# ECE 2025, Spring 2003, Problem Set #2 Solutions

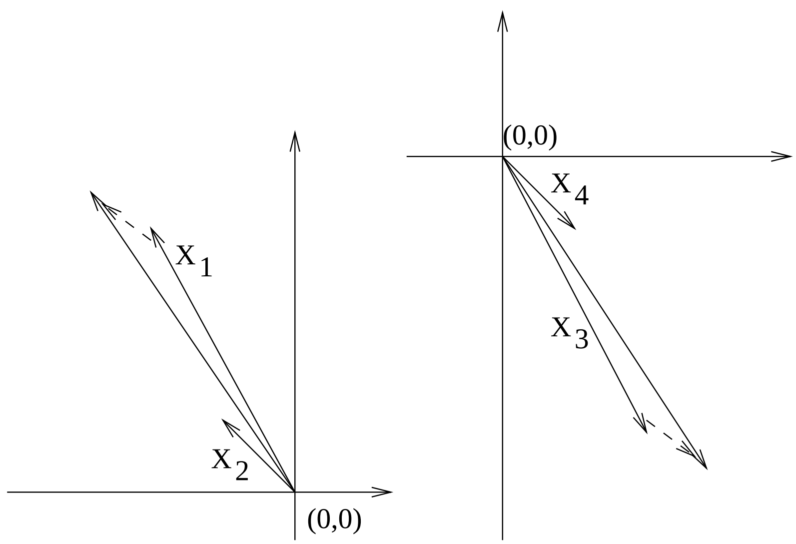
- Problem 2.1

(a) $x_a(t) = 3\cos(22\pi t - 4\pi/3) + \cos(22\pi t + 3\pi/4) - 3\cos(55\pi t - 4\pi/3) - \cos(55\pi t + 3\pi/4)$

$$\begin{aligned}
X_1 &= 3\ e^{-j4\pi/3} = -1.5000 + j2.5981 \\
X_2 &= e^{j3\pi/4} = -0.7071 + j0.7071 \\
X_1 + X_2 &= -2.2071 + j3.3052 = 3.9744\ e^{j2.1596} = 3.9744\ e^{j22\pi \times 0.0312} \\
X_3 &= 3\ e^{j(\pi - 4\pi/3)} = 1.5000 - j2.5981 \\
X_4 &= e^{j(\pi + 3\pi/4)} = 0.7071 - j0.7071 \\
X_3 + X_4 &= 2.2071 - j3.3052 = 3.9744\ e^{-j0.9820} = 3.9744\ e^{j(55\pi \times 0.0057)}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
x_a(t) &= 3.9744\cos(22\pi t + 2.1596) + 3.9744\cos(55\pi t - 0.9820) \\
&= 3.9744\cos(22\pi(t + 0.0312)) + 3.9744\cos(55\pi(t - 0.0057)).
\end{aligned}$$

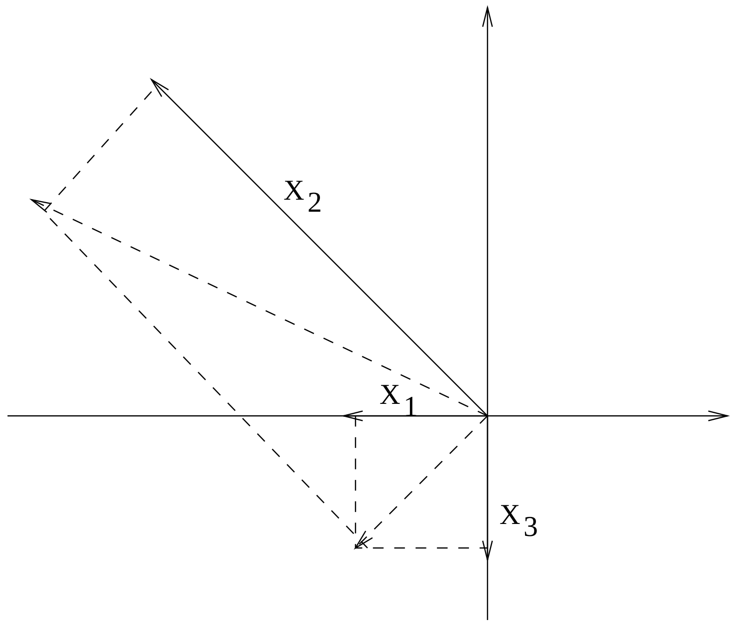



(b) $x_b(t) = \sqrt{2}\cos(89\pi t + 101\pi) + 5\cos(89\pi t - 101.25\pi) + \sqrt{2}\cos(89\pi t + 101.5\pi)$

