

Lecture 19
Continuous-Time Signals
and Systems
28-Mar-03

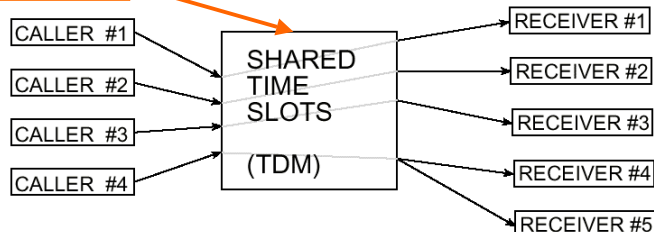
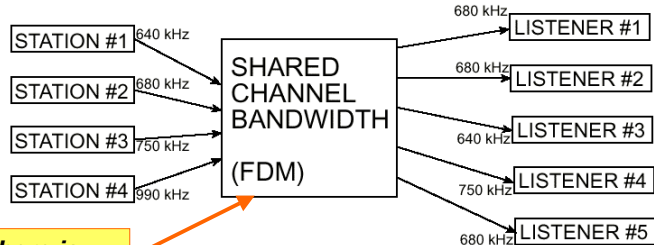
Info: Web-CT, Lab, HW

- Quiz #2: resolve grades by today
- Quiz #3 will be 11-April (Friday)
- Lab #10 on PeZ
- Lab #11 will be on convolution

The way communication systems work

It's an analog world out here.

These days, there is A lot of digital in here.



Lecture

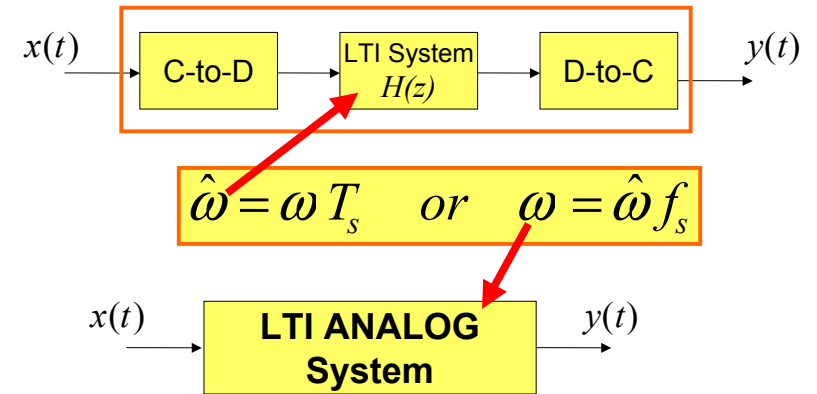
READING ASSIGNMENTS

- This Lecture:
 - Chapter 9, Sects 9-1 to 9-5
- Other Reading:
 - Recitation: Ch. 9, all
 - Next Lecture: Chapter 9, Sects 9-6 to 9-8

LECTURE OBJECTIVES

- Bye bye to D-T Systems for a while
- The **UNIT IMPULSE** signal
 - Definition
 - Properties
- Continuous-time signals and systems
 - Example systems
 - Review: **L**inearity and **T**ime-**I**nvariance
 - Convolution integral: **impulse** response

