GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
FINAL EXAM

DATE: July 30, 2002 COURSE: ECE 2025

NAME: SOLUTIONS, VER. 2 STUDENT #: ____________
LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

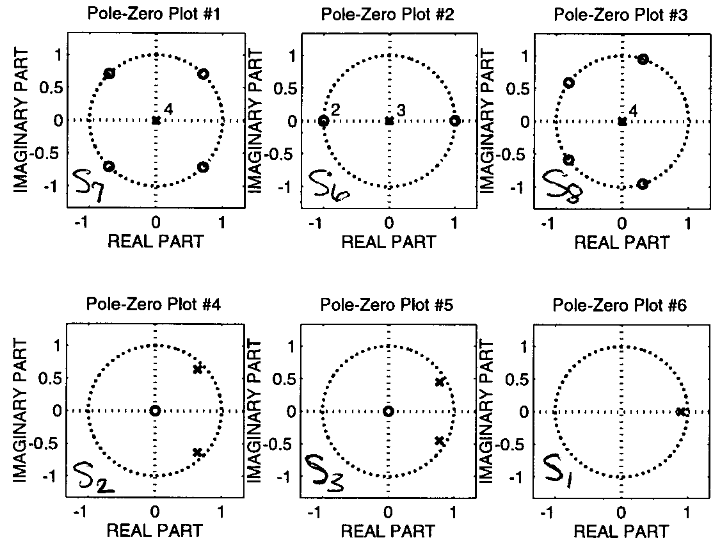
L05:Mon-4:00pm (Bordelon) L01:Tues-10:00am (Hunt) L02:Tues-12:00pm (Bordelon)

L03:Tues-2:00pm (Bordelon) L04:Tues-4:00pm (Brown) L06:Tues-6:00pm (Brown)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning clearly to receive partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	

Problem SUMMER-02-F.1:



For each of the pole-zero plots (#1, #2, $\cdots$, #6), determine which one of the following systems (specified by either $H(z)$, a difference equation, or a MATLAB statement) matches the pole-zero plot.

$\mathcal{S}_1$: $y[n] = 0.9y[n-1] + x[n-1]$

$\mathcal{S}_2$: $H(z) = \dfrac{z^{-1}}{1 - 1.273z^{-1} + 0.81z^{-2}}$

$\mathcal{S}_3$: `y = filter([0, 1], [1, -1.559, 0.81]);`

$\mathcal{S}_4$: $H(z) = \dfrac{z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$

$\mathcal{S}_5$: $H(z) = (1 + z^{-1})^3$

$\mathcal{S}_6$: $H(z) = 1 + z^{-1} - z^{-2} - z^{-3}$

$\mathcal{S}_7$: `y = conv([1, 0, 0, 0, 1], x);`

$\mathcal{S}_8$: $y[n] = \displaystyle\sum_{k=0}^{4} x[n-k]$

$\mathcal{S}_9$: $y[n] = x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4]$

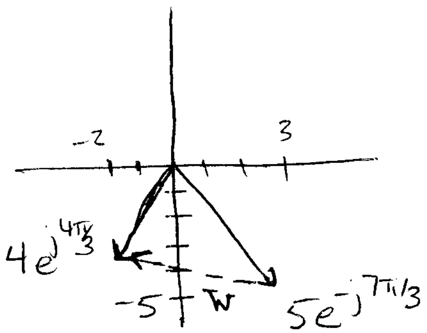
Problem SUMMER-02-F.2:

Solve the following complex number equations:

(a) Solve for $w[n]$ in the following equation by drawing a $\boxed{\text{phasor diagram}}$.

$$w[n] + 5\cos(9n - 7\pi/3) = 4\cos(9n + 4\pi/3)$$

Express $w[n]$ in the form $w[n] = A\cos(\omega_o n + \phi)$.



