ECE 2025 Spring 2003
Problem Set #9

Problem 9.1

(a) $y[n]=\frac{1}{2}(x[n-2]-x[n-4])$

Impulse response: $h[n]=0.5\delta[n-2]-0.5\delta[n-4]$

System function: $H(z)=\frac{Y(z)}{X(z)}=0.5z^{-2}-0.5z^{-4}$

Frequency response:

$$H(e^{j\hat{\omega}})=0.5e^{-2j\hat{\omega}}-0.5e^{-j\cdot 4\hat{\omega}}$$

(b) $h[n]=\delta[n]+2\delta[n-1]+2\delta[n-3]+\delta[n-4]$

Difference equation:

$$y[n]=x[n]*h[n]$$
$$=x[n]+2x[n-1]+2x[n-3]+x[n-4]$$

Frequency response:

$$H(e^{j\hat{\omega}})=\sum_n h[n]e^{-j\hat{\omega}n}=1+2e^{-j\hat{\omega}}+2\cdot e^{-3j\hat{\omega}}+e^{-4j\hat{\omega}}$$

System function:

$$H(z)=\sum_n h[n]z^{-n}=1+2z^{-1}+2z^{-3}+z^{-4}$$

(c) $H(e^{j\hat{\omega}})=[3-2\cos\hat{\omega}]e^{-j3\hat{\omega}}$

$$=[3-(e^{j\hat{\omega}}+e^{-j\hat{\omega}})]e^{-3j\hat{\omega}}$$
$$=3e^{-3j\hat{\omega}}-e^{-2j\hat{\omega}}-e^{-4j\hat{\omega}}$$

$h[n] = 3\delta[n-3] - \delta[n-2] - \delta[n-4]$

Difference equation:

$y[n] = 3x[n-3] - x[n-2] - x[n-4]$

System function:

$H(z) = 3z^{-3} - z^{-2} - z^{-4}$

Problem 9.2

(a) $H(z) = z^{-3}$

Impulse response: $h[n] = \delta[n-3]$

Difference equation: $y[n] = x[n-3]$

Frequency response: $H(e^{j\hat{\omega}}) = e^{-3j\hat{\omega}}$

(b) $H(z) = z^{-1} + 3z^{-4} + z^{-7}$

Impulse response: $h[n] = \delta[n-1] + 3\delta[n-4] + \delta[n-7]$

Difference equation: $y[n] = x[n-1] + 3x[n-4] + x[n-7]$

Frequency response: $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + 3e^{-4j\hat{\omega}} + e^{-j7\hat{\omega}}$

(c) $H(z) = \dfrac{1+z^{-5}}{1+z^{-1}} = \dfrac{(1+z^{-1})(1-z^{-1}+z^{-2}-z^{-3}+z^{-4})}{1+z^{-1}}$

$= 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}$

Impulse response:

$$h[n]=\delta[n]-\delta[n-1]+\delta[n-2]-\delta[n-3]+\delta[n-4]$$

Difference equation:

$$y[n]=x[n]-x[n-1]+x[n-2]-x[n-3]+x[n-4]$$

Frequency response:

$$H(e^{j\hat{\omega}})=1-e^{-j\hat{\omega}}+e^{-2j\hat{\omega}}-e^{-3j\hat{\omega}}+e^{-4j\hat{\omega}}$$

Problem 9.3

(a) $H(z)=7$

Impulse response: $h[n]=7\delta[n]$

Difference equation: $y[n]=7x[n]$

Frequency response: $H(e^{j\hat{\omega}})=7$

(b) $H(z)=z^{2}-z^{-2}$

Impulse response: $h[n]=\delta[n+2]-\delta[n-2]$

Difference equation: $y[n]=x[n+2]-x[n-2]$

Frequency response:

$$H(e^{j\hat{\omega}})=e^{2j\hat{\omega}}-e^{-2j\hat{\omega}}$$

(c) $H(z)=(1-z^{-1})\underbrace{(1-0.5e^{j\frac{\pi}{3}}z^{-1})(1-0.5e^{-j\frac{\pi}{3}}z^{-1})}$

$$1-\underbrace{(0.5e^{j\frac{\pi}{3}}+0.5e^{-j\frac{\pi}{3}})}z^{-1}+0.25z^{-2}$$

$$2*0.5*\cos(\tfrac{\pi}{3})=0.5$$

Therefore,

$$H(z)=(1-z^{-1})(1-0.5z^{-1}+0.25z^{-2})$$

$$=1-1.5z^{-1}+0.75z^{-2}-0.25z^{-3}$$

Impulse response:

$$h[n]=\delta[n]-1.5\delta[n-1]+0.75\delta[n-2]-0.25\delta[n-3]$$

Difference equation:

$$y[n]=x[n]-1.5x[n-1]+0.75x[n-2]-0.25x[n-3]$$

Frequency response:

$$H(e^{j\hat{\omega}})=1-1.5e^{-j\hat{\omega}}+0.75e^{-2j\hat{\omega}}-0.25e^{-3j\hat{\omega}}$$

Problem 9.4

$$x[n]=x(t)\Big|_{t=nT_s}=x(t)\Big|_{t=\frac{n}{8000}}$$

$$x[n]=2\cos\left(\frac{6000\pi n}{8000}-\frac{\pi}{4}\right)+11\cos\left(\frac{12000\pi}{8000}n-\frac{\pi}{3}\right)$$

$$=2\cos\left(\frac{3\pi}{4}n-\frac{\pi}{4}\right)+11\cos\left(1.5\pi n-\frac{\pi}{3