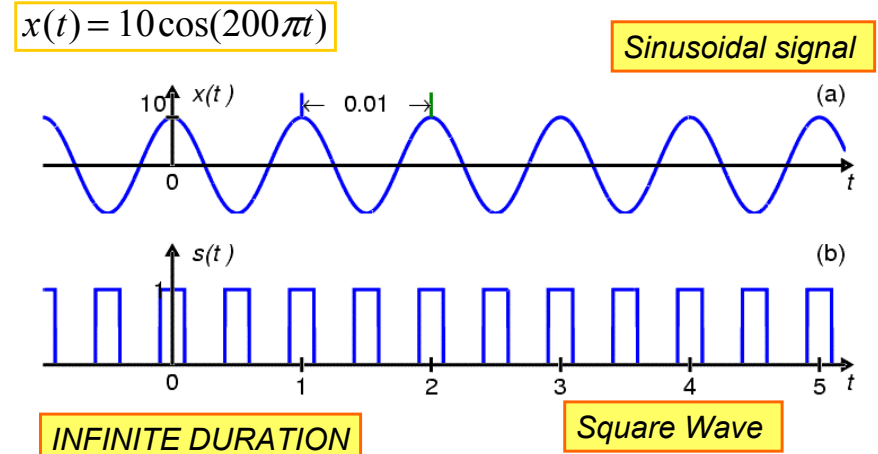
D-T Filtering of C-T Signals



ANALOG SIGNALS $x(t)$

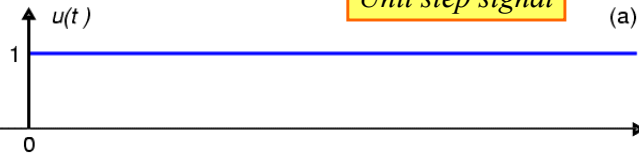
- INFINITE LENGTH
 - SINUSOIDS: $(t = \text{time in secs})$
 - PERIODIC SIGNALS
 - ONE-SIDED, e.g., for $t > 0$
 - UNIT STEP: $u(t)$
- FINITE LENGTH
 - SQUARE PULSE
- IMPULSE SIGNAL: $\delta(t)$
- DISCRETE-TIME: $x[n]$ is list of numbers**

CT Signals: PERIODIC

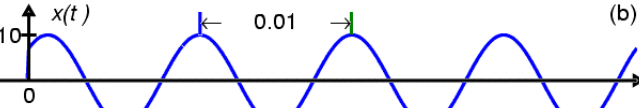


CT Signals: ONE-SIDED

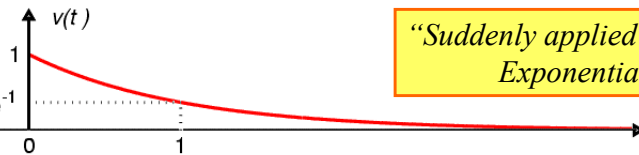
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



One-Sided
Sinusoid



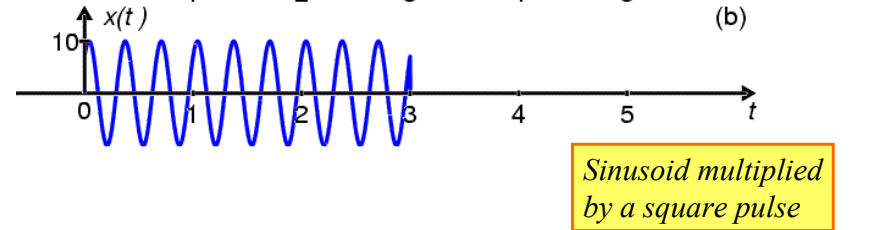
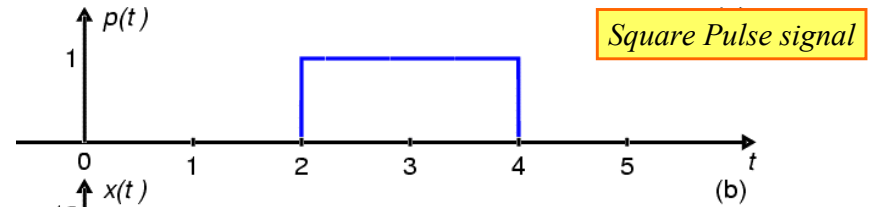
$$v(t) = e^{-t}u(t)$$



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CT Signals: FINITE LENGTH

$$p(t) = u(t-2) - u(t-4)$$



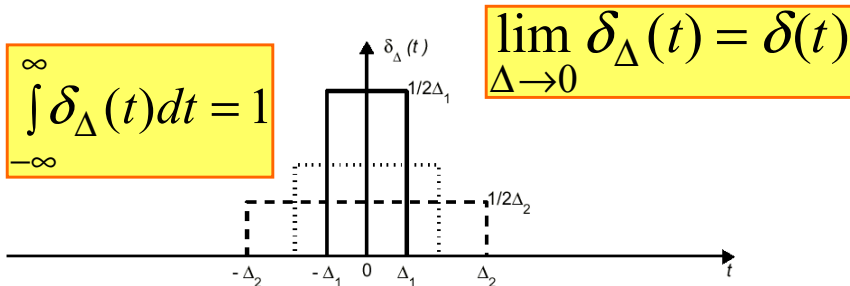
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What is an Impulse?

- A signal that is concentrated at one point.



