

ECE-2025

Spring-2003

Lecture 13

Digital Filtering of Analog Signals

24-Feb-03

Info: Web-CT, Lab, HW

- Quiz #2 on 14-March (Friday)
 - Coverage: HW #4, #5, #6, #7, and #8
- New Lab version for Lab #6
 - Changed **Section 3.4**; posted on Saturday
 - **Combine row detector with column detector**
 - Original is acceptable; new version works much better
- Don't print "mostly black" images
 - Convert black to white, and white to black
 - Use **colormap(1-gray(256))** for "negative"

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Perseverance

- **A lowly virtue whereby mediocrity achieves a glorious success...**A. Bierce
- **Bear in mind, if you are going to amount to anything, that your success does not depend upon the brilliance and the impetuosity with which you take hold, but upon the ever lasting and sanctified bull doggedness with which you hang on after you have taken hold...**Dr. A. B. Meldrum

LECTURE

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 6, Sections 6-6, 6-7 & 6-8
- Other Reading:
 - Recitation: Chapter 6
 - **FREQUENCY RESPONSE EXAMPLES**
 - Next Lecture: Chapter 7

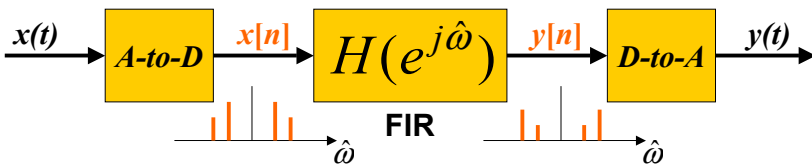
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LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of $x[n]$ thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- UNIFICATION:** How does Frequency Response affect $x(t)$ to produce $y(t)$?



TIME & FREQUENCY

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M h[k] x[n-k]$$

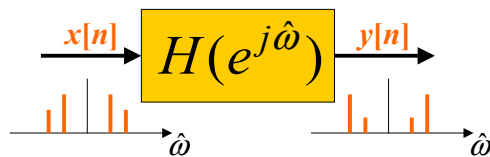
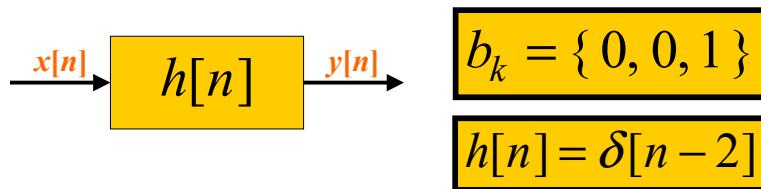
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \dots$$

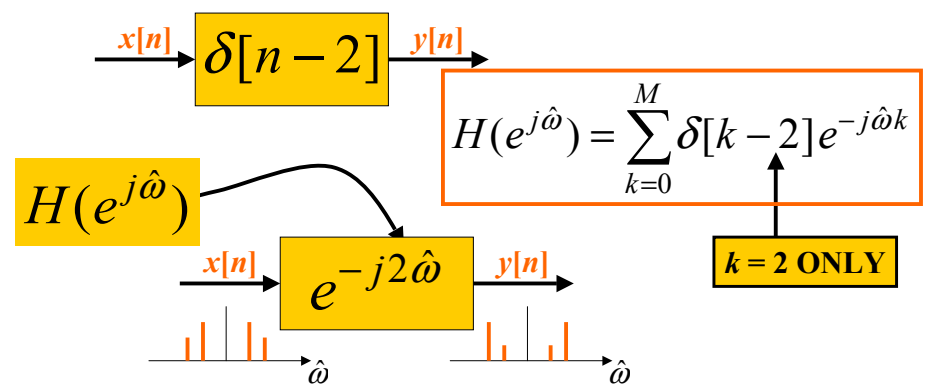
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



GENERAL DELAY PROPERTY

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n - n_d]$

$$h[n] = \delta[n - n_d]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M \delta[k - n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

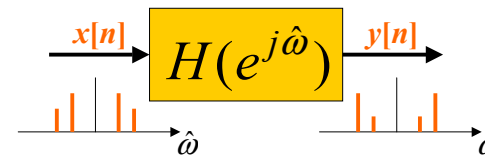
ONLY ONE
non-ZERO TERM
for k at $k = n_d$

FREQ DOMAIN --> TIME ??

