Problem #1

a) The two frequencies are

$$\left.\begin{array}{l} f_1 = 60 = 2^2 \cdot 3 \cdot 5 \\ f_2 = 175 = 5^2 \cdot 7 \end{array}\right\} \text{The greatest common divisor} = 5$$

so the fundamental frequency is $f_0 = 5\,Hz$, and $T_0 = 1/5$ sec.

b) $\omega_0 = 2\pi f_0 = 10\pi$ rad/sec.

c) The d.c. value is equal to the amplitude of the spectral line at $\omega = 0$:

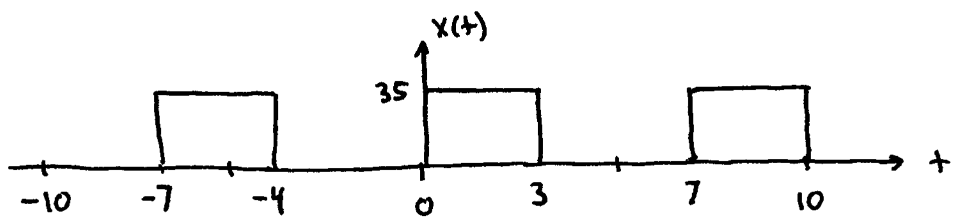
$$\text{d.c. value} = 11e^{j\pi} = -11$$

d) Note that with a fundamental frequency of 5 Hz, this signal contains a d.c. term along with the 12th and 35th harmonics. Thus,

	F.S. coefficient
$k = 35$	$4e^{-j\pi/2}$
$k = 12$	$6e^{-j\pi/3}$
$k = 0$	-11
$k = -12$	$6e^{j\pi/3}$
$k = -35$	$4e^{j\pi/2}$

Problem #2

a)



b) Since the period is $T_0 = 7$, then

$$a_0 = \frac{1}{7}\int_0^7 x(t)\,dt = \frac{1}{7}\int_0^3 35\,dt = \frac{105}{7} = 15$$

c) For the Fourier series coefficients we have

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-jk\omega_0 t}\,dt = \frac{1}{7}\int_0^3 35\, e^{-jk2\pi t/7}\,dt$$

$$= \frac{35}{7}\,\frac{1}{-jk2\pi/7}\, e^{-jk2\pi t/7}\Big|_0^3 = \frac{35}{-jk2\pi}\left[e^{-jk6\pi/7} - 1\right]$$

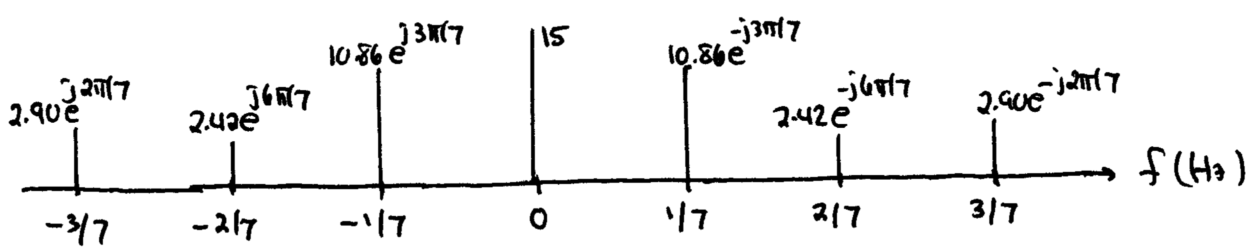
$$= \frac{35}{jk2\pi}\left[1 - e^{-jk6\pi/7}\right] = \frac{35}{jk2\pi}\, e^{-jk3\pi/7}\underbrace{\left[e^{jk3\pi/7} - e^{-jk3\pi/7}\right]}_{2j\sin(3\pi k/7)}$$

$$= \frac{35}{k\pi}\, e^{-jk3\pi/7}\sin(3\pi k/7)$$

d) With $f_0 = 1/7$ Hz, if we want to plot the spectrum over the frequency range

$$-\tfrac{1}{2} < f < \tfrac{1}{2} \text{ Hz}$$

then we need to plot the spectral lines corresponding to the Fourier series coefficients for $|k| \leq 3$.



