EE-2025 Spring-2003

Lecture 7
Fourier Series & Spectrum
3-Feb-2003

General Info

- ***Quiz Dates: 14-March & 11-April (Fridays)***
- ***Help Sessions: M1, M6, T6 and W4:30***
  - Every week
- **New Ch3 has new Fourier Series**
  - PDF file posted to WebCT
  - Replaces pp. 62-66 in DSP First Ch 3
- Prob Set #4 due This Week

Lab Info

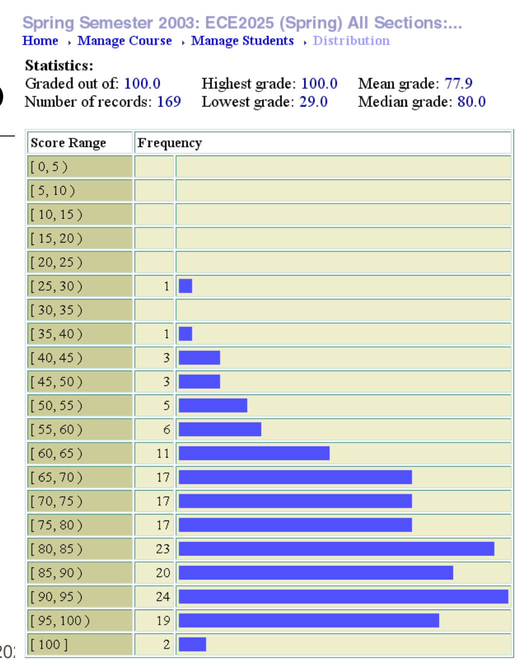
- Lab #3 Report
  - Turn in during your lab time
  - Late? –10 points per day
  - Finish INSTRUCTOR VERIFICATION in Lab
    - Come prepared with some preliminary code
  - ERRATA ? ALWAYS Check the Bulletin Board
- Lab #4 (Music Synthesis) has been posted
  - Bring Headphones to lab for the next few weeks
  - Read the Pre-Lab and do it before lab

Quiz #1 Info

45/169 above 90

Median was 80

Below 60? trouble



Spring Semester 2003: ECE2025 (Spring) All Sections:...

Home › Manage Course › Manage Students › Distribution

Statistics:

Graded out of: 100.0 Highest grade: 100.0 Mean grade: 77.9

Number of records: 169 Lowest grade: 29.0 Median grade: 80.0

Score Range	Frequency
[0, 5)	
[5, 10)	
[10, 15)	
[15, 20)	
[20, 25)	
[25, 30)	1
[30, 35)	
[35, 40)	1
[40, 45)	3
[45, 50)	3
[50, 55)	5
[55, 60)	6
[60, 65)	11
[65, 70)	17
[70, 75)	17
[75, 80)	17
[80, 85)	23
[85, 90)	20
[90, 95)	24
[95, 100)	19
[100]	2

READING ASSIGNMENTS

- This Lecture:
  - **Fourier Series in Ch 3, Sects 3-4, 3-5 & 3-6**
    - Replaces pp. 62-66 in Ch 3 in DSP First
    - Notation: a_k for Fourier Series
- Other Reading:
  - Next Lecture: Sampling

LECTURE OBJECTIVES

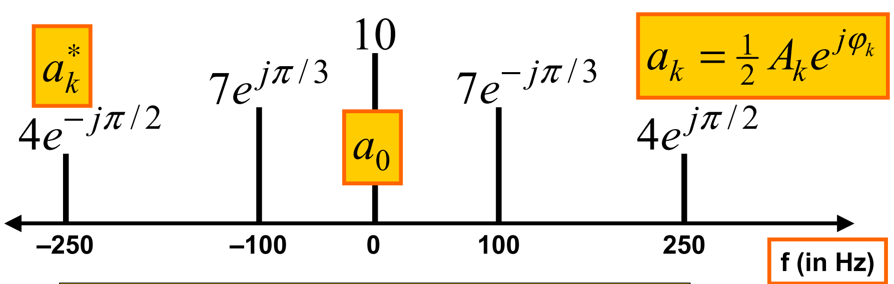
- **ANALYSIS** via Fourier Series
  - For **PERIODIC** signals: $x(t+T_0) = x(t)$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j(2\pi k/T_0)t}dt$$

