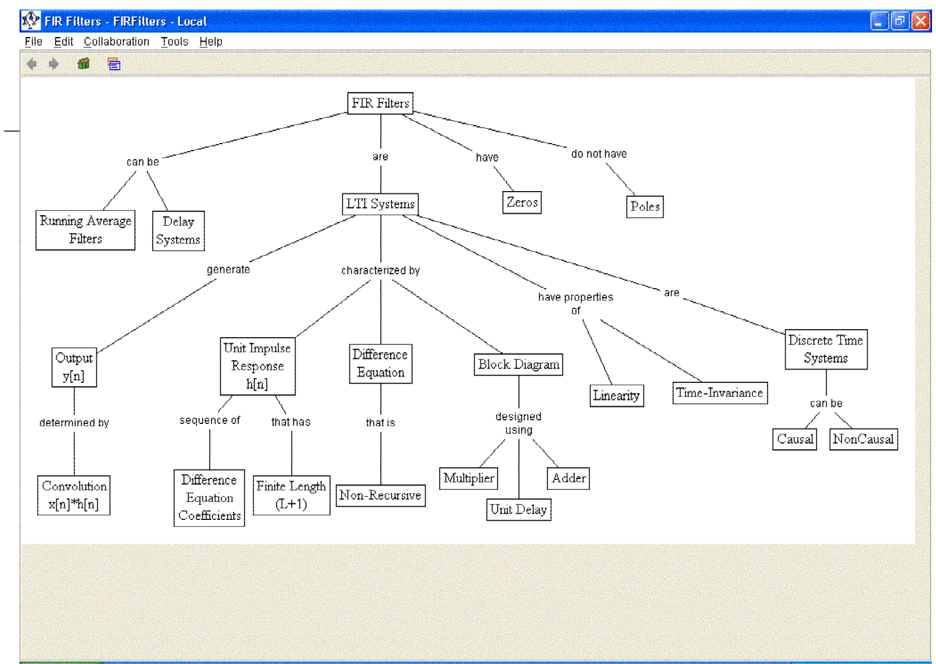
EE-2025 Spring-2003

Lecture 11
Linearity, Time-Invariance and Convolution
17-Feb-03

Info: Web-CT, Lab, HW

- UTILIZE OFFICE HOURS
- Prepare for on-line Pre-Post-Labs
  - Run MATLAB GUIs for Lab #6
- Labs #5 and #6: Image Processing
  - Sampling & Zooming
  - Edge Enhancement for Images
- Quiz #2 on 14-March
  - Problem Sets #4, #5, #6, #7 and #8



DEBUGGING

- "Any Fool" can write code
- Debugging is the interesting part
  - It takes brain power !!!
- HOWEVER,
  - Assume the stupid mistake is the problem

LECTURE

READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, Sections 5-5 and 5-6
    - Section 5-4 will be covered, but not "in depth"
- Other Reading:
  - Recitation: Ch. 5, Sects 5-6, 5-7 & 5-8
    - CONVOLUTION
  - Next Lecture: start Chapter 6

LECTURE OBJECTIVES

- GENERAL PROPERTIES of FILTERS
  - LINEARITY
  - TIME-INVARIANCE
  - ==> **CONVOLUTION**

LTI SYSTEMS

- BLOCK DIAGRAM REPRESENTATION
  - Components for Hardware
  - Connect Simple Filters Together to Build More Complicated Systems

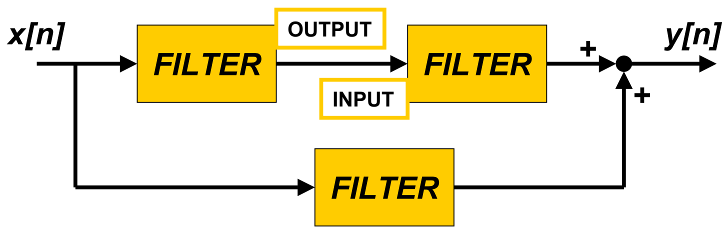
OVERVIEW

- IMPULSE RESPONSE, $h[n]$
  - FIR case: same as $\{b_k\}$
- CONVOLUTION
  - GENERAL: $y[n] = h[n] * x[n]$
  - GENERAL CLASS of SYSTEMS
  - LINEAR and TIME-INVARIANT
- ALL **LTI** systems have $h[n]$ & use convolution