$$a_0 = 15$$

$$a_1 = \frac{35}{\pi} e^{-j3\pi/7}\sin(3\pi/7) = 10.86\, e^{-j3\pi/7} = a_{-1}^*$$

$$a_2 = \frac{35}{2\pi} e^{-j6\pi/7}\sin(6\pi/7) = 2.42\, e^{-j6\pi/7} = a_{-2}^*$$

$$a_3 = \frac{35}{3\pi} e^{-j9\pi/7}\sin(9\pi/7) = -2.90 e^{-j9\pi/7} = 2.90\, e^{-j2\pi/7} = a_{-3}^*$$

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Problem #3

a) we know that $$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk(2\pi/T)t}dt$$

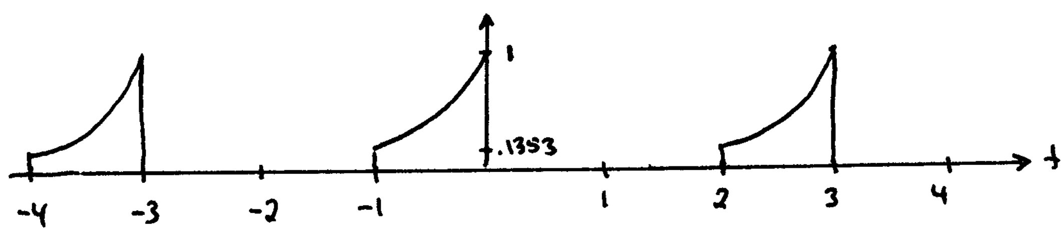
where the integral may be over any interval covering a period. we are given

$$a_k = \frac{1}{3}\int_{-1}^{0} e^{2t}e^{-j(2\pi/3)kt}dt$$

As we see, $T=3$ and $x(t)=e^{2t}$ for $-1 \le t \le 0$. Therefore, $x(t)$ must be zero for $0 \le t \le 2$, which is the remainder of the period. So,

$$x(t) = \begin{cases} e^{2t} & -1 \le t \le 0 \\ 0 & 0 < t \le 2 \end{cases}$$

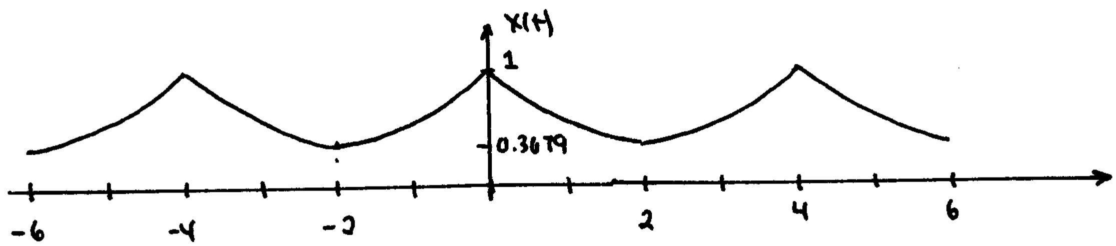
b)



c) $$a_0 = \frac{1}{3}\int_{-1}^{2} x(t)dt = \frac{1}{3}\int_{-1}^{0} e^{2t}dt = \frac{1}{6}e^{2t}\Big|_{-1}^{0} = \frac{1}{6}\left[1-e^{-2}\right] = 0.1441$$

Problem #4 : $x(t) = e^{-|t|/2}$ for $-2 \le t < 2$.