- START with $H(e^{j\hat{\omega}})$ and find $h[n]$ or b_k



$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$



FREQ DOMAIN --> TIME

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega}) \quad \text{EULER's Formula}$$

$$= 7e^{-j2\hat{\omega}} (0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n - 1] + 3.5\delta[n - 3]$$

$$b_k = \{0, 3.5, 3.5\}$$

PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
 - OUTPUT has SAME FREQUENCY
 - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
 - MAGNITUDE vs. Frequency
 - PHASE vs. Freq
 - PLOTTING

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$

MAG

PHASE

FREQ. RESPONSE PLOTS

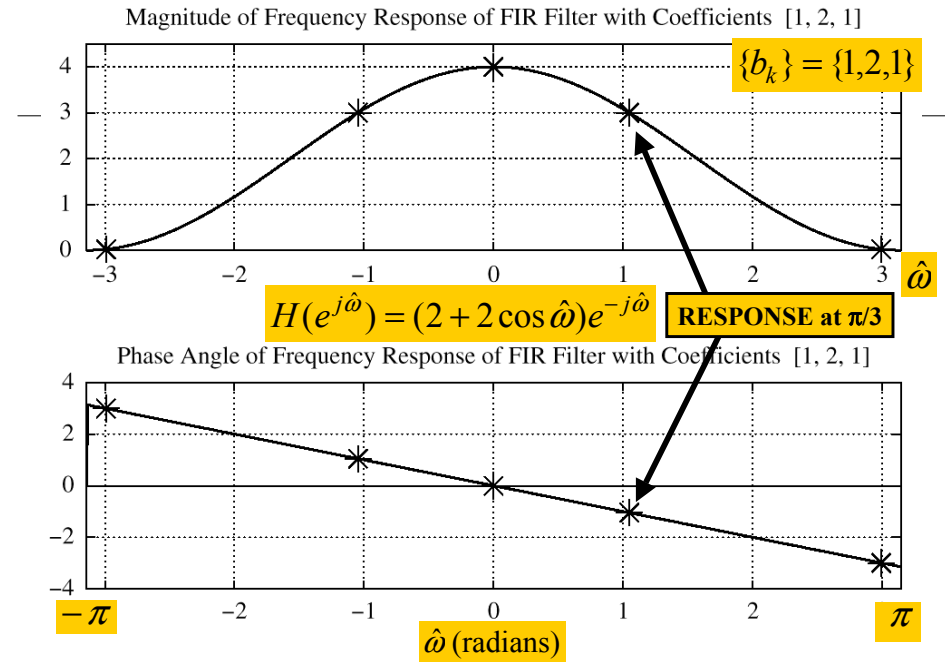
- DENSE GRID (**ww**) from $-\pi$ to $+\pi$
 - **ww** = `-pi:(pi/100):pi;`
- **yy** = `freqz(bb,1,ww)`
 - VECTOR **bb** contains Filter Coefficients
 - DSP-First: **yy** = `freqz(bb,1,ww)`

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

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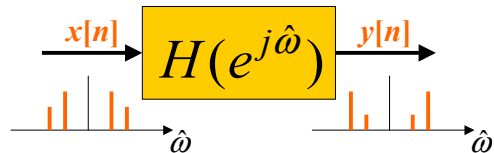
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EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

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EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @ \hat{\omega} = \pi/3$$

$$y[n] = (3e^{-j\pi/3}) \times 2e^{j\pi/4} e^{j(\pi/3)n} = 6e^{-j\pi/12} e^{j(\pi/3)n}$$

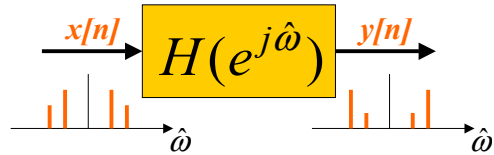
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EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

EX: COSINE INPUT (ans-1)

Find $y[n]$ when $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2 \cos(\frac{\pi}{3}n + \frac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6 \cos(\frac{\pi}{3}n - \frac{\pi}{12})$$

