ECE 2025 - Spring 2003

Problem Set #6 Solutions

(6.1) (a) $x[n] = 7\cos(0.5\pi n + \pi/4) = x(t)\Big|_{t = n/f_s}$

For $f_s = 400$ Hz an no aliasing,

$$x(t) = x[n]\Big|_{n = tf_s} = 7\cos(0.5\pi(400t) + \pi/4)$$

$$= 7\cos(2\pi(100)t + \pi/4)$$

$\Rightarrow$ All frequencies that will alias to $x[n]$: $100 + 400k$, $k = 1, 2, \cdots$

All frequencies that will fold to $x[n]$: $-100 + 400k$, $k = 1, 2, \cdots$

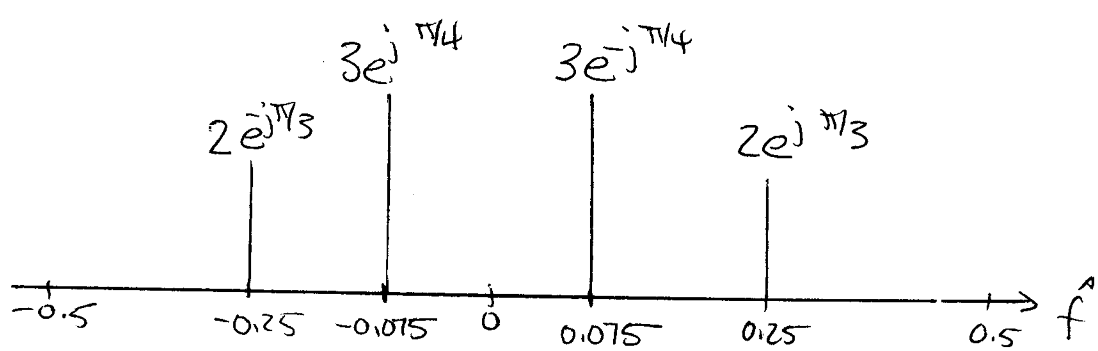
$\Rightarrow$ 2 inputs with frequency between 400 and 800 Hz :

$x_1(t) = 7\cos(2\pi(500)t + \pi/4)$ (aliases)

$x_2(t) = 7\cos(2\pi(700)t - \pi/4)$ (folds)

(b) Need $f_s > 2(100)$ samples/sec.

(c) $\hat{f} = f/f_s$ for $f_s = 400$ Hz.



6.2 (a) $x[n] = x(n/f_s)$

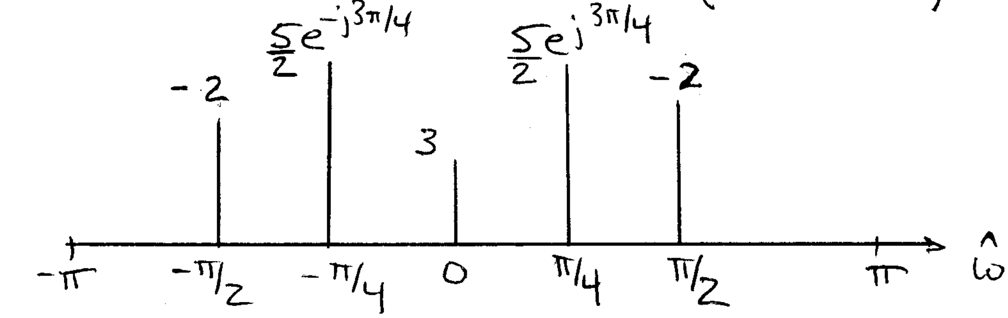
$$= 3 + 4\cos\left(2\pi(2000)\frac{n}{8000} - \pi\right) + 5\cos\left(2\pi(7000)\frac{n}{8000} - 3\pi/4\right)$$

$$= 3 + 4\cos(0.5\pi n - \pi) + 5\cos\left(\frac{14}{8}\pi n - 3\pi/4\right)$$

$$= 3 + 4\cos(0.5\pi n - \pi) + 5\cos\left(\left[\frac{14}{8}\pi - 2\pi\right]n - 3\pi/4\right)$$

