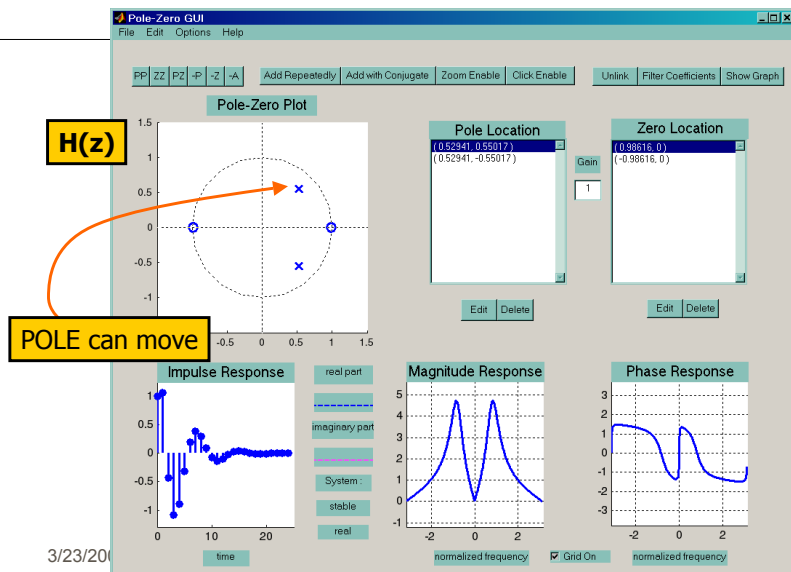


Lecture 18
3 Domains for IIR Systems
24-Mar-03

Info: Web-CT, Lab, HW

- Quiz #3 is 11-April (Friday)
- Prob Set #10 is due this week
- Lab #9 is DTMF (Touch-Tone) Lab
- Lab #10 is PeZ
- Quiz #2: Resolve any grade changes by Friday (28-Mar)

PeZ Demo: Pole-Zero Placing



Education

- The only product where the consumer tries to get as **LITTLE** as possible for his/her money !
- STREAMED Lectures are available on the web
- Linked from WebCT

Lecture

READING ASSIGNMENTS

- This Lecture:
 - Chapter 8, all
- Other Reading:
 - Recitation: Ch. 8, all
 - POLES & ZEROS
 - Next Lecture: Chapter 9

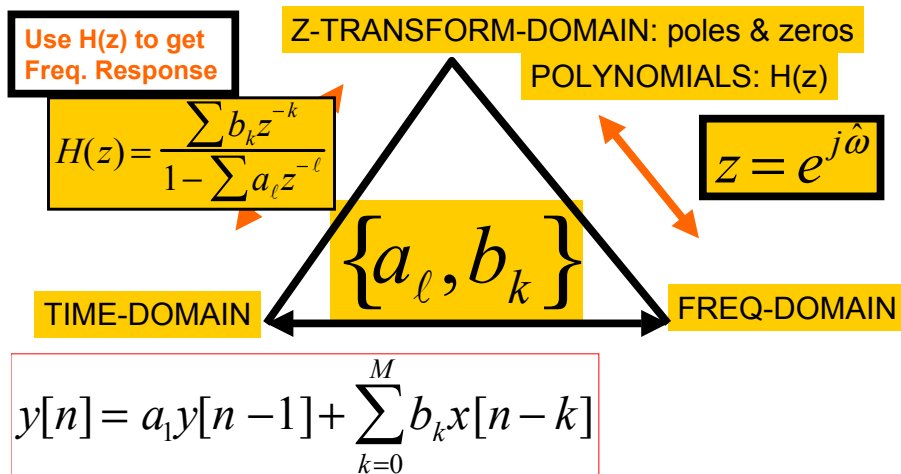
LECTURE OBJECTIVES

- **SECOND-ORDER** IIR FILTERS
 - TWO FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^2 b_k x[n-k]$$

- H(z) can have **COMPLEX POLES** & ZEROS
- **THREE-DOMAIN APPROACH**
 - BPFs have POLES NEAR THE UNIT CIRCLE

THREE DOMAINS



Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS			
	$x[n]$	\iff	$X(z)$
1.	$ax_1[n] + bx_2[n]$	\iff	$aX_1(z) + bX_2(z)$
2.	$x[n - n_0]$	\iff	$z^{-n_0} X(z)$
3.	$y[n] = x[n] * h[n]$	\iff	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	\iff	1
5.	$\delta[n - n_0]$	\iff	z^{-n_0}
6.	$a^n u[n]$	\iff	$\frac{1}{1 - az^{-1}}$

SECOND-ORDER FILTERS

- Two FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

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MORE POLES

- Denominator is QUADRATIC

- 2 Poles: REAL
- or COMPLEX CONJUGATES

$$\frac{a_1 \pm \sqrt{a_1^2 + 4a_2}}{2}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 - a_1 z - a_2}$$

PROPERTY OF REAL POLYNOMIALS

A polynomial of degree N has N roots. If all the coefficients of the polynomial are real, the roots either must be real, or must occur in complex conjugate pairs.