DIGITAL FILTERING

- CONCENTRATE on the FILTER (DSP)
- DISCRETE-TIME SIGNALS
  - FUNCTIONS of n, the "time index"
  - INPUT $x[n]$
  - OUTPUT $y[n]$

BUILDING BLOCKS



- BUILD UP COMPLICATED FILTERS
  - FROM SIMPLE MODULES
  - Ex: FILTER MODULE MIGHT BE 3-pt FIR

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $b_k = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$
$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

MATLAB for FIR FILTER

- `yy = conv(bb,xx)`
  - VECTOR `bb` contains Filter Coefficients
  - DSP-First: `yy = firfilt(bb,xx)`
- FILTER COEFFICIENTS $\{b_k\}$

`conv2()` *for images*

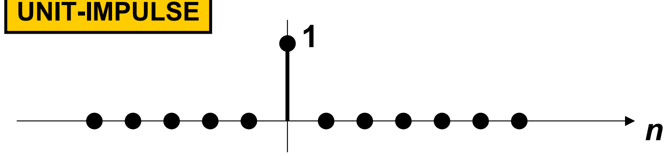
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

SPECIAL INPUT SIGNALS

- x[n] = SINUSOID **FREQUENCY RESPONSE**
- x[n] has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

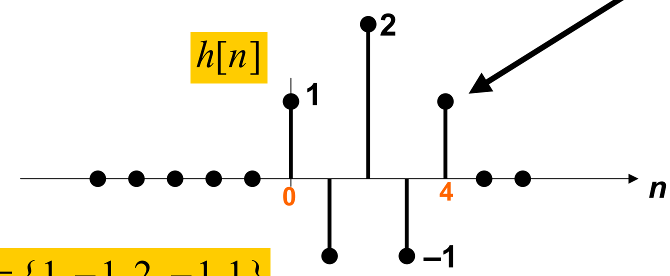
n	$n<0$	0	1	2	3	$\dots$	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	$\dots$	b_M	0	0

$$h[n]=\sum_{k=0}^{M} b_k \delta[n-k]$$

MATH FORMULA for h[n]

- Use SHIFTED IMPULSES to write $h[n]$

$$h[n]=\delta[n]-\delta[n-1]+2\delta[n-2]-\delta[n-3]+\delta[n-4]$$



$$b_k=\{1,-1,2,-1,1\}$$

LTI: Convolution Sum

- **Output = Convolution of x[n] & h[n]**
  - NOTATION: $y[n]=h[n]*x[n]$
  - Here is the FIR case:

FINITE LIMITS

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k]$$

Same as b_k

FINITE LIMITS

CONVOLUTION Example

$$h[n]=\delta[n]-\delta[n-1]+2\delta[n-2]-\delta[n-3]+\delta[n-4]$$
$$x[n]=u[n]$$

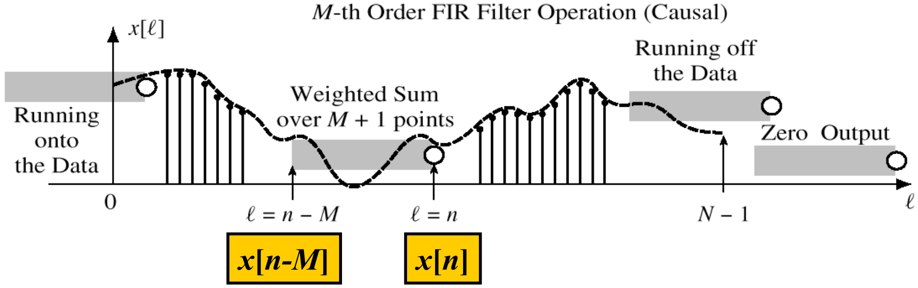
n	-1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	-1	2	-1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	-1	-1	-1	-1	-1	-1	-1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	-1	-1	-1	-1	-1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