$$= 3 + 4\cos(0.5\pi n - \pi) + 5\cos\left(-\frac{\pi}{4}n - 3\pi/4\right)$$

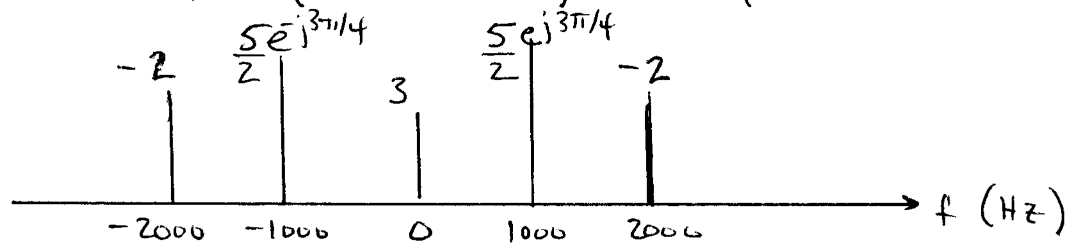
$$x[n] = 3 + 4\cos(0.5\pi n - \pi) + 5\cos\left(\frac{\pi}{4}n + 3\pi/4\right)$$



(b) $y(t) = x[t f_s]$

$$= 3 + 4\cos(0.5\pi(8000t) - \pi) + 5\cos\left(\frac{\pi}{4}(8000t) + 3\pi/4\right)$$

$$= 3 + 4\cos(2\pi(2000)t - \pi) + 5\cos(2\pi(1000)t + 3\pi/4)$$



(c) $x(t)$ has a fundamental period of $\frac{1}{1000}$.
So, if we sample with $T_s = \frac{1}{1000}$ (i.e., $f_s = 1000$ samples/sec.), then all the samples of $x[n]$ will be the same:

$$x[n] = 3 - 4 + 5\cos(-3\pi/4) = -1 + 5(-\sqrt{2}/2)$$

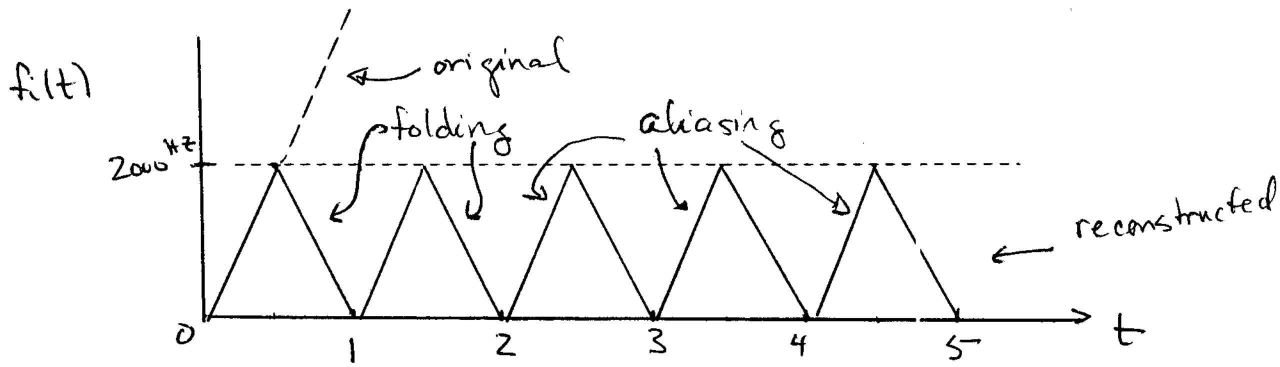
$$\Rightarrow y(t) = -1 + 5(-\sqrt{2}/2)$$