$$\begin{aligned}
X_1 &= \sqrt{2}\ e^{j101\pi} = -1.4142 \\
X_2 &= 5\ e^{-j101.25\pi} = -3.5355 + j3.5355 \\
X_3 &= \sqrt{2}\ e^{j101.5\pi} = -j1.4142 \\
X_1 + X_2 + X_3 &= -4.9497 + j2.1213 = 5.3852\ e^{j2.7367} = 5.3852\ e^{j89\pi \times 0.0098}
\end{aligned}$$

Therefore,

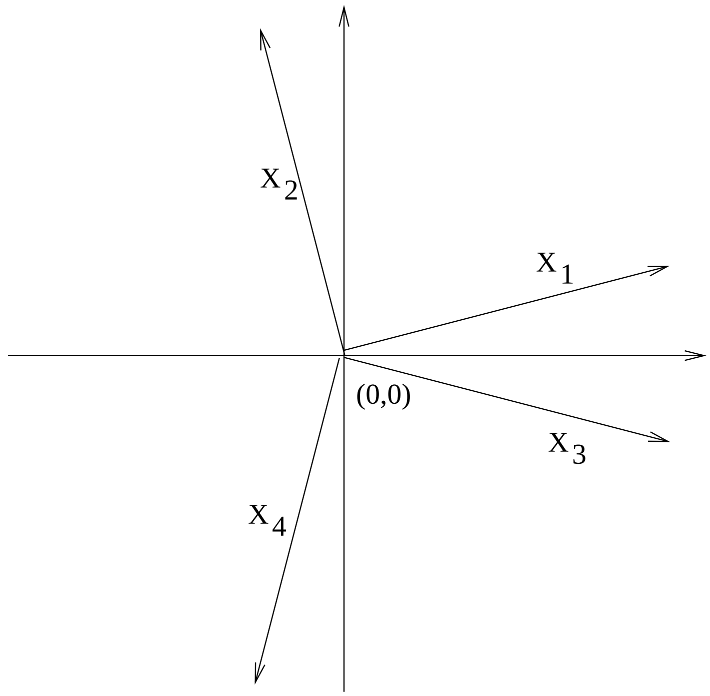
$$x_b(t) = 5.3852\cos(89\pi t + 2.7367) = 5.3852\cos(89\pi(t + 0.0098)).$$




(c) $x_c(t) = 7\cos(\pi t + \pi/12) + 7\cos(\pi t + 7\pi/12) + 7\cos(\pi t - 5\pi/12) + 7\cos(\pi t - 11\pi/12)$

$$\begin{aligned}
X_1 &= 7\ e^{j\pi/12} = 6.7615 + j1.8117 \\
X_2 &= 7\ e^{j7\pi/12} = -1.8117 + j6.7615 \\
X_3 &= 7\ e^{-j5\pi/12} = 1.8117 - j6.7615 \\
X_4 &= 7\ e^{-j11\pi/12} = -6.7615 - j1.8117 \\
X_1 + X_2 + X_3 + X_4 &= 0.
\end{aligned}$$

Therefore, $x_c(t) = 0$.




- Problem 2.2

$$x(t) = \sqrt{3}\cos(\omega_0 t - 2\pi/3) + 3\cos(\omega_0 t + 3\pi/2).$$

$$\begin{aligned} X_1 &= \sqrt{3}\ e^{-j2\pi/3} = -0.8660 - j1.5000 \\ X_2 &= 3\ e^{j3\pi/2} = -3j \\ X_1 + X_2 &= -0.8660 - j4.5000 = 4.5826\ e^{-j1.7609} \end{aligned}$$

Therefore,

$$x(t) = 4.5826\cos(\omega_0 t - 1.7609).$$

(a) Find $z_1(t)$ such that $\text{Re}\{z_1(t)\} = 3\cos(\omega_0 t + 3\pi/2)$.

$$z_1(t) = 3\ e^{j(\omega_0 t + 3\pi/2)} = 3\ e^{j3\pi/2}\ e^{j\omega_0 t} = -3j\ e^{j\omega_0 t}.$$

(b) Find $z(t)$ such that $x(t) = \text{Re}\{z(t)\}$.