GENERAL FIR FILTER

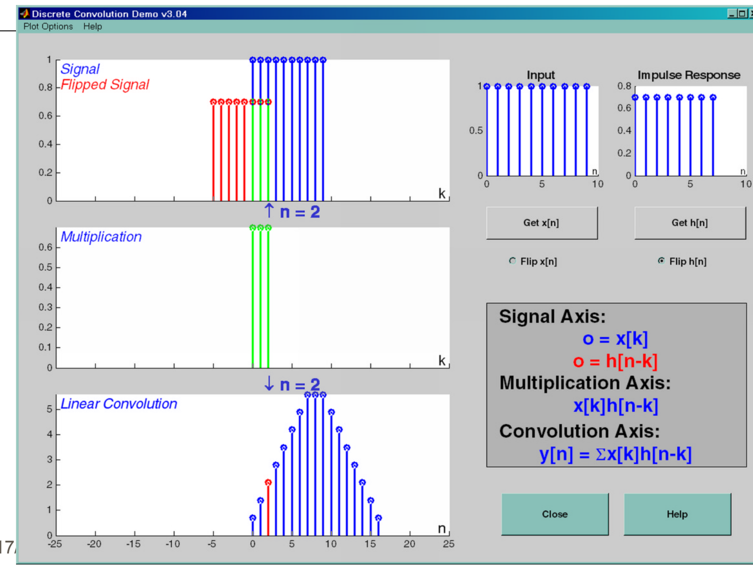
- SLIDE a Length-L WINDOW over x[n]

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

M-th Order FIR Filter Operation (Causal)



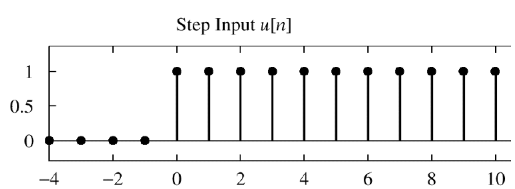
DCONVDEMO: MATLAB GUI



POP QUIZ

- FIR Filter is "FIRST DIFFERENCE"
  - $y[n] = x[n] - x[n-1]$
- INPUT is "UNIT STEP"

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



- Find $y[n]$

$$y[n] = u[n] - u[n-1] = \delta[n]$$

HARDWARE STRUCTURES

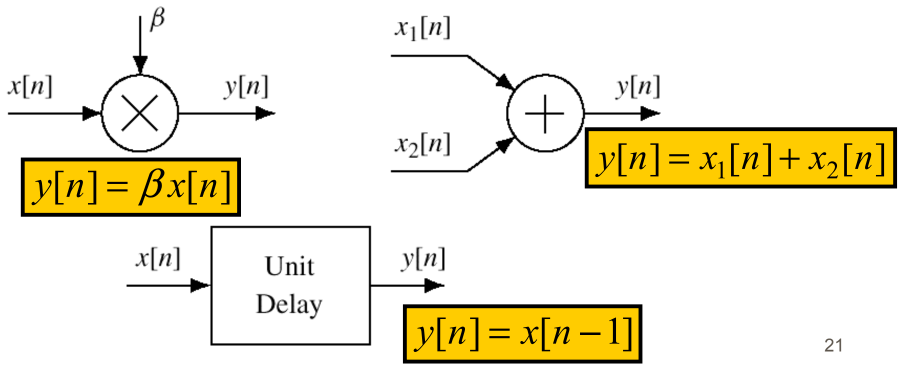
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- INTERNAL STRUCTURE of "FILTER"
  - WHAT COMPONENTS ARE NEEDED?
  - HOW DO WE "HOOK" THEM TOGETHER?
- SIGNAL FLOW GRAPH NOTATION