a)



b) $$a_0 = \frac{1}{4}\int_{-2}^{2} x(t)dt = \frac{1}{4}\int_{-2}^{2} e^{-|t|/2}dt = \frac{1}{2}\int_0^2 e^{-t/2}dt$$

$$= \frac{1}{2}(-2)e^{-t/2}\Big|_0^2 = 1-e^{-1} = 0.6321$$

c) $$a_k = \frac{1}{4}\int_{-2}^{2} x(t)\, e^{-jk(2\pi/4)t}\, dt = \frac{1}{4}\int_{-2}^{2} e^{-|t|/2}\, e^{-jk\pi t/2}\, dt$$

d) $$a_k = \frac{1}{4}\int_{0}^{2} e^{-(1+jk\pi)t/2}\, dt + \frac{1}{4}\int_{-2}^{0} e^{-(-1+jk\pi)t/2}\, dt$$

$$= \frac{1}{4}\,\frac{-2}{1+jk\pi}\, e^{-(1+jk\pi)t/2}\Big|_{0}^{2} + \frac{1}{4}\,\frac{-2}{-1+jk\pi}\, e^{(1-jk\pi)t/2}\Big|_{-2}^{0}$$

$$= \frac{-1/2}{1+jk\pi}\left[e^{-(1+jk\pi)} - 1\right] + \frac{-1/2}{-1+jk\pi}\left[1 - e^{-(1-jk\pi)}\right]$$

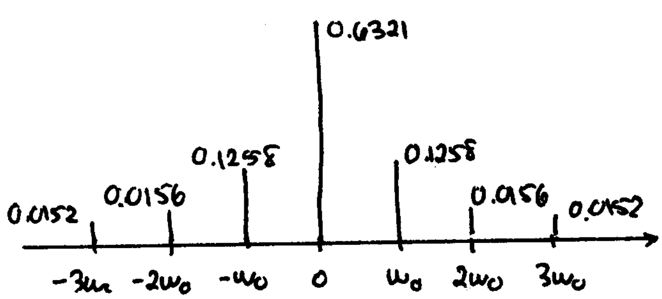
$$= \frac{1/2}{1+jk\pi} + \frac{1/2}{1-jk\pi} - \frac{1}{2}e^{-1}\left[\frac{e^{-jk\pi}}{1+jk\pi} + \frac{e^{jk\pi}}{1-jk\pi}\right]$$

$$= \frac{1}{1+(k\pi)^2} - \frac{\frac{1}{2}e^{-1}}{1+(k\pi)^2}\left[(1-jk\pi)e^{-jk\pi} + (1+jk\pi)e^{jk\pi}\right]$$

Note that
$$e^{jk\pi} = e^{-jk\pi} = (-1)^k$$
Therefore, the term in brackets simplifies to $2(-1)^k$ and we have

$$a_k = \frac{1}{1+(k\pi)^2}\left[1 - e^{-1}(-1)^k\right]$$

e)



$a_0 = 0.6321$

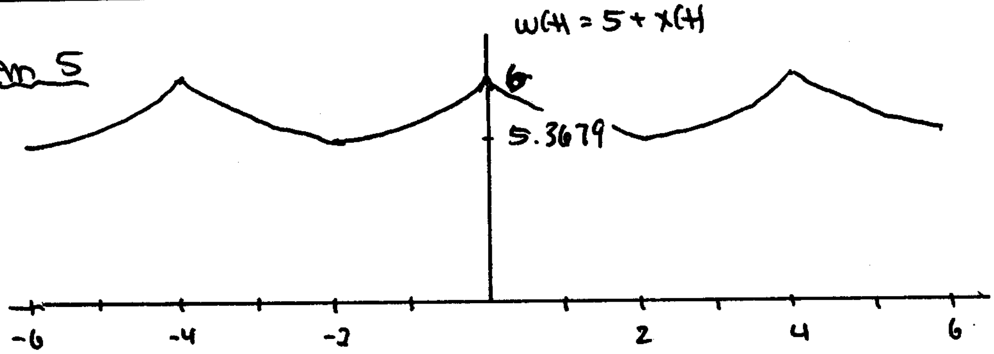
$a_1 = \frac{1}{1+\pi^2}(1+e^{-1}) = 0.1258$