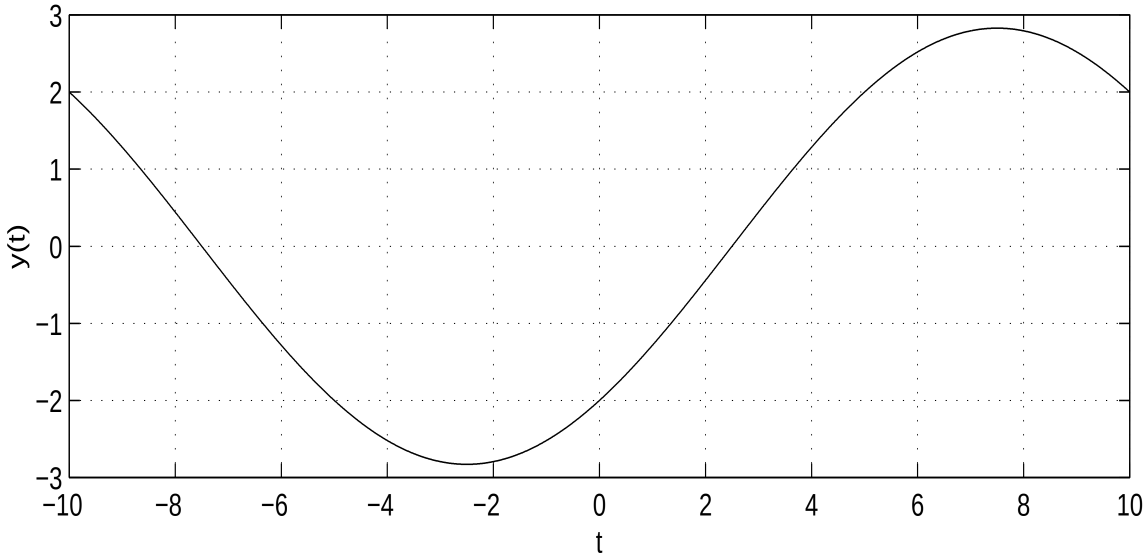
$$z(t) = 4.5826\ e^{j(\omega_0 t - 1.7609)}.$$

Its complex amplitude is $4.5826\ e^{-j1.7609}$.

(c) $\omega_0 = 0.1\pi$ rad/sec. Plot $\text{Re}\{(-2-2j)e^{j\omega_0 t}\}$ over $-10 \le t \le 10$.

First, we find $-2-2j = 2.8284\ e^{-j2.3562}$. Let

$$y(t) = \text{Re}\{(-2-2j)e^{j\omega_0 t}\} = 2.8284\cos(0.1\pi t - 2.3562) = 2.8284\cos(0.1\pi(t - 7.5)).$$




From $\omega_0 = 2\pi f = 0.1\pi$, we find $f = 0.05$Hz, and hence the period $T = 1/f = 20$ sec. The interval $-10 \le t \le 10$ includes exactly one period.

- Problem 2.3

  $z(t) = Z\ e^{j5\pi t}$ where $Z = 3\ e^{j\pi/4}$.

  (a) Evaluate $\int_0^{0.1} z^2(t)\ dt$.

  First, we have

$$z^2(t) = Z^2\ e^{j10\pi t} = 9\ e^{j\pi/2}\ e^{j10\pi t} = 9j\ e^{j10\pi t}.$$

  Therefore,

$$\begin{aligned} & \int_0^{0.1} z^2(t)\ dt \\ = & \int_0^{0.1} 9j\ e^{j10\pi t}\ dt \\ = & \int_0^{0.1} \frac{9j}{j10\pi}\ de^{j10\pi t} \\ = & \frac{9}{10\pi}\ e^{j10\pi t}\Big|_{t=0}^{0.1} \\ = & \frac{9}{10\pi}\left(e^{j\pi} - 1\right) \\ = & -\frac{9}{5\pi} = -0.5730. \end{aligned}$$

  (b) Evaluate $\int_{-0.1}^{0.1} z^2(t)\ dt$.