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TWO COMPLEX POLES

- Find Impulse Response ?

- Can OSCILLATE vs. n

- “RESONANCE”

$$(p_k)^n = (re^{j\theta})^n = r^n e^{jn\theta}$$

- Find **FREQUENCY RESPONSE**

- Depends on Pole Location
- Close to the Unit Circle?
 - Make **BANDPASS FILTER**

$$\text{pole} = re^{j\theta}$$

$$r \rightarrow 1?$$

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2nd ORDER EXAMPLE

$$h[n] = (0.9)^n \cos\left(\frac{\pi}{3}n\right)u[n] = (0.9)^n \frac{1}{2}(e^{j\pi n/3} + e^{-j\pi n/3})u[n]$$

$$H(z) = \frac{0.5}{1 - 0.9e^{j\pi/3}z^{-1}} + \frac{0.5}{1 - 0.9e^{-j\pi/3}z^{-1}}$$

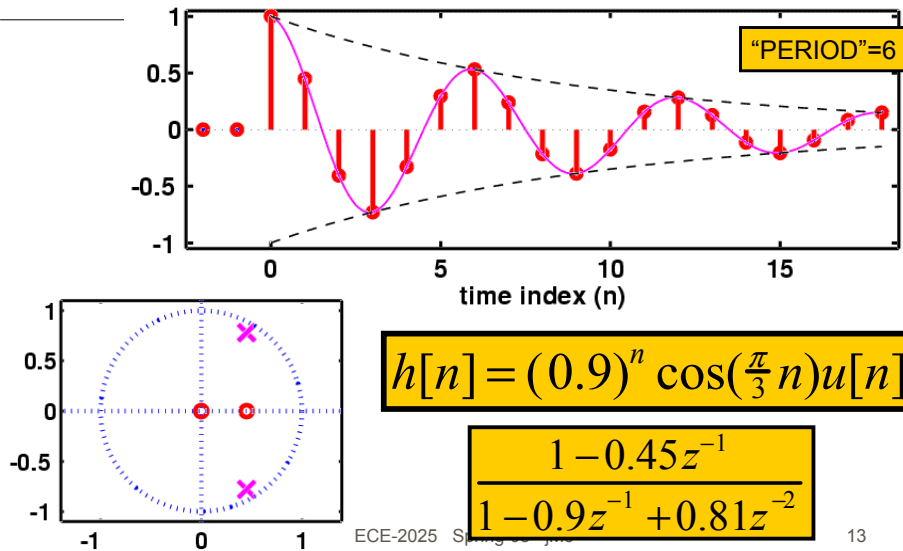
$$H(z) = \frac{1 - 0.9\cos\left(\frac{\pi}{3}\right)z^{-1}}{(1 - 0.9e^{j\pi/3}z^{-1})(1 - 0.9e^{-j\pi/3}z^{-1})}$$

$$H(z) = \frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

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h[n]: Decays & Oscillates



2nd ORDER Z-transform PAIR

$$h[n] = r^n \cos(\theta n)u[n]$$

GENERAL ENTRY for
z-Transform TABLE

$$H(z) = \frac{1 - r \cos \theta z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

$$h[n] = A r^n \cos(\theta n + \varphi)u[n]$$

$$H(z) = A \frac{\cos \varphi - r \cos(\theta - \varphi)z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