HARDWARE ATOMS

- Add, Multiply & Store

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$



FIR STRUCTURE

- Direct Form

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

SIGNAL FLOW GRAPH

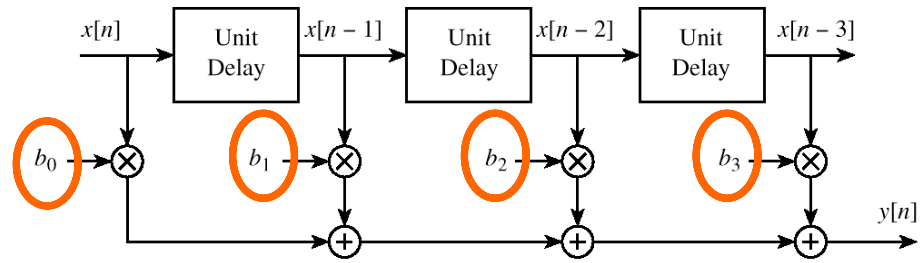
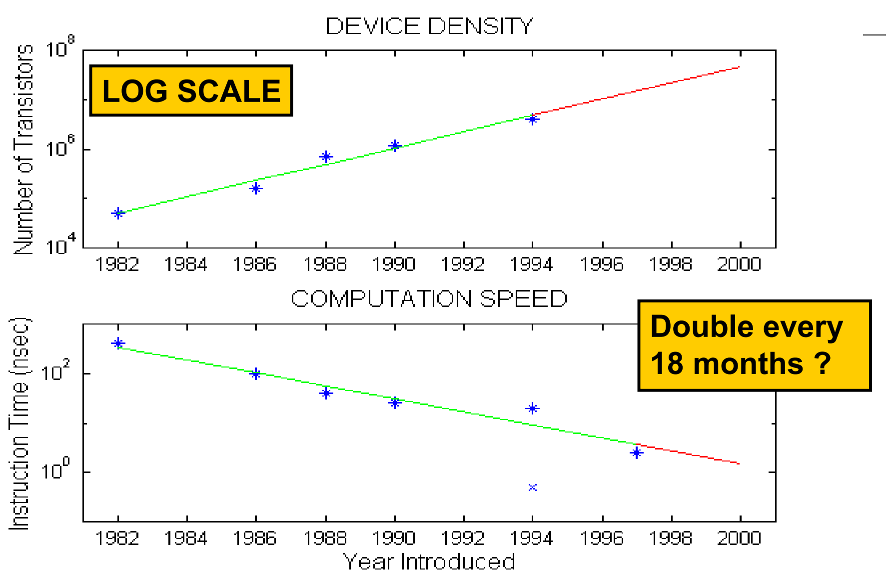


Figure 5.13 Block-diagram structure for the Mth order FIR filter.

Moore's Law for TI DSPs



SYSTEM PROPERTIES

- MATHEMATICAL DESCRIPTION
- TIME-INVARIANCE
- LINEARITY
- CAUSALITY
  - "No output prior to input"

TIME-INVARIANCE

- IDEA:
  - "Time-Shifting the input will cause the same time-shift in the output"
- EQUIVALENTLY,
  - We can prove that
    - The time origin (n=0) is picked arbitrary

TESTING Time-Invariance

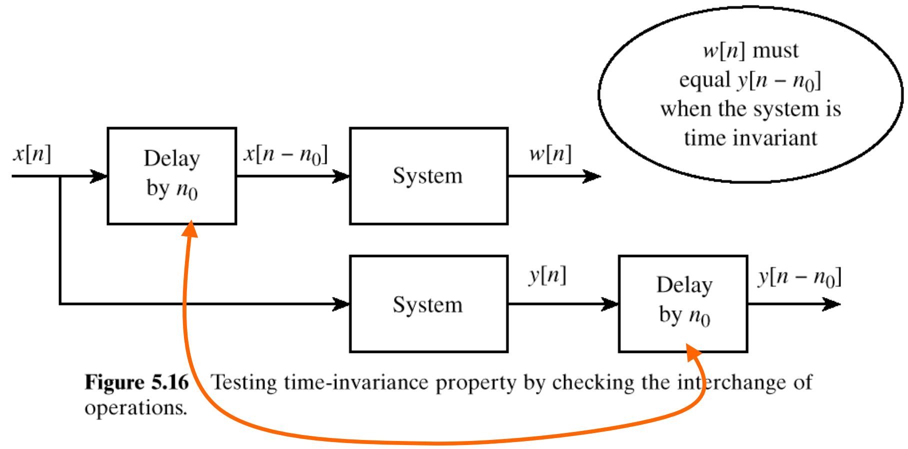


Figure 5.16 Testing time-invariance property by checking the interchange of operations.