First, let us substitute $z^2(t) = 9j\ e^{j10\pi t}$.

$$\begin{aligned}
& \int_{-0.1}^{0.1} z^2(t)\ dt \\
= & \int_{-0.1}^{0.1} 9j\ e^{j10\pi t}\ dt \\
= & \int_{-0.1}^{0.1} \frac{9j}{j10\pi}\ de^{j10\pi t} \\
= & \frac{9}{10\pi}\ e^{j10\pi t}\Big|_{t=-0.1}^{0.1} \\
= & \frac{9}{10\pi}\left(e^{j\pi} - e^{-j\pi}\right) = 0.
\end{aligned}$$

(c) Evaluate $\int_0^2 z^*(t) z(t)\ dt$.

We find, $z^*(t)z(t) = |z(t)|^2 = 3^2 = 9$.

Therefore, $\int_0^2 z^*(t)z(t)\ dt = \int_0^2 9\ dt = 18$.

- Problem 2.4

$$\begin{aligned}
2\cos(\omega_0 t + 2\pi/3) &= A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(\omega_0 t + \phi_2) \quad (1) \\
2\cos(\omega_0 t + \pi) &= A_1 \cos(\omega_0 t + \phi_1) - A_2 \cos(\omega_0 t + \phi_2) \quad (2)
\end{aligned}$$

From (1)+(2), we obtain

$$2\cos(\omega_0 t + 2\pi/3) + 2\cos(\omega_0 t + \pi) = 2A_1 \cos(\omega_0 t + \phi_1).$$

Equivalent complex-number equation:

$$A_1\ e^{j\phi_1} = e^{j2\pi/3} + e^{j\pi}.$$

From (1)-(2), we obtain

$$2\cos(\omega_0 t + 2\pi/3) - 2\cos(\omega_0 t + \pi) = 2A_2 \cos(\omega_0 t + \phi_2).$$

Equivalent complex-number equation:

$$A_2\ e^{j\phi_2} = e^{j2\pi/3} - e^{j\pi}.$$

Let

$$\begin{aligned}
X_1 &= e^{j2\pi/3} = -0.5 + j0.8660, \\
X_2 &= e^{j\pi} = -1,
\end{aligned}$$

then

$$
\begin{aligned}
X_1 + X_2 &= -1.5 + j0.8660 = 1.7320\ e^{j2.6180}, \\
X_1 - X_2 &= 0.5 + j0.8660 = e^{j1.0472}.
\end{aligned}
$$

It follows then that

$$A_1\ e^{j\phi_1} = 1.7320\ e^{j2.6180},$$

$$A_2\ e^{j\phi_2} = e^{j1.0472}.$$

Answers $A_1 = 1.7321$ and $A_2 = 1$ are unique.

Answers $\phi_1 = 2.6180 + 2\pi k$ and $\phi_2 = 1.0472 + 2\pi k$ (where $k$ is any integer), are not unique.

- Problem 2.5

(a)

$$
\begin{aligned}
x(t) &= 9 + 10\cos(20\pi t - \pi/3) + 7\cos(50\pi t + \pi/4) \\
&= 9\ e^{j0} + 5\ e^{-j\pi/3}\ e^{j20\pi t} + 5\ e^{j\pi/3}\ e^{-j20\pi t} \\
&\quad +3.5\ e^{j\pi/4}\ e^{j50\pi t} + 3.5\ e^{-j\pi/4}\ e^{-j50\pi t}.
\end{aligned}
$$

(b) Spectrum representation of $x(t)$ as shown in the plot with

$$a_0 = 9,$$

$$\omega_1 = 20\pi, \quad a_1 = 5\ e^{-j\pi/3}, \quad a_{-1} = 5\ e^{j\pi/3},$$

$$\omega_2 = 50\pi, \quad a_2 = 3.5\ e^{j\pi/4}, \quad a_{-2} = 3.5\ e^{-j\pi/4}.$$