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2nd ORDER EX: n-Domain

$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

$$y[n] = 0.9y[n-1] - 0.81y[n-2] + x[n] - 0.45x[n-1]$$

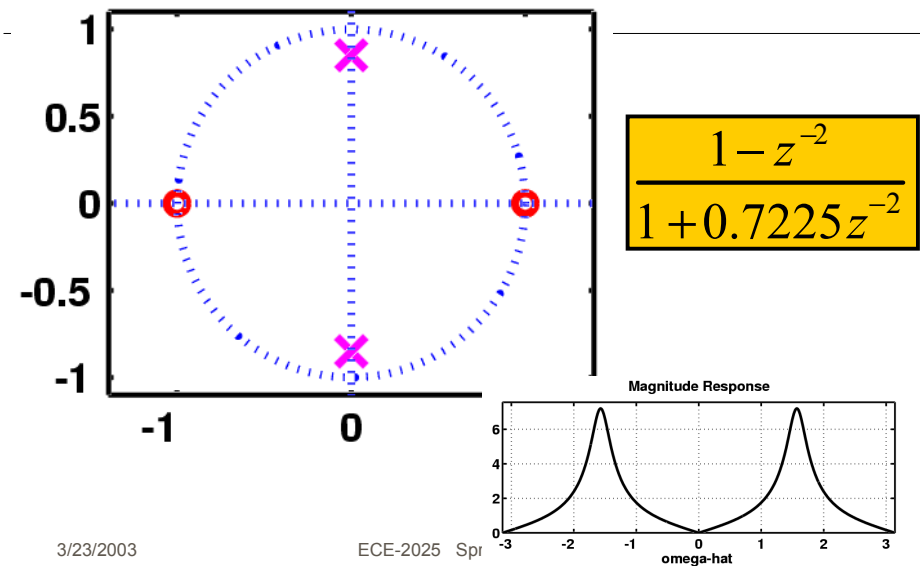
```
aa = [ 1, -0.9, 0.81 ];
bb = [ 1, -0.45 ];
nn = -2:19;
hh = filter( bb, aa, (nn==0) );
HH = freqz( bb, aa, [-pi,pi/100:pi] );
```

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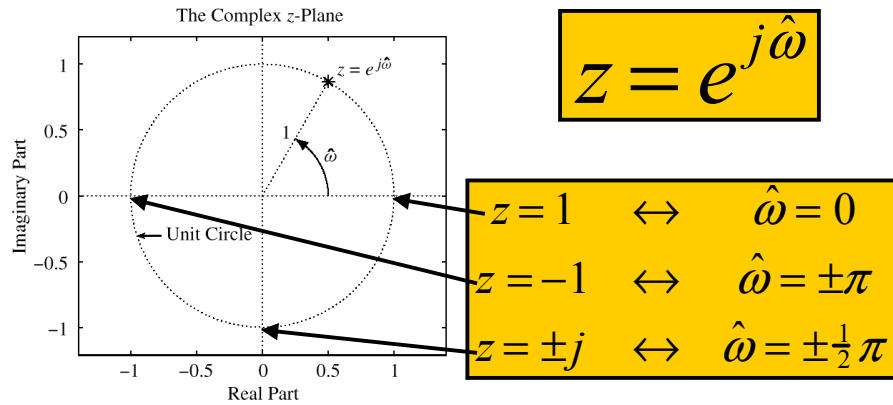
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Complex POLE-ZERO PLOT