$$W + 5e^{-j7\pi/3} = 4e^{j4\pi/3}$$

$$W = 4.5826\, e^{j0.9395\pi}$$

$$w[n] = 4.5826\cos(9n + 0.9395\pi)$$

(b) Determine $\boxed{\text{all}}$ the solutions of the following polynomial equation:

$$(1 - z^{-1})(1 - 27z^{-3}) = 0$$

$$1 - z^{-1} = 0 \Rightarrow z = 1$$

$$1 - 27z^{-3} = 0 \Rightarrow z^3 = 27$$

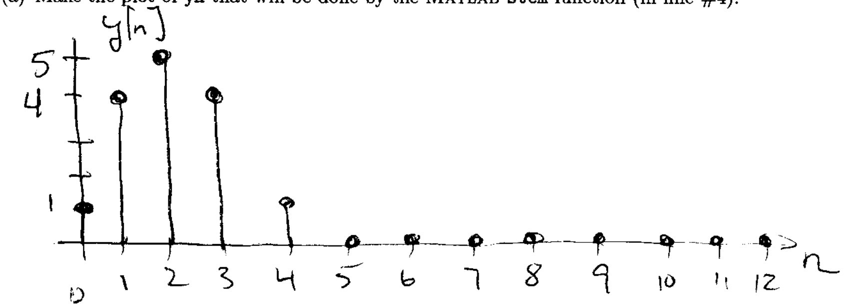
$$z = 3,\ 3e^{j2\pi/3},\ 3e^{j4\pi/3}$$

Problem SUMMER-02-F.3:

The following MATLAB code will compute a time response and the frequency response of a digital filter:

```
bb = [ 1 3 1 ];     aa = [ 1 ];
xn = [ ones(1,3), zeros(1,10) ];
yn = filter( bb, aa, xn );
subplot(2,1,1), stem( [0:12], yn, 'filled' );   %--- TIME RESPONSE
%
w = -pi : (pi/100) : pi;
H = freqz( bb, aa, w);
subplot(2,1,2), plot( w, abs(H) )    %--- FREQUENCY RESPONSE
```

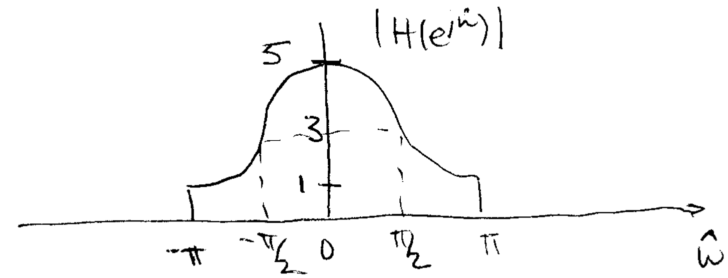
(a) Make the plot of yn that will be done by the MATLAB stem function (in line #4).



(b) Again referring to the MATLAB code above, make the plot of the magnitude response versus $\hat{\omega}$ over the range $-\pi \le \hat{\omega} \le \pi$. Justify by giving a $\boxed{\text{simple}}$ formula for the frequency response $H(e^{j\hat{\omega}})$.

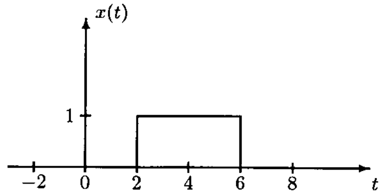
$$H(e^{j\hat{\omega}}) = 1 + 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

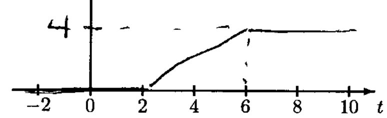
$$= e^{-j\hat{\omega}}\left[3 + 2\cos(\hat{\omega})\right]$$



Problem SUMMER-02-F.4:

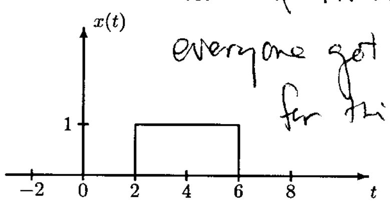
(a) If $h(t) = u(t)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis.*



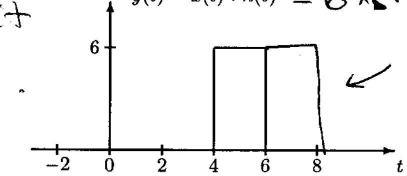


(b) Given that $y(t) = x(t) * h(t)$, find $h(t)$. $h(t) = \boxed{6\delta(t-2)}$

Because of mistake, everyone got credit for this part.

