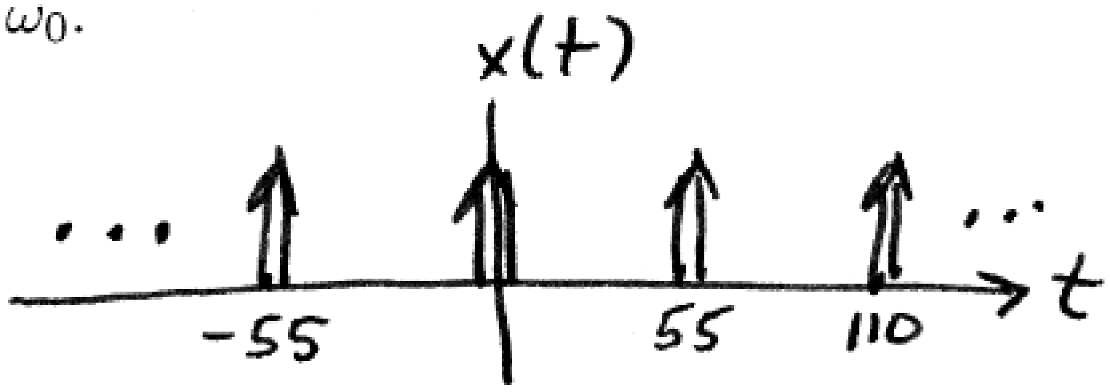
Problem 5: (20%)

Assume that $x(t)$ is the periodic function given by

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 55k) = \sum_{k=-\infty}^{\infty} \frac{1}{55} e^{j\omega_0 kt}.$$

(a) Determine the value of the fundamental frequency ω_0.

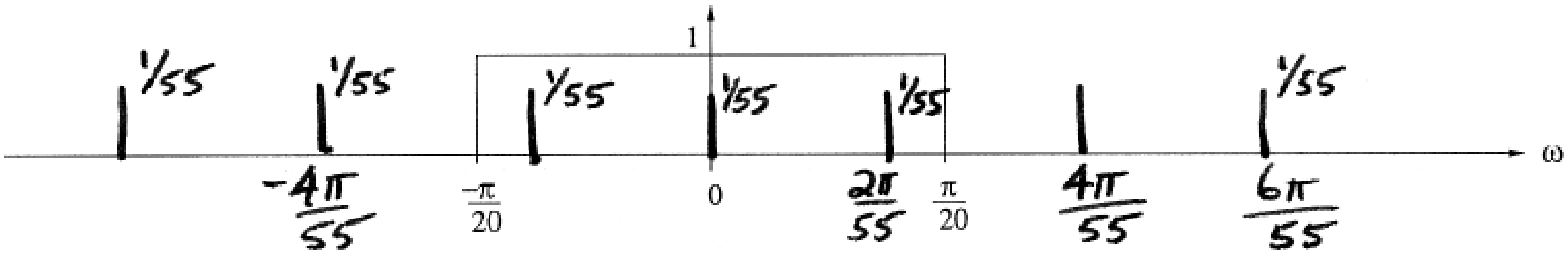
$$\boxed{\omega_0 = 2\pi/T = 2\pi/55 \text{ rad/s}}$$



period = 55 sec.

(b) Suppose that $x(t)$ is the input to an LTI system with the frequency response illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| \leq \frac{\pi}{20} \\ 0 & |\omega| > \frac{\pi}{20} \end{cases}$$



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. Your answer should be expressed in terms of only real quantities. (Hint: Plot the spectrum of $x(t)$ on the plot of the frequency response.)

$$\boxed{y(t) = \frac{1}{55} + \frac{2}{55}\cos\left(\frac{2\pi}{55}t\right)}$$

The LPF passes 3 components in the spectrum, so

$$y(t) = \frac{1}{55} + \frac{1}{55}e^{j\frac{2\pi t}{55}} + \frac{1}{55}e^{-j\frac{2\pi}{55}t}$$

$$= \frac{1}{55} + \frac{2}{55}\cos\left(\frac{2\pi}{55}t\right)$$