UNIT CIRCLE

- MAPPING BETWEEN z and $\hat{\omega}$



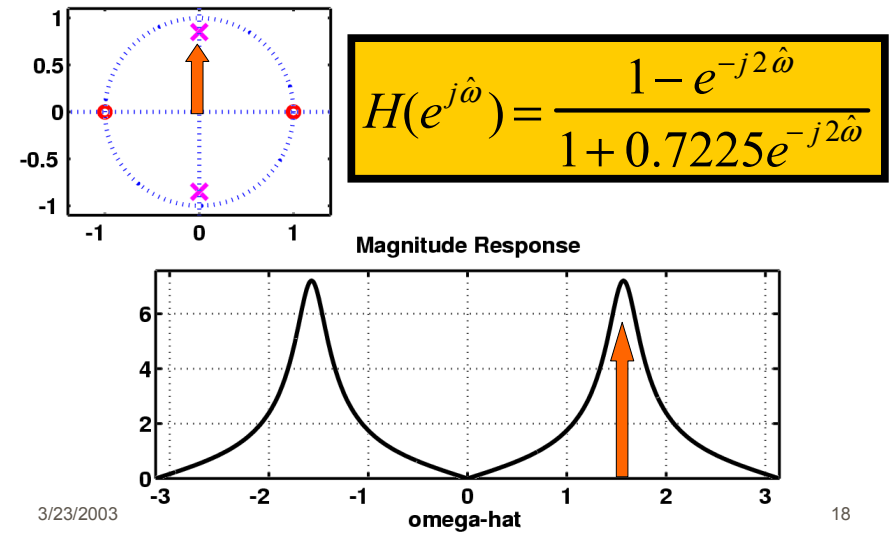
$$z = e^{j\hat{\omega}}$$

$$z = 1 \leftrightarrow \hat{\omega} = 0$$

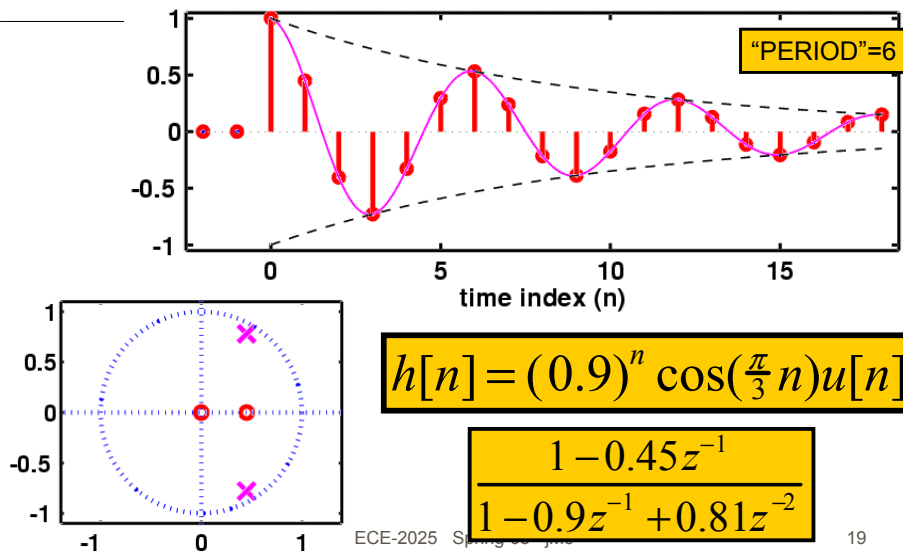
$$z = -1 \leftrightarrow \hat{\omega} = \pm\pi$$

$$z = \pm j \leftrightarrow \hat{\omega} = \pm\frac{1}{2}\pi$$

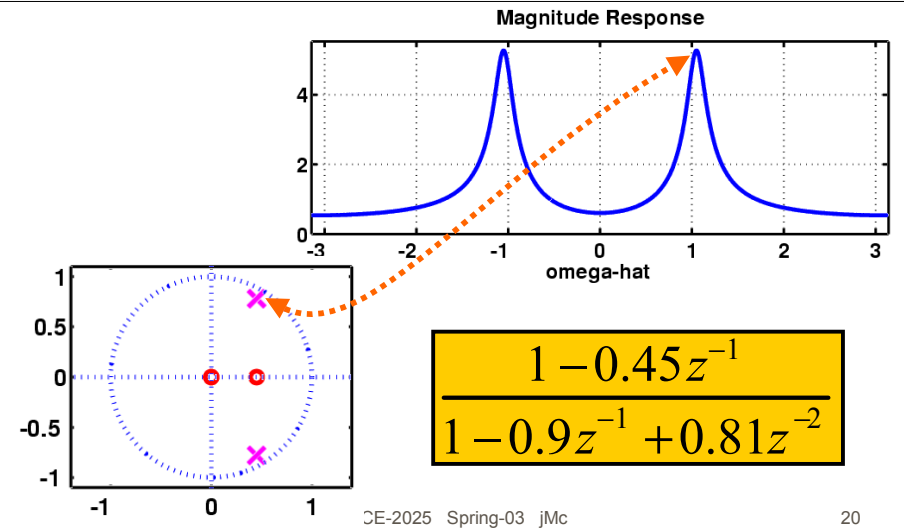
FREQUENCY RESPONSE from POLE-ZERO PLOT



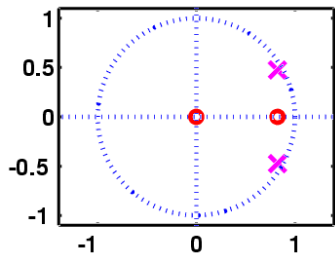
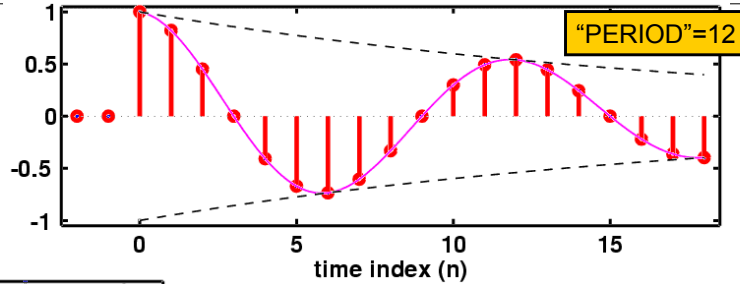
h[n]: Decays & Oscillates