- **SPECTRUM** from Fourier Series
  - a_k is Complex Amplitude for k-th Harmonic

SPECTRUM DIAGRAM

- Recall Complex Amplitude vs. Freq



$$x(t) = a_0 + \sum_{k=1}^{N}\left\{ a_k\, e^{j2\pi f_k t} + a_k^*\, e^{-j2\pi f_k t}\right\}$$

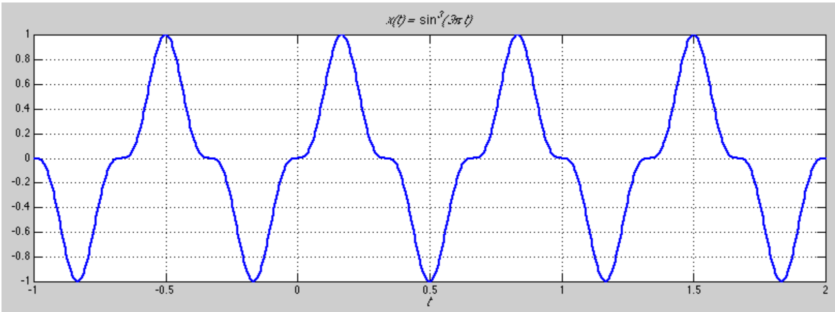
Harmonic Signal

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$$

PERIOD/FREQUENCY of COMPLEX EXPONENTIAL:

$$2\pi(f_0) = \omega_0 = \frac{2\pi}{T_0} \quad \text{or} \quad T_0 = \frac{1}{f_0}$$

Example $x(t)=\sin^3(3\pi t)$



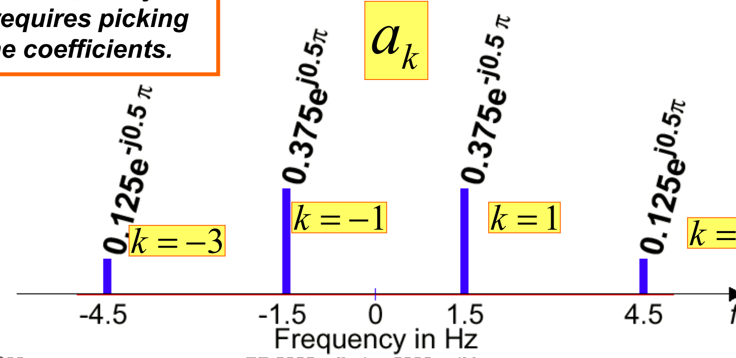
$$x(t)=\left(\frac{j}{8}\right)e^{j9\pi t}+\left(\frac{-3j}{8}\right)e^{j3\pi t}+\left(\frac{3j}{8}\right)e^{-j3\pi t}+\left(\frac{-j}{8}\right)e^{-j9\pi t}$$

Example $x(t)=\sin^3(3\pi t)$

$$x(t)=\left(\frac{j}{8}\right)e^{j9\pi t}+\left(\frac{-3j}{8}\right)e^{j3\pi t}+\left(\frac{3j}{8}\right)e^{-j3\pi t}+\left(\frac{-j}{8}\right)e^{-j9\pi t}$$

In this case, analysis just requires picking off the coefficients.