(6.3) (a) $x(t) = \cos(-4000\pi t^2) = \cos(4000\pi t^2)$

$$w_i(t) = \frac{d}{dt}\left[4000\pi t^2\right] = 8000\pi t$$

$$\Rightarrow f_i(t) = 4000t \quad , \quad 0 \le t \le 5$$

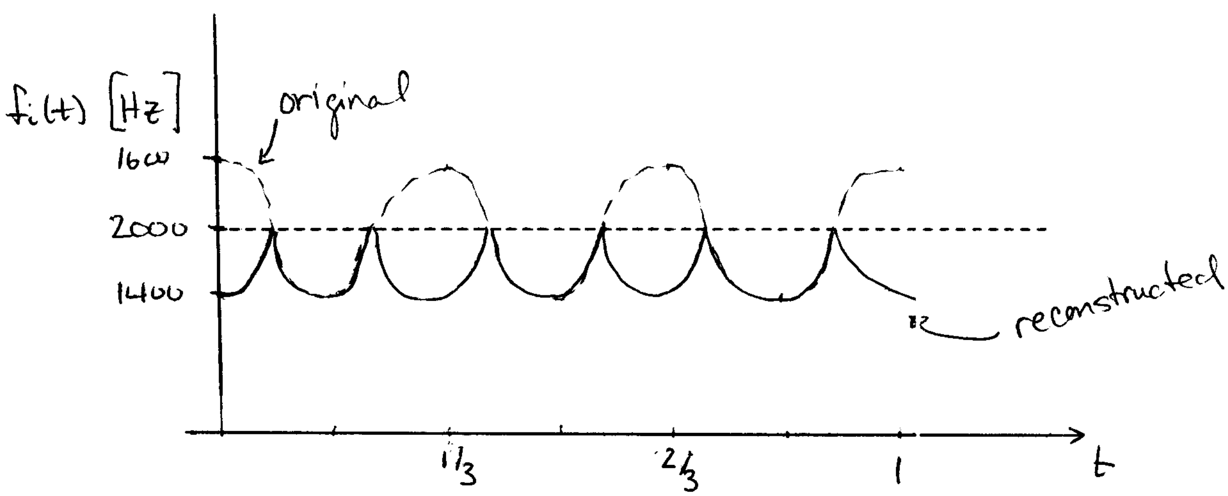
If $f_s = 4000$ Hz, then reconstructed $f_i(t)$ will be less than 2000 Hz.



(b) $$w_i(t) = \frac{d}{dt}\left[4000\pi t + 200\sin(6\pi t)\right]$$

$$= 4000\pi + 1200\pi\cos(6\pi t)$$

$$\Rightarrow f_i(t) = 2000 + 600\cos(6\pi t)$$



(6.4) Model the rotating wheel by $e^{j\omega_0 t}$ for some $\omega_0 > 0$.

Because of the 8 spokes, the signal looks like $e^{j8\omega_0 t} = e^{j2\pi 8 f_0 t}$ since all the spokes look the same.

Fundamental period of this signal is $\frac{1}{8f_0}$. So,

for $f_s = \frac{8f_0}{k}$, $k = 1, 2, 3, \cdots$, the wheel appears to be stationary.

Since $f_s = 30$, $f_0 = \frac{30k}{8}$ rotations/sec. are all the rotation rates that appear stationary.

With a diameter of 2 feet, the wheel travels 2π feet/rotation.

$$\Rightarrow \text{speed} = 2\pi\left(\frac{30k}{8}\right) = \frac{30\pi k}{4} \text{ feet/sec.}$$

$$\text{OR speed} = \left(\frac{15\pi k}{2} \text{ ft/sec}\right) \times \left(\frac{1}{5280} \text{ miles/ft}\right) \times \left(3600 \frac{\text{sec}}{\text{hour}}\right)$$

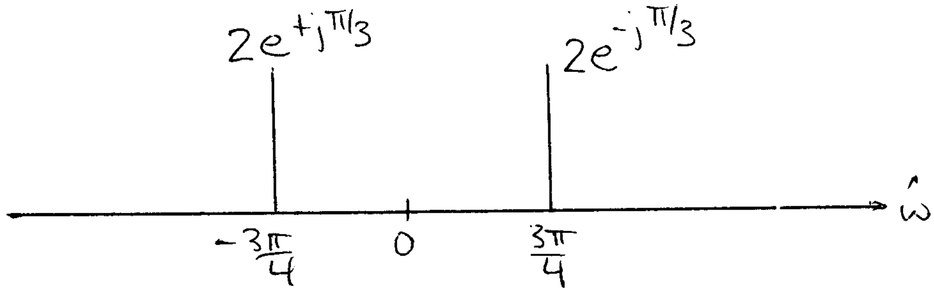
$$= \frac{225\pi k}{44} \text{ mph.}, \quad k = 1, 2, 3, \cdots$$

6.5 (a) $x[n] = x(n/f_s) = 2e^{j\left(44\pi\left(\frac{n}{16}\right) - \pi/3\right)} + 2e^{j\left(20\pi\left(\frac{n}{16}\right) + \pi/3\right)}$

$$= 2e^{j\left(\frac{11\pi n}{4} - \pi/3\right)} + 2e^{j\left(\frac{5\pi n}{4} + \pi/3\right)}$$

$$= 2e^{j\left(\left(\frac{11\pi}{4} - 2\pi\right)n - \pi/3\right)} + 2e^{j\left(\left(\frac{5\pi}{4} - 2\pi\right)n + \pi/3\right)}$$

$$x[n] = 2e^{j\left(\frac{3\pi n}{4} - \pi/3\right)} + 2e^{j\left(-\frac{3\pi n}{4} + \pi/3\right)}$$