SINUSOID thru FIR

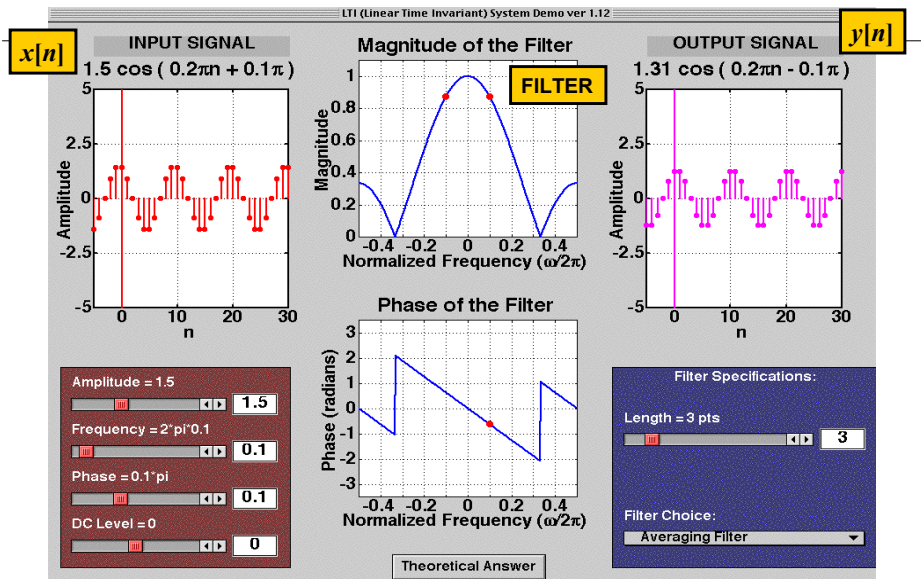
- IF $H^*(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

$$x[n] = A \cos(\omega_1 t + \phi)$$

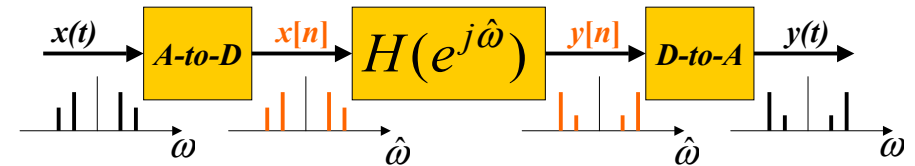
$$\Rightarrow y[n] = A \left| H(e^{j\hat{\omega}_1}) \right| \cos(\omega_1 t + \phi + \angle H(e^{j\hat{\omega}_1}))$$



LTI Demo with Sinusoids

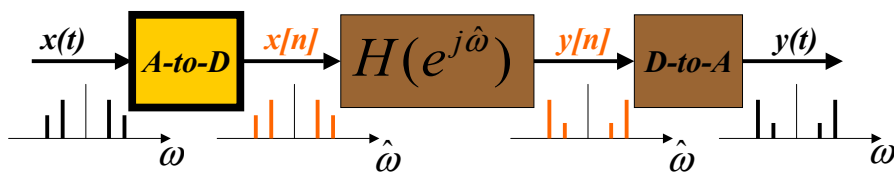


DIGITAL "FILTERING"



- ω ■ SPECTRUM of $x(t)$ (SUM of SINUSOIDS)
- $\hat{\omega}$ ■ SPECTRUM of $x[n]$
 - Is ALIASING a PROBLEM ?
- $\hat{\omega}$ ■ SPECTRUM $y[n]$ (FIR Gain or Nulls)
- ω ■ Then, OUTPUT $y(t)$ = SUM of SINUSOIDS

FREQUENCY SCALING



■ TIME SAMPLING:

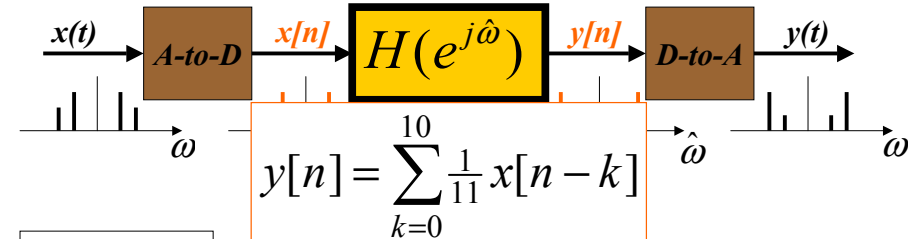
■ IF **NO** ALIASING:

■ FREQUENCY SCALING

$$t = nT_s$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

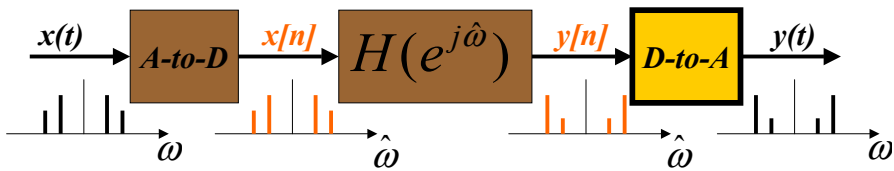
11-pt AVERAGER Example



$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2} \hat{\omega})}{11 \sin(\frac{1}{2} \hat{\omega})} e^{-j5\hat{\omega}} \quad ?$$

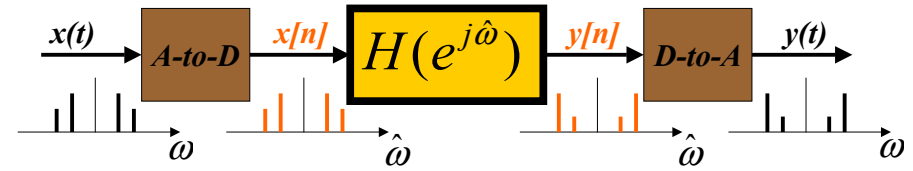
$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{1}{2}\pi)$$

D-A FREQUENCY SCALING



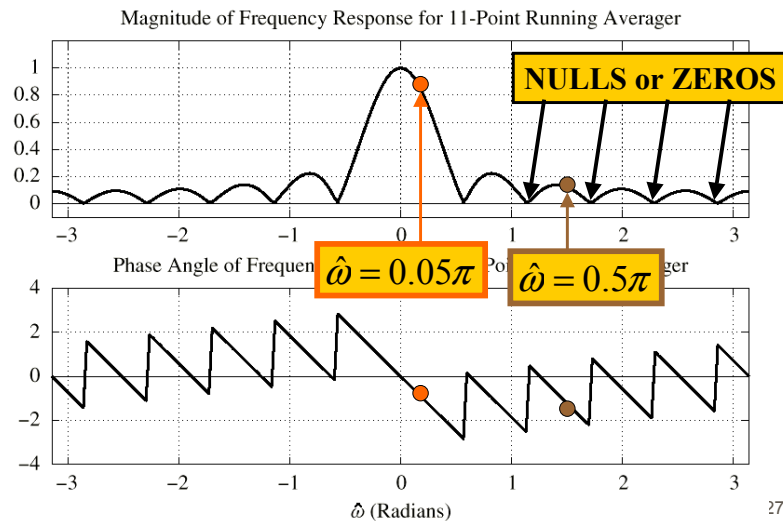
- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow tf_s$
- RECONSTRUCT up to $0.5f_s$
 - FREQUENCY SCALING $\omega = \hat{\omega} f_s$

TRACK the FREQUENCIES



- | | | | | |
|--------|----------|-------------------|----------|--------|
| 250 Hz | 0.5π | $H(e^{j0.5\pi})$ | 0.5π | 250 Hz |
| 25 Hz | $.05\pi$ | $H(e^{j0.05\pi})$ | $.05\pi$ | 25 Hz |
- $F_s = 1000 \text{ Hz}$ **NO new freqs**

11-pt AVERAGER



EVALUATE Freq. Response

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2} \hat{\omega})}{11 \sin(\frac{1}{2} \hat{\omega})} e^{-j5\hat{\omega}}$$

At $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2} (0.5\pi))}{11 \sin(\frac{1}{2} (0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11 \sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