a_k



STRATEGY: x(t) → a_k

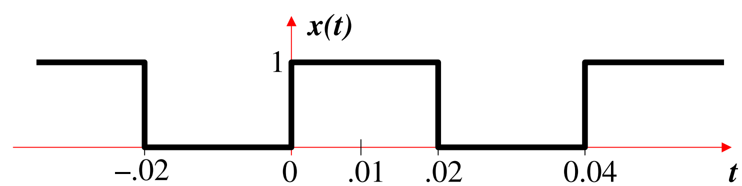
- **<u>ANALYSIS</u>**
  - Get representation from the signal
  - Works for **<u>PERIODIC</u>** Signals
- Fourier Series
  - Answer is: an INTEGRAL over one period

$$a_k=\frac{1}{T_0}\int_0^{T_0} x(t)e^{-j\omega_0 kt}\,dt$$

SQUARE WAVE EXAMPLE

$$x(t)=\begin{cases}1 & 0\le t<\frac{1}{2}T_0\\ 0 & \frac{1}{2}T_0\le t<T_0\end{cases}$$

for $T_0=0.04$ sec.



FS for a SQUARE WAVE $\{a_k\}$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j(2\pi/T_0)kt}dt \qquad (k \neq 0)$$

$$a_k = \frac{1}{.04}\int_0^{.02} 1e^{-j(2\pi/.04)kt}dt = \frac{1}{.04(-j2\pi k/.04)}e^{-j(2\pi/.04)kt}\Big|_0^{.02}$$

$$= \frac{1}{(-j2\pi k)}(e^{-j(\pi)k} - 1) = \frac{1-(-1)^k}{j2\pi k}$$

DC Coefficient: a_0

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j(2\pi/T_0)kt}dt \qquad (k = 0)$$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt = \frac{1}{T_0}(\text{Area})$$

$$a_0 = \frac{1}{.04}\int_0^{.02} 1\,dt = \frac{1}{.04}(.02 - 0) = \tfrac{1}{2}$$

Fourier Coefficients a_k

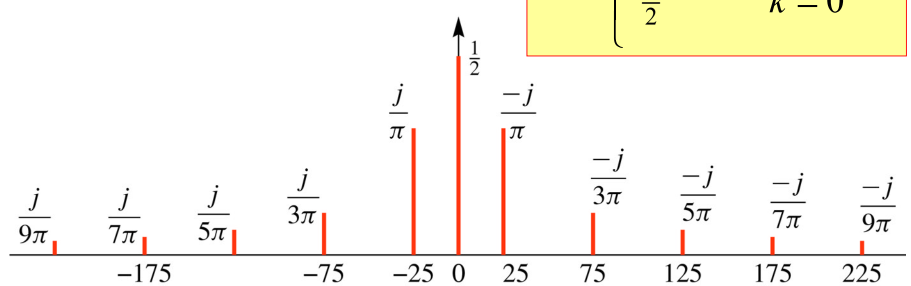
- a_k is a function of k
  - Complex Amplitude for k-th Harmonic
  - This one doesn't depend on the period, T_0

$$a_k = \frac{1-(-1)^k}{j2\pi k} = \begin{cases} \frac{1}{j\pi k} & k = \pm 1, \pm 3, \ldots \\ 0 & k = \pm 2, \pm 4, \ldots \\ \frac{1}{2} & k = 0 \end{cases}$$

Spectrum from Fourier Series

$$\omega_0 = 2\pi/(0.04) = 2\pi(25)$$

$$a_k = \begin{cases} \frac{-j}{\pi k} & k = \pm 1, \pm 3, \ldots \\ 0 & k = \pm 2, \pm 4, \ldots \\ \frac{1}{2} & k = 0 \end{cases}$$



Fourier Series Synthesis

- HOW do you **APPROXIMATE** x(t) ?