(b) $x[n] = 4\cos\left(\frac{3\pi n}{4} - \pi/3\right)$

$$\Rightarrow y(t) = x[f_s t] = 4\cos\left(\frac{3\pi}{4}\cdot 16t - \pi/3\right)$$

$$= 4\cos\left(12\pi t - \pi/3\right)$$

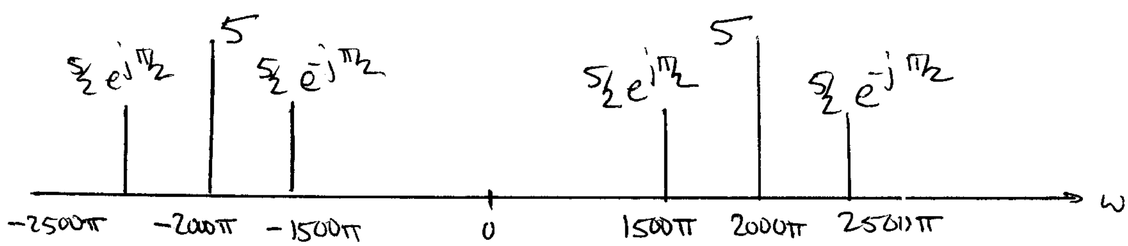
Completely real because the complex exponentials in $x(t)$ alias to terms in $x[n]$ that are at $\pm\frac{3\pi}{4}$ in normalized frequency with complex amplitudes that are complex conjugates of each other.

(6.6) (a) $x(t) = \left[10 + 10\cos(500\pi t - \pi/2)\right]\cos(2000\pi t)$

$$= \left[10 + 5e^{-j\pi/2}e^{j500\pi t} + 5e^{j\pi/2}e^{-j500\pi t}\right]\left(\tfrac{1}{2}e^{j2000\pi t} + \tfrac{1}{2}e^{-j2000\pi t}\right)$$

$$= \tfrac{5}{2}e^{j\pi/2}e^{-j2500\pi t} + 5e^{-j2000\pi t} + \tfrac{5}{2}e^{-j\pi/2}e^{-j1500\pi t}$$

$$+ \tfrac{5}{2}e^{j\pi/2}e^{j1500\pi t} + 5e^{j2000\pi t} + \tfrac{5}{2}e^{-j\pi/2}e^{j2500\pi t}$$

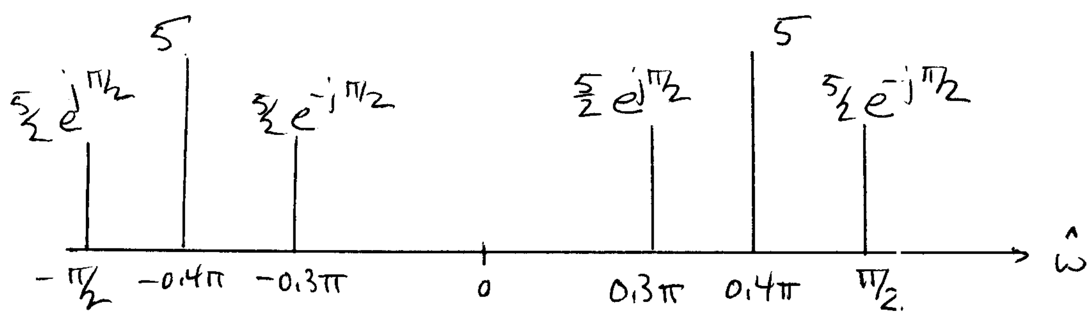


(b) Highest frequency = 1250 Hz.