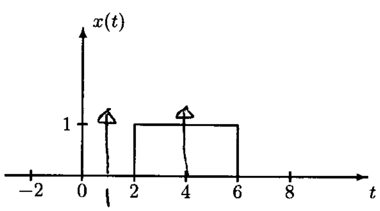


$y(t) = x(t) * h(t) = 6x(t-2)$

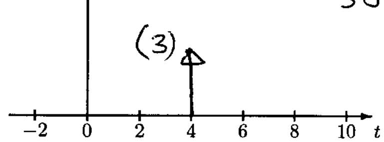


Should have been this.

(c) Plot $y(t) = 3x(t)[\delta(t-1) + \delta(t-4)]$ on the graph on the right.

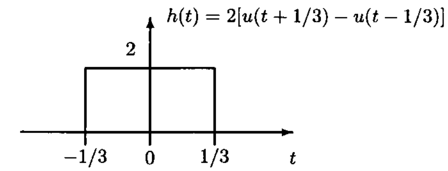


$y(t) = 3x(t)[\delta(t-1) + \delta(t-4)] = 3\delta(t-4)$



Problem SUMMER-02-F.5:

The impulse response of an LTI system is

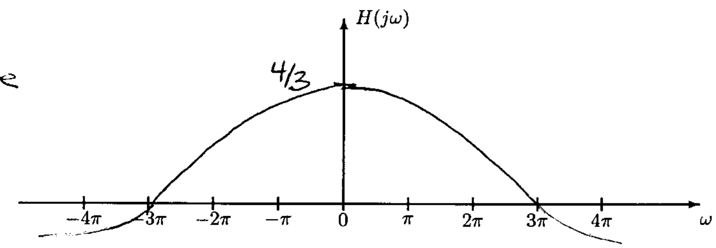


(a) Determine the frequency response $H(j\omega)$ of the system.

$\longleftrightarrow \frac{2\sin(\omega T)}{\omega}$

$$\Rightarrow H(j\omega) = \frac{4\sin(\omega/3)}{\omega}$$

(b) Make a carefully labeled sketch of $H(j\omega)$ on the axes below.



(c) Is this LTI system causal? Explain your answer to receive full credit.

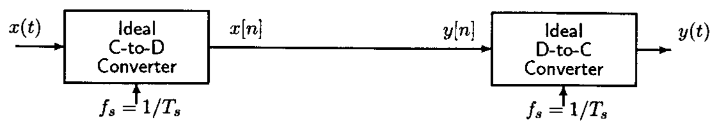
No, $h(t) \neq 0$ for all $t < 0$.

(d) Is this LTI system stable? Explain your answer to receive full credit.

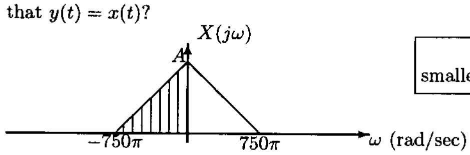
Yes. $\int_{-\infty}^{\infty} |h(t)|\,dt = 4/3 < \infty$

Problem SUMMER-02-F.6:

Consider the following system for sampling and reconstruction of a continuous-time signal:



(a) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that $y(t) = x(t)$?



smallest f_s = 750 samples/sec

$f_{max} = 375$ Hz

(b) In this part, the input signal is $x(t) = 5 + 5\cos(150\pi t + \pi/2)$. If the sampling rate is $\boxed{f_s = 100 \text{ samples/sec}}$, what is the corresponding output $y(t)$?

$$x[n] = 5 + 5\cos\left(\frac{150}{100}\pi n + \pi/2\right)$$

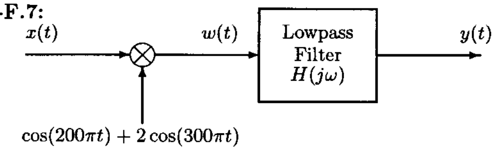
$$= 5 + 5\cos(1.5\pi n + \pi/2)$$

$$= 5 + 5\cos(0.5\pi n - \pi/2)$$

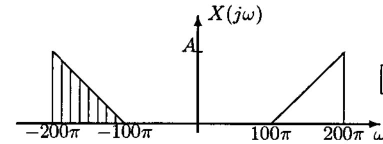
$$\Rightarrow y(t) = 5 + 5\cos(50\pi t - \pi/2)$$

$$\boxed{y(t) = 5 + 5\cos(50\pi t - \pi/2)}$$

Problem SUMMER-02-F.7:



In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$, as depicted in $\boxed{\text{Figure A}}$ below.