LINEAR SYSTEM

- LINEARITY = Two Properties
- SCALING
  - "Doubling x[n] will double y[n]"
- SUPERPOSITION:
  - "Adding two inputs gives an output that is the sum of the individual outputs"

TESTING LINEARITY

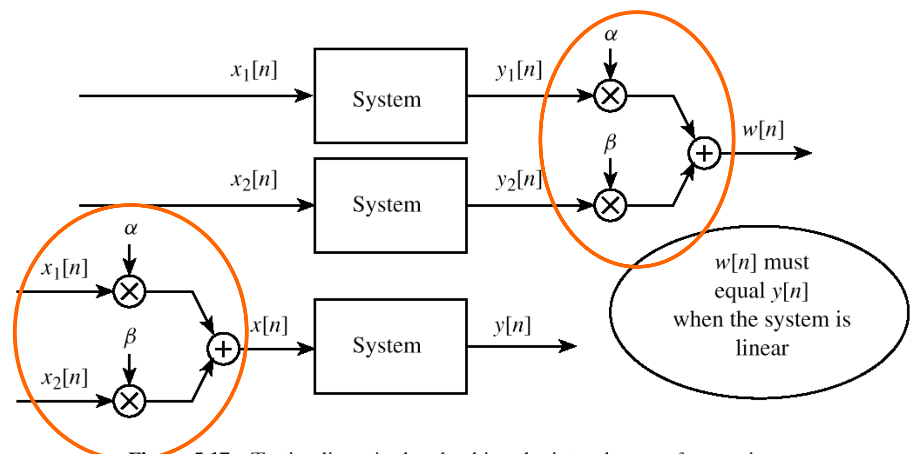


Figure 5.17 Testing linearity by checking the interchange of operations.

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - IMPULSE RESPONSE h[n]
  - CONVOLUTION: y[n] = x[n]*h[n]
    - The “rule”defining the system can ALWAYS be re-written as convolution
- FIR Example: h[n] is same as b_k

POP QUIZ

- FIR Filter is “FIRST DIFFERENCE”
  - $y[n] = x[n] - x[n-1]$
- Write output as a convolution
  - Need impulse response

$$h[n] = \delta[n] - \delta[n-1]$$

  - Then, another way to compute the output:

$$y[n] = \left(\delta[n] - \delta[n-1]\right) * x[n]$$

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$

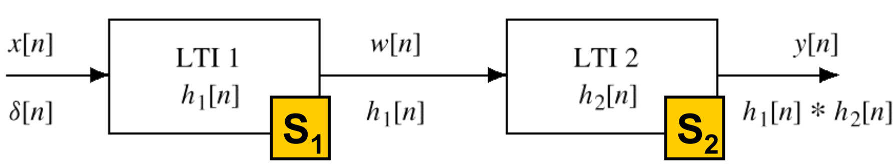


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- Find “overall” h[n] for a cascade ?

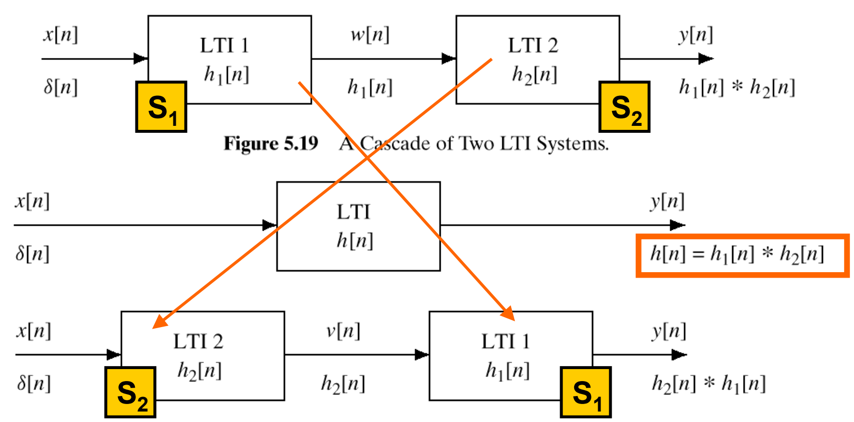


Figure 5.19 A Cascade of Two LTI Systems.

Figure 5.20 Switching the order of cascaded LTI systems.