Complex POLE-ZERO PLOT



h[n]: Decays & Oscillates



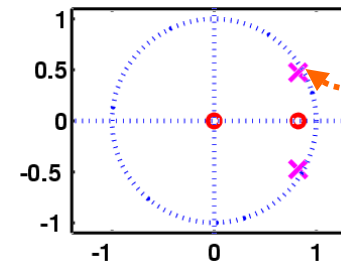
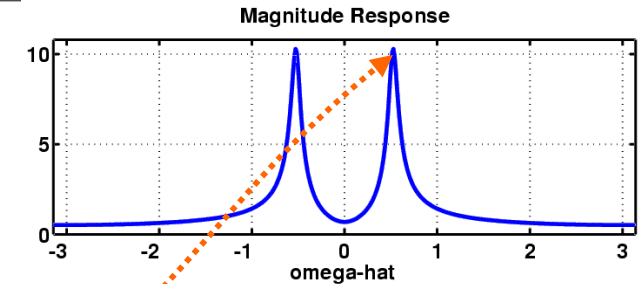
$$h[n] = (0.95)^n \cos\left(\frac{\pi}{6} n\right) u[n]$$

$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$

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Complex POLE-ZERO PLOT



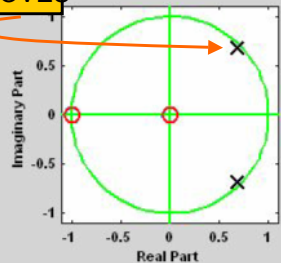
$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$

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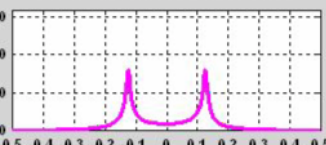
3 DOMAINS MOVIE: IIR

POLE MOVES

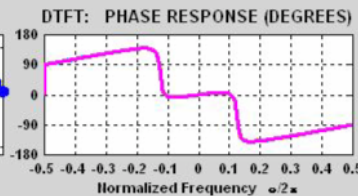
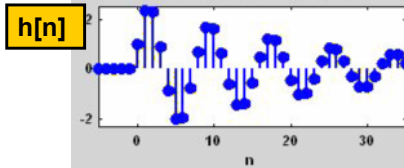


$$\frac{1 + z^{-1}}{1 - 1.36z^{-1} + 0.918z^{-2}}$$

H(z)



H(ω)



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THREE INPUTS

- Given:

$$H(z) = \frac{5}{1 + 0.8z^{-1}}$$

- Find the output, $y[n]$

- When

$$x[n] = \cos(0.2\pi n)$$

$$x[n] = u[n]$$

$$x[n] = \cos(0.2\pi n)u[n]$$

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SINUSOID ANSWER

- Given:

$$H(z) = \frac{5}{1 + 0.8z^{-1}}$$

- The input:

$$x[n] = \cos(0.2\pi n)$$

- Then $y[n]$

$$y[n] = M \cos(0.2\pi n + \psi)$$

$$H(e^{j0.2\pi}) = \frac{5}{1 + 0.8e^{-j0.2\pi}} = 2.919e^{j0.089\pi}$$

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Step Response

$$Y(z) = H(z)X(z) = \left(\frac{5}{1 + 0.8z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right)$$

Partial Fraction Expansion

$$Y(z) = \frac{A}{1 + 0.8z^{-1}} + \frac{B}{1 - z^{-1}} = \frac{(A + B) + (.8B - A)z^{-1}}{(1 + 0.8z^{-1})(1 - z^{-1})}$$

$$\Rightarrow (A + B) = 5 \quad \text{and} \quad (.8B - A) = 0$$

$$Y(z) = \frac{A}{1 + 0.8z^{-1}} + \frac{B}{1 - z^{-1}}$$

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Step Response

$$Y(z) = \frac{20}{9} \frac{1}{1 + 0.8z^{-1}} + \frac{25}{9} \frac{1}{1 - z^{-1}}$$

$$y[n] = \frac{20}{9} (-0.8)^n u[n] + \frac{25}{9} u[n]$$

$$y[n] \rightarrow \frac{25}{9} \quad \text{as} \quad n \rightarrow \infty$$

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Stability

- Nec. & suff. condition: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

$$h[n] = b(a)^n u[n] \Leftrightarrow H(z) = \frac{b}{1 - az^{-1}}$$

$$\sum_{n=0}^{\infty} |b||a|^n < \infty \quad \text{if} \quad |a| < 1 \Rightarrow$$