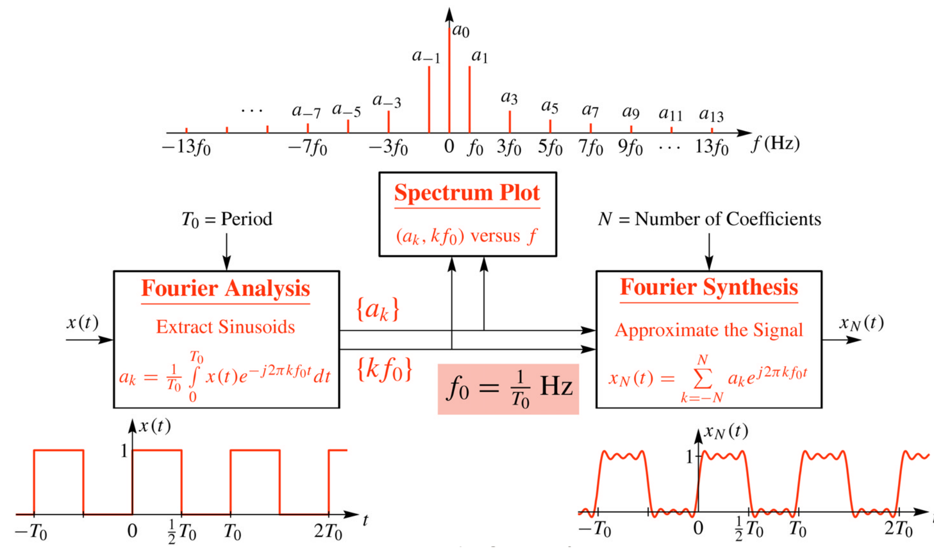
$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j(2\pi/T_0)kt}\,dt$$

- Use FINITE number of coefficients

$$x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi k f_0 t}$$

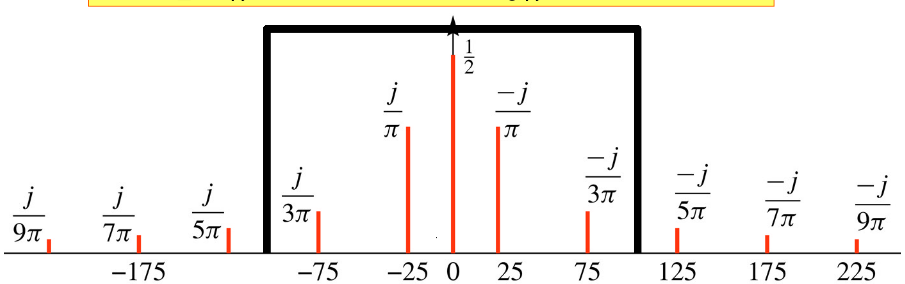
$a_{-k} = a_k^*$ when $x(t)$ is real

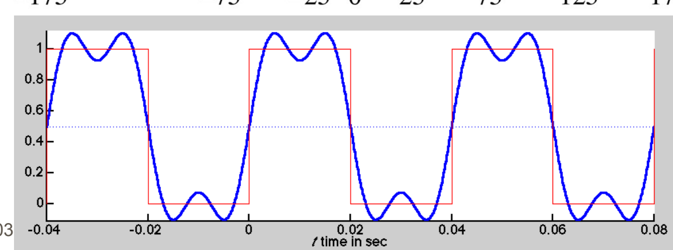
Fourier Series Synthesis



Synthesis: 1st & 3rd Harmonics

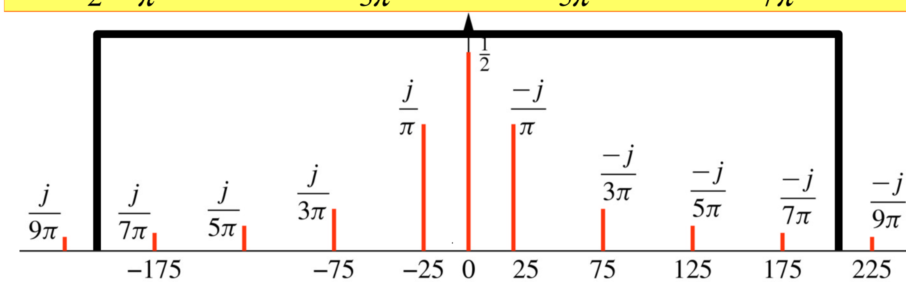
$$y(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi(25)t - \tfrac{\pi}{2}) + \frac{2}{3\pi}\cos(2\pi(75)t - \tfrac{\pi}{2})$$