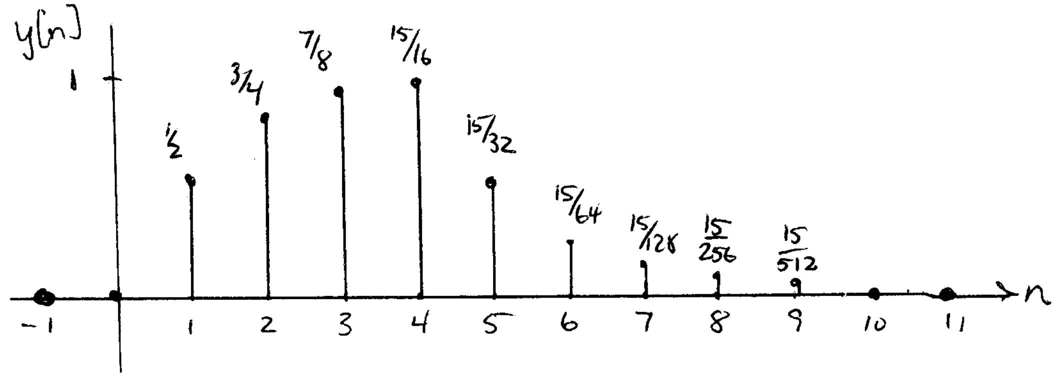
$\Rightarrow$ Need $f_s > 2500$ samples/sec.

(c) $f_s = 5000 > 2500 \Rightarrow$ no aliasing.

So, in each case $\hat{\omega} = \omega / f_s$.

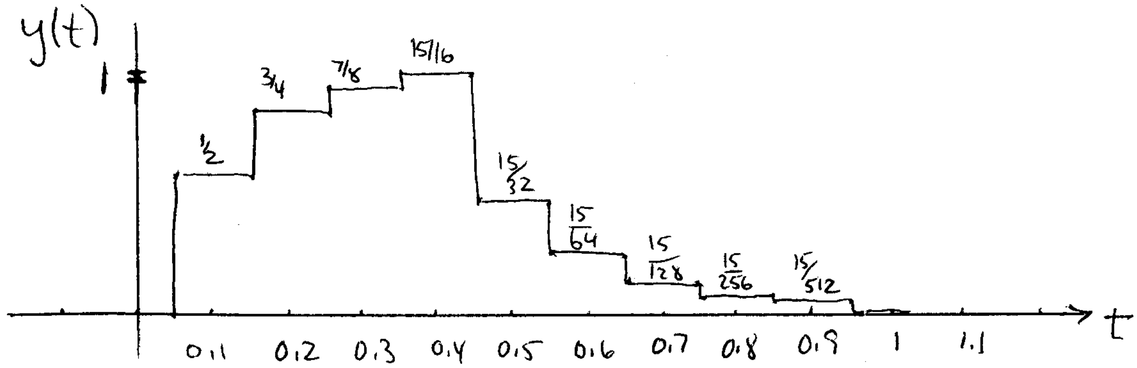


6.7 (a)



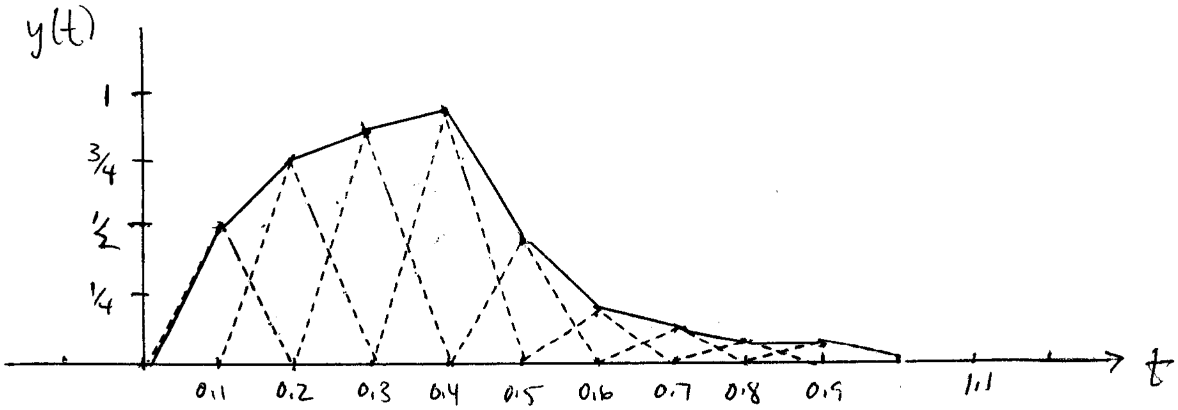
y[n]
1
1/2
3/4
7/8
15/16
15/32
15/64
15/128
15/256
15/512
-1
1
2
3
4
5
6
7
8
9
10
11
n

(b)



y(t)
1
1/2
3/4
7/8
15/16
15/32
15/64
15/128
15/256
15/512
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
t

(c)



y(t)
1
3/4
1/2
1/4
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.1
t