$a_2 = \frac{1}{1+4\pi^2}(1-e^{-1}) = 0.0156$

$a_3 = \frac{1}{1+9\pi^2}(1+e^{-1}) = 0.0152$

Problem 5

a)



By adding a constant, all we do is shift the plot up or down by the constant, in this case up by 5.

b)
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkt(\pi/2)} \qquad \text{and}$$

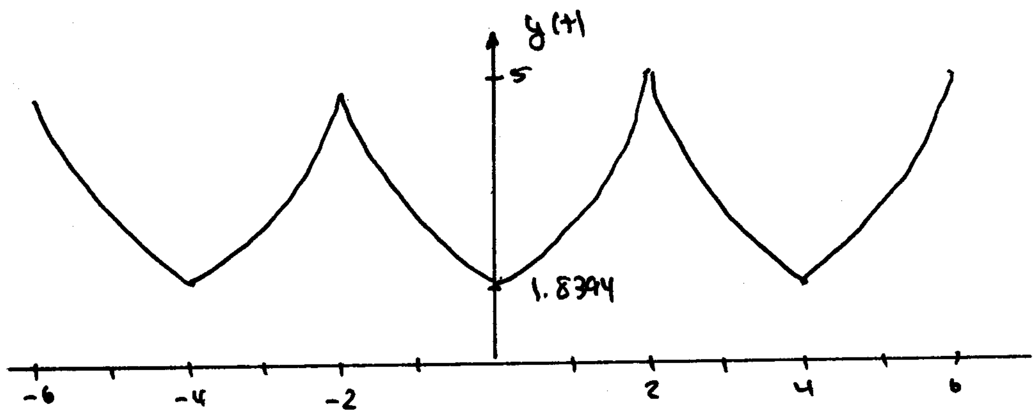
$$w(t) = x(t) + 5 = 5 + \sum_{k=-\infty}^{\infty} a_k e^{jkt(\pi/2)}$$

$$= (5 + a_0) + \sum_{k \neq 0} a_k e^{jkt(\pi/2)}$$

So we see that the F.S. coefficients of w(t) are the same as those for x(t), except for k=0 where

$$a_0 \longrightarrow 5 + a_0$$

c)



Here, $y(t) = 5x(t-2)$. So, we scale the amplitude of x(t) by 5, and we shift (delay) x(t) by two.

d) With $y(t) = 5x(t-2)$, Note that the F.S. coefficients are

$$a_k = \frac{1}{4}\int_{-2}^{2} 5x(t-2)e^{-jkt(\pi/2)}dt = 5\cdot\frac{1}{4}\int_{-4}^{0} x(u)e^{-jk(u+2)(\pi/2)}du$$

$$= 5e^{-jk\pi}\cdot\underbrace{\frac{1}{4}\int_{-4}^{0} x(u)e^{-jku(\pi/2)}du}_{\text{F.S. coef. for } x(t)\,!}$$

Therefore, the F.S. coefficients for $y(t)$ are found by multiplying the F.S. coefficients for $x(t)$ by $5e^{-jk\pi}$, so

$$a_k = 5e^{-jk\pi}\left[\frac{1}{1+(k\pi)^2}\left\{1-e^{-1}(-1)^k\right\}\right]$$

$$= \frac{5}{1+(k\pi)^2}\left[(-1)^k - e^{-1}\right]$$

(Again we have used the fact that $e^{-jk\pi} = (-1)^k$).

The F.S. coefficients for $k=0,1,2,$ and 3 are

$$a_0 = 5\cdot(0.6321) \qquad [\text{five times that for } x(t)]$$

$$a_1 = \frac{5}{1+\pi^2}(-1-e^{-1}) = a_{-1}$$

$$a_2 = \frac{5}{1+4\pi^2}(1-e^{-1}) = a_{-2}$$

$$a_3 = \frac{5}{1+9\pi^2}(-1-e^{-1}) = a_{-3}$$