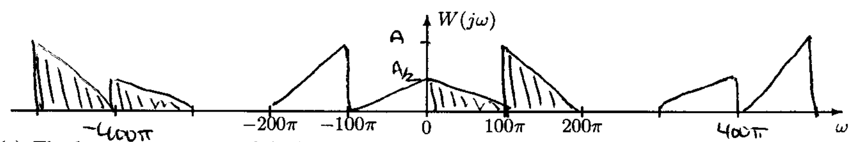


$\boxed{\text{Figure A}}$

(a) First give the general equation that expresses $W(j\omega)$, the Fourier transform of $w(t) = x(t)[\cos(200\pi t) + 2\cos(300\pi t)]$, in terms of $X(j\omega)$.

$$\boxed{W(j\omega) = \tfrac{1}{2}X(j(\omega+200\pi)) + \tfrac{1}{2}X(j(\omega-200\pi)) + X(j(\omega+300\pi)) + X(j(\omega-300\pi))}$$

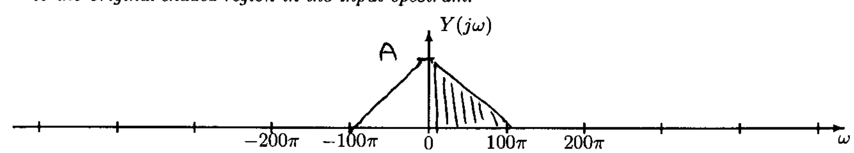
(b) Now **carefully** plot the Fourier transform $W(j\omega)$ for the specific input $x(t)$ whose Fourier transform $X(j\omega)$ is given above in $\boxed{\text{Figure A}}$. *Note that part of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding shaded region or regions in your plot of $W(j\omega)$, and be sure to carefully label both amplitudes and frequencies.*



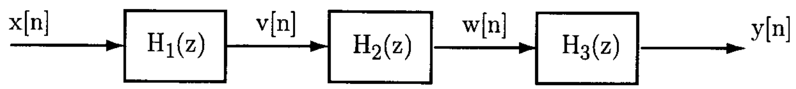
(c) The frequency response of the lowpass filter is

$$H(j\omega) = \begin{cases} 2 & |\omega| \le 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$

Plot the Fourier transform $Y(j\omega)$ below for the $X(j\omega)$ given in $\boxed{\text{Figure A}}$ above. *Be sure to carefully label both amplitudes and frequencies and be sure to shade the region corresponding to the original shaded region in the input spectrum.*



Problem SUMMER-02-F.8:
In the following cascade of systems, all systems are defined by their system functions.



$$H_1(z) = \frac{2}{1+\frac{1}{3}z^{-1}} \qquad H_2(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} \qquad H_3(z) = \frac{2}{1-\frac{1}{2}z^{-1}}$$

(a) Determine the unknown coefficients $\{b_k\}$ so that the output signal $y[n]$ will be $y[n] = 2\delta[n]$ when the input signal $x[n]$ is an impulse, i.e., $x[n] = \delta[n]$.

WANT $H_1(z)H_2(z)H_3(z) = 2$

$$H_1(z)H_2(z)H_3(z) = \frac{4(b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3})}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$= \frac{4(b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3})}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

$\Rightarrow b_0 = \frac{1}{2},\ b_1 = -\frac{1}{12},\ b_2 = -\frac{1}{12},\ b_3 = 0$

(b) Using part (a), determine whether the following statement is true or false:
"For any input signal $x[n]$, the output is always $y[n] = 2x[n]$."
Give a solid reason to back up your choice of true or false.

True. Overall system is LTI with

$h[n] = 2\delta[n] \Rightarrow y[n] = 2\delta[n] * x[n]$
$= 2x[n]$

for any input $x[n]$.