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Defining the Impulse

- Assume the properties apply to the limit:

$$\lim_{\Delta \rightarrow 0} \delta_{\Delta}(t) = \delta(t)$$

- One "INTUITIVE" definition is:

$$\delta(t) = 0, \quad t \neq 0 \quad \text{Concentrated at } t=0$$

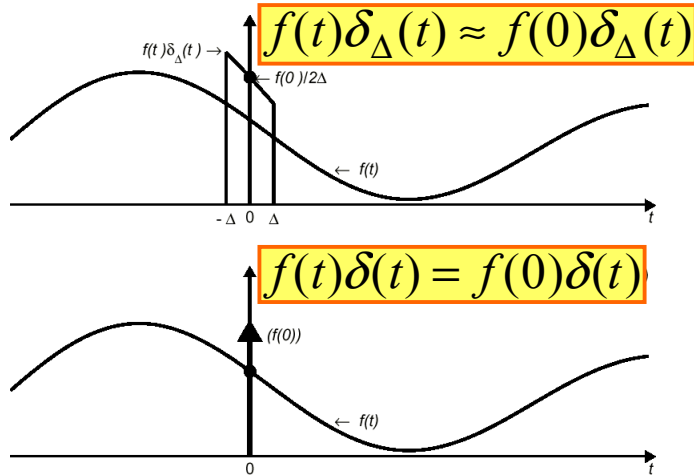
$$\int_{-\infty}^{\infty} \delta(\tau) d\tau = 1 \quad \text{Unit area}$$

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Sampling Property



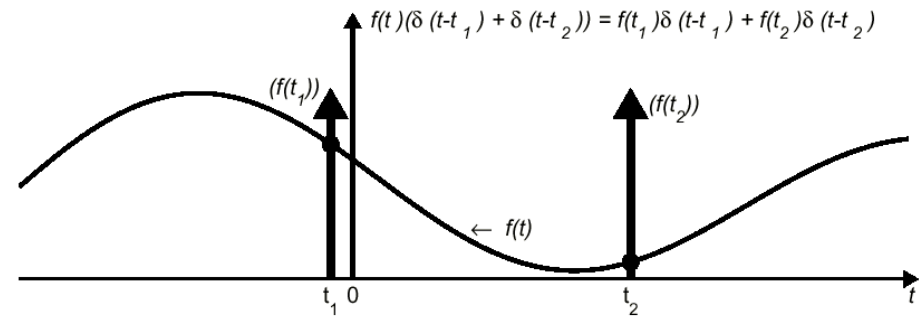
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General Sampling Property

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$



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Properties of the Impulse

$$\delta(t - t_0) = 0, \quad t \neq t_0 \quad \text{Concentrated at one time}$$

$$\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1 \quad \text{Unit area}$$

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0) \quad \text{Sampling Property}$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_0) dt = f(t_0) \quad \text{Extract one value of } f(t)$$

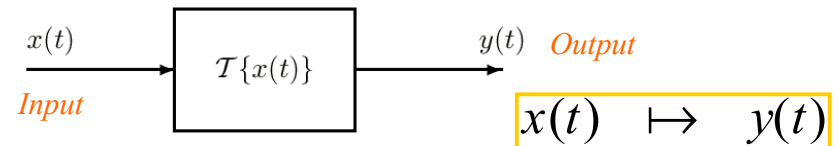
$$\frac{du(t)}{dt} = \delta(t) \quad \text{Derivative of unit step}$$

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Continuous-Time Systems



Examples:

- Delay $y(t) = x(t - t_d)$
- Modulator $y(t) = [A + x(t)]\cos \omega_c t$
- Integrator $y(t) = \int_{-\infty}^t x(\tau) d\tau$

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CT BUILDING BLOCKS

- INTEGRATOR (CIRCUITS)
- DIFFERENTIATOR
- DELAY by t_0
- MODULATOR (e.g., AM Radio)
- MULTIPLIER & ADDER

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Ideal Delay:

- Mathematical Definition:

$$y(t) = x(t - t_d)$$

- To find the IMPULSE RESPONSE, $h(t)$, let $x(t)$ be an impulse, so

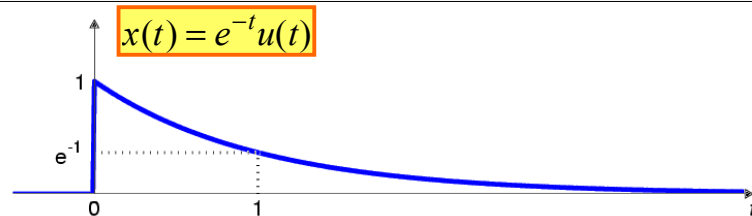
$$h(t) = \delta(t - t_d)$$

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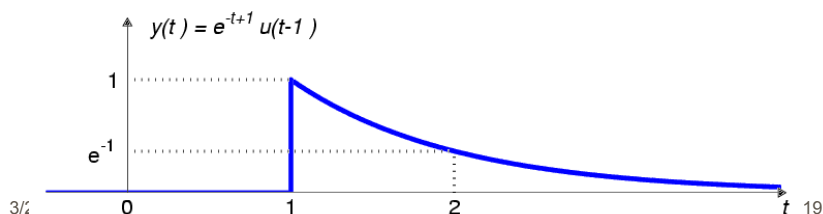
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Output of Ideal Delay of 1 sec



$$y(t) = x(t - 1) = e^{-(t-1)}u(t-1)$$



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0

1

2

t 19

Integrator:

- Mathematical Definition:

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

Running Integral

- To find the IMPULSE RESPONSE, $h(t)$, let $x(t)$ be an impulse, so

$$h(t) = \int_{-\infty}^t \delta(\tau) d\tau = u(t)$$

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Integrator:

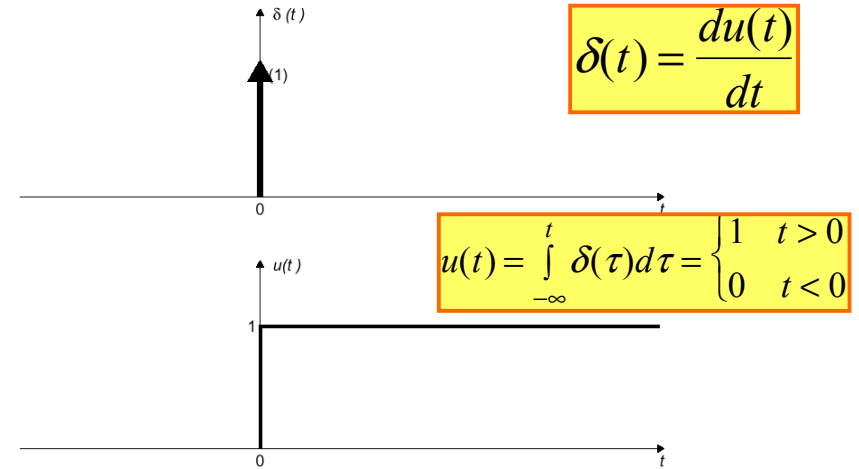
$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

- Integrate the impulse

$$\int_{-\infty}^t \delta(\tau) d\tau = u(t)$$

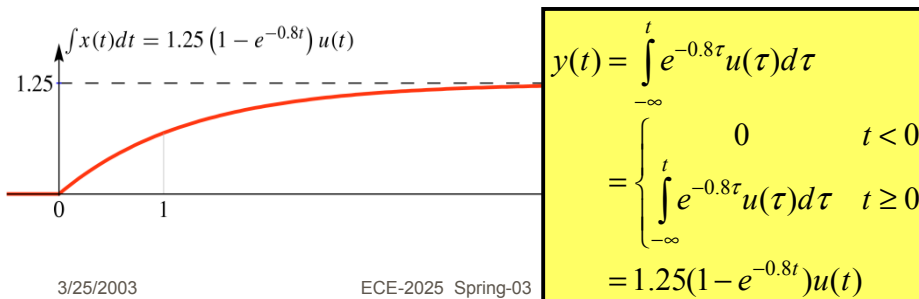
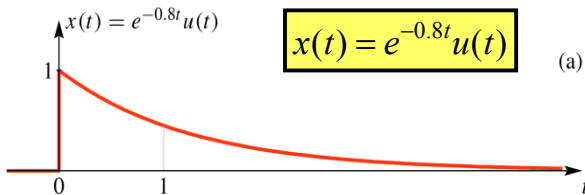
- IF $t < 0$, we get zero
- IF $t > 0$, we get one
 - Thus we have $h(t) = u(t)$ for the integrator

Graphical Representation



Output of Integrator

$$y(t) = \int_{-\infty}^t x(\tau) d\tau = x(t) * u(t)$$



Differentiator:

- Mathematical Definition:

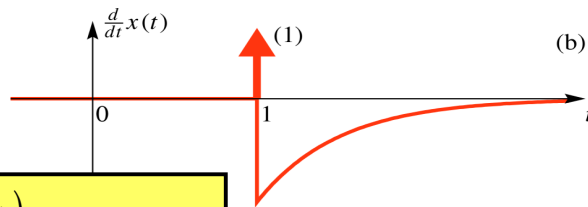
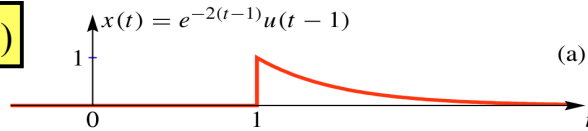
$$y(t) = \frac{dx(t)}{dt}$$