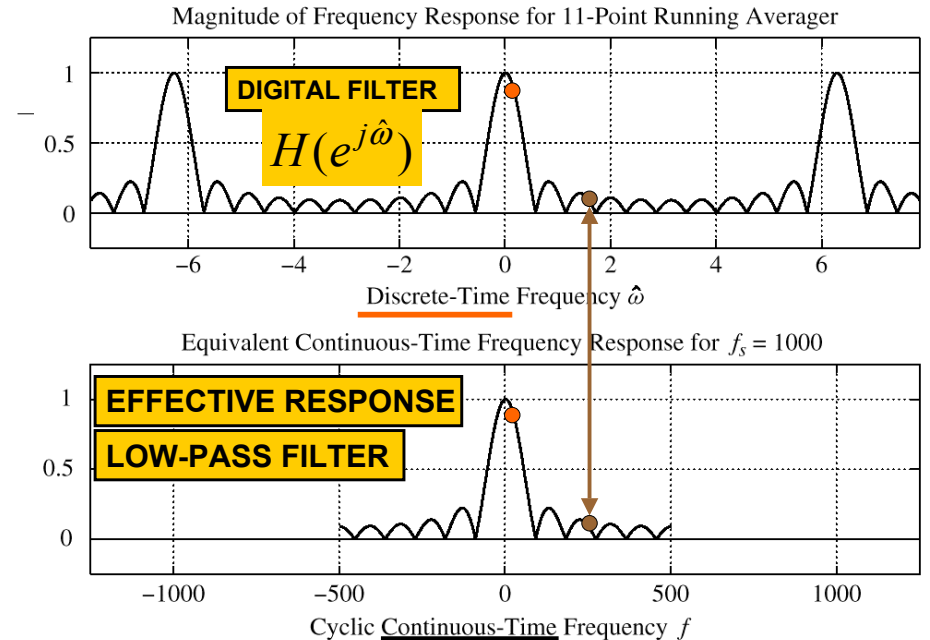
EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$$\begin{aligned}
 H(e^{j2\pi(25)/1000}) &= \frac{\sin(\pi(25)(11)/1000)}{11 \sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000} \\
 &= 0.8811 e^{-j\pi/4} \\
 H(e^{j2\pi(250)/1000}) &= \frac{\sin(\pi(250)(11)/1000)}{11 \sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000} \\
 &= 0.0909 e^{-j\pi/2}
 \end{aligned}$$

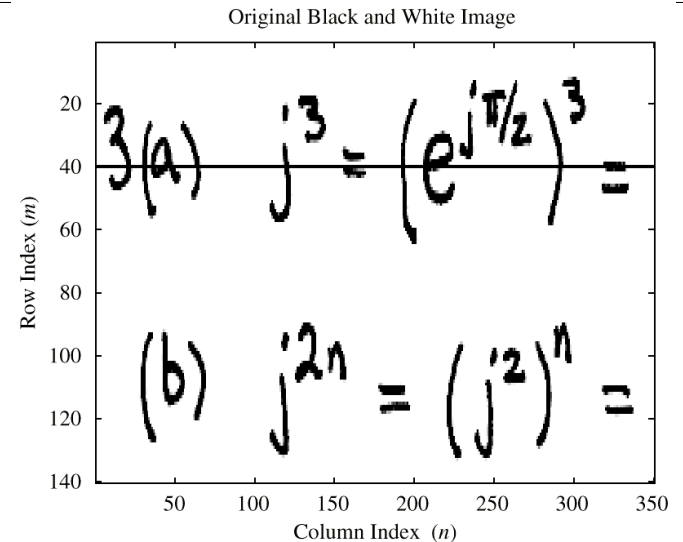
$y(t) = 0.8811 \cos(2\pi(25)t - \pi/4) + 0.0909 \sin(2\pi(250)t - \pi/2)$



FILTER TYPES

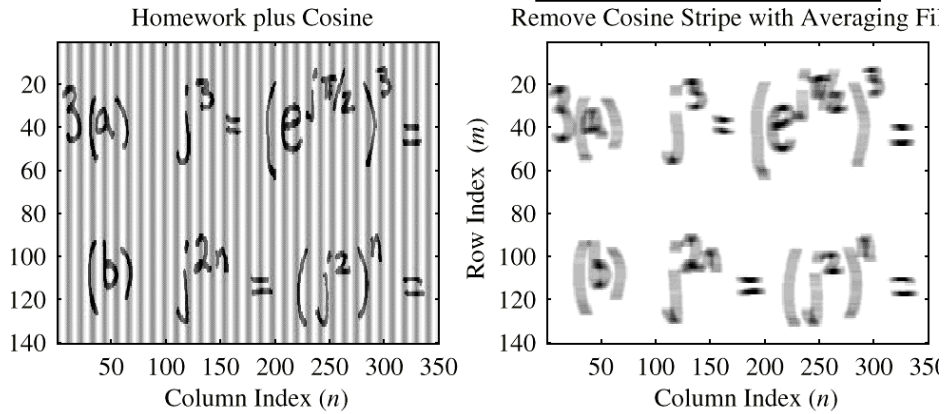
- LOW-PASS FILTER (LPF)
 - BLURRING
 - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (HPF)
 - SHARPENING for IMAGES
 - BOOSTS THE HIGHS
 - REMOVES DC
- BAND-PASS FILTER (BPF)

B & W IMAGE

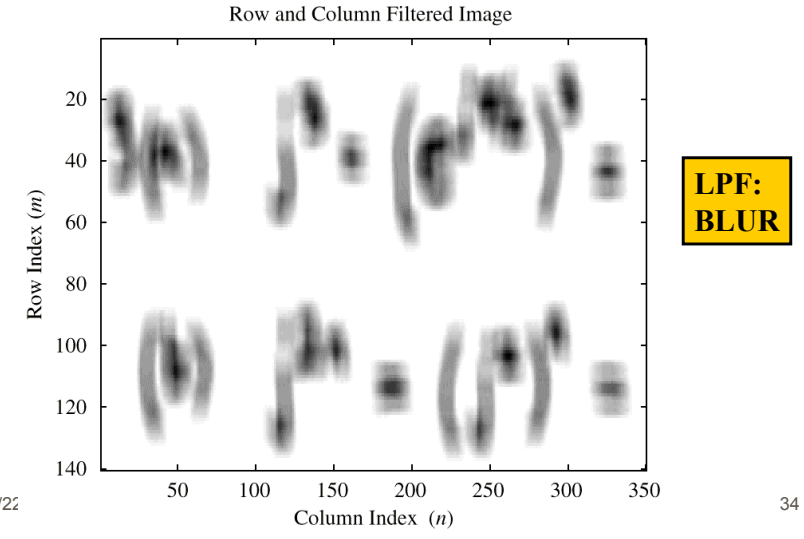


B&W IMAGE with COSINE

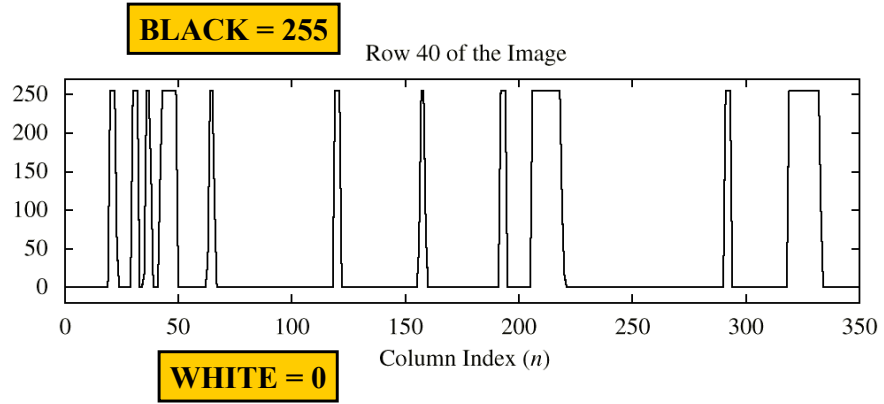
FILTERED: 11-pt AVG



FILTERED B&W IMAGE



ROW of B&W IMAGE



FILTERED ROW of IMAGE