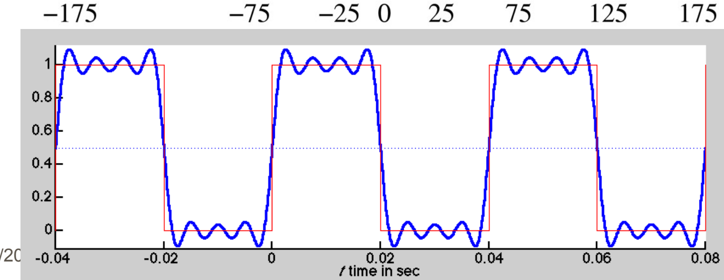




Synthesis: up to 7th Harmonic

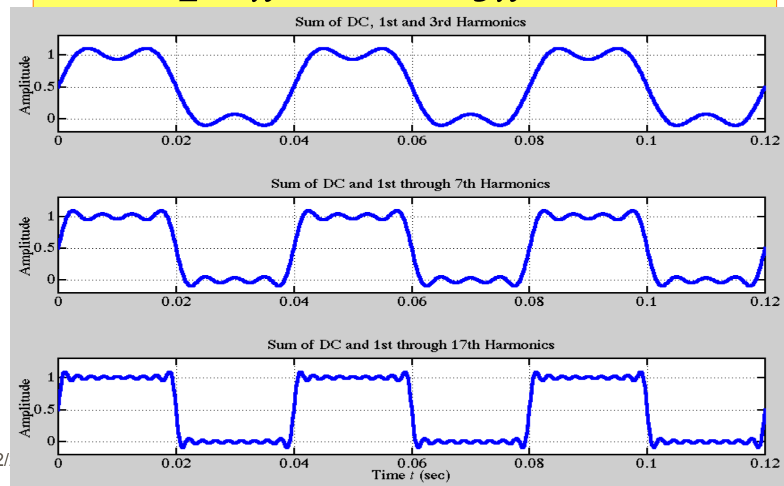
$$y(t) = \frac{1}{2} + \frac{2}{\pi}\cos(50\pi t - \tfrac{\pi}{2}) + \frac{2}{3\pi}\sin(150\pi) + \frac{2}{5\pi}\sin(250\pi t) + \frac{2}{7\pi}\sin(350\pi)$$





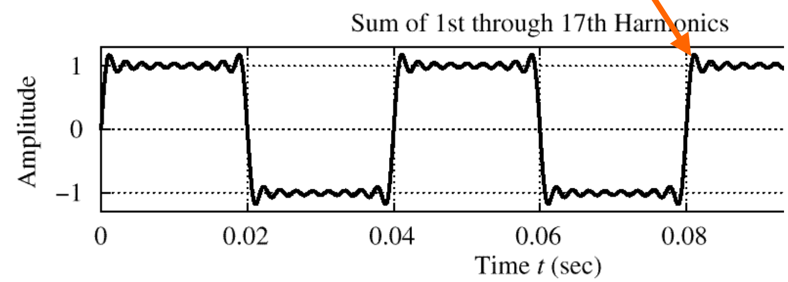
Fourier Synthesis

$$x_N(t) = \frac{1}{2} + \frac{2}{\pi}\sin(\omega_0 t) + \frac{2}{3\pi}\sin(3\omega_0 t) + \ldots$$



Gibbs' Phenomenon

- Convergence at DISCONTINUITY of x(t)
  - There is always an overshoot
  - 9% for the Square Wave case



Fourier Series Demos

- Fourier Series Java Applet
  - Greg Slabaugh
    - Interactive
  - http://users.ece.gatech.edu/~slabaugh/java/fourier/fourier.html
- MATLAB GUI: fseriesdemo
  - http://users.ece.gatech.edu/mcclella/matlabGUIs/index.html

fseriesdemo GUI