- To find $h(t)$, let $x(t)$ be an impulse, so

$$h(t) = \frac{d\delta(t)}{dt} = \delta^{(1)}(t) \quad \text{Doublet}$$

Differentiator Output: $y(t) = \frac{dx(t)}{dt}$

$$x(t) = e^{-2(t-1)}u(t-1)$$



$$\begin{aligned} y(t) &= \frac{d}{dt} (e^{-2(t-1)}u(t-1)) \\ &= -2e^{-2(t-1)}u(t-1) + e^{-2(t-1)}\delta(t-1) \\ &= -2e^{-2(t-1)}u(t-1) + 1\delta(t-1) \end{aligned}$$

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Linear and Time-Invariant (LTI) Systems

- If a continuous-time system is both linear and time-invariant, then the output $y(t)$ is related to the input $x(t)$ by a **convolution integral**

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

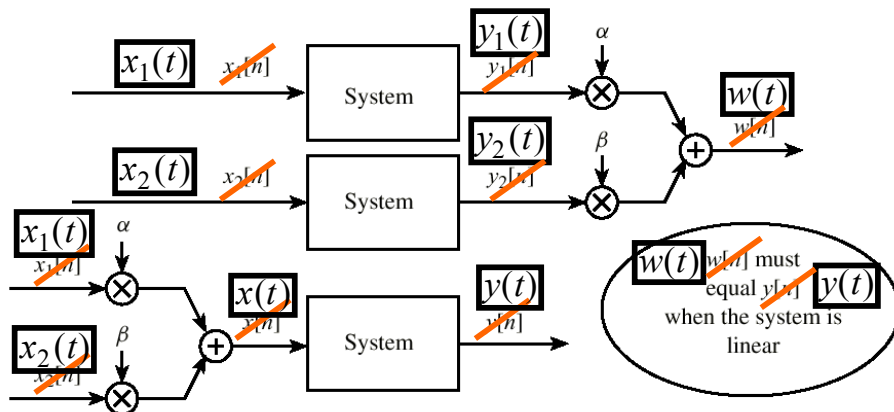
where $h(t)$ is the **impulse response** of the system.

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Testing for Linearity

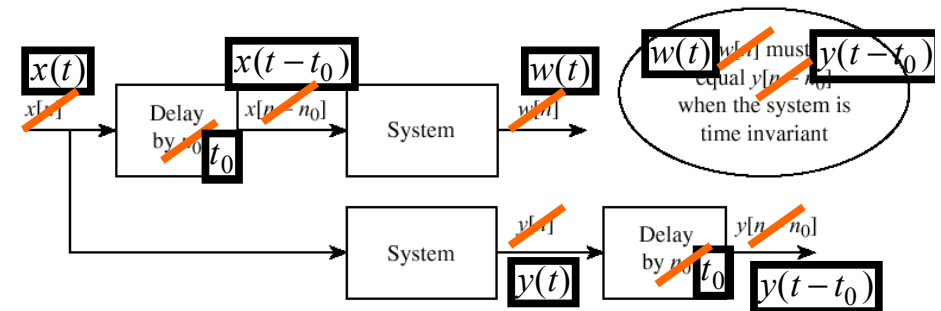


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Testing Time-Invariance



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Ideal Delay: $y(t) = x(t - t_d)$

- Linear

$$ax_1(t - t_d) + bx_2(t - t_d) = ay_1(t) + by_2(t)$$

- and Time-Invariant

$$w(t) = x((t - t_0) - t_d)$$

$$y(t - t_0) = x((t - t_0) - t_d)$$

Integrator: $y(t) = \int_{-\infty}^t x(\tau) d\tau$

- Linear

$$\int_{-\infty}^t [ax_1(\tau) + bx_2(\tau)] d\tau = ay_1(t) + by_2(t)$$

- And Time-Invariant

$$w(t) = \int_{-\infty}^t x(\tau - t_0) d\tau \quad \text{let } \sigma = \tau - t_0$$

$$\Rightarrow w(t) = \int_{-\infty}^{t-t_0} x(\sigma) d\sigma = y(t - t_0)$$

Modulator: $y(t) = [A + x(t)] \cos \omega_c t$

- Not** linear--obvious because

$$[A + ax_1(t) + bx_2(t)] \neq$$

$$[A + ax_1(t)] + [A + bx_2(t)]$$

- Not** time-invariant

$$w(t) = [A + x(t - t_0)] \cos \omega_c t \neq y(t - t_0)$$