Pole must be
Inside unit circle

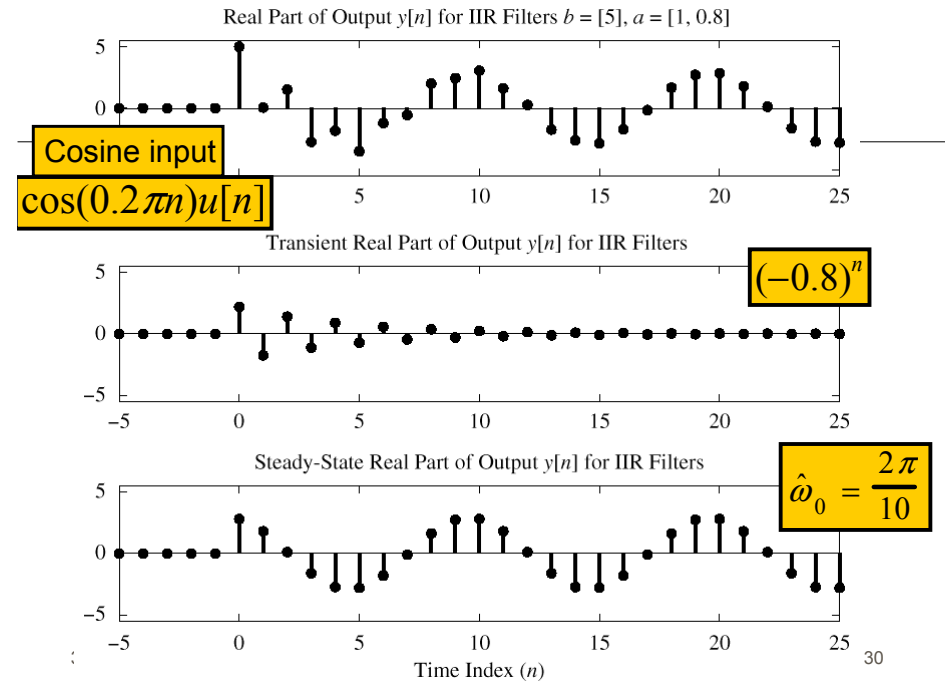
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SINUSOID starting at n=0

- We'll look at an example in MATLAB
 - $\cos(0.2\pi n)$
 - Pole at -0.8 , so a^n is $(-0.8)^n$
- There are two components:
 - TRANSIENT
 - Start-up region just after $n=0$; $(-0.8)^n$
 - STEADY-STATE
 - Eventually, $y[n]$ looks sinusoidal.
 - **Magnitude & Phase from Frequency Response**



STABILITY

- When Does the TRANSIENT DIE OUT ?

STEADY-STATE RESPONSE AND STABILITY

A stable system is one that does not “blow up.” This intuitive statement can be formalized by saying that the output of a stable system can always be bounded ($|y[n]| < M_y$) whenever the input is bounded ($|x[n]| < M_x$).³

$$y[n] = a_1 y[n - 1] + b_0 x[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

$$h[n] = b_0 a_1^n u[n]$$

need $|a_1| < 1$

STABILITY CONDITION

- ALL POLES INSIDE the UNIT CIRCLE
- UNSTABLE EXAMPLE: POLE @ $z=1.1$

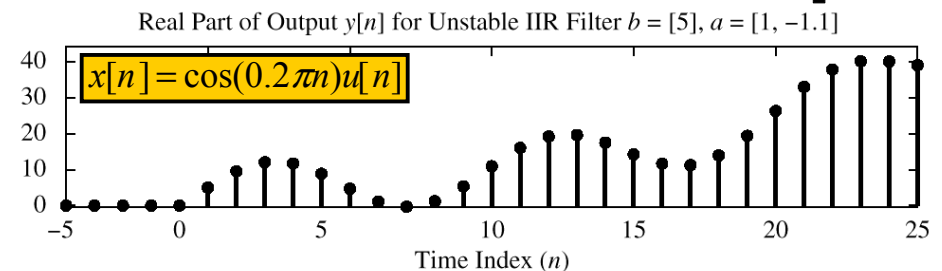


Figure 8.15 Illustration of an unstable IIR system. Pole is at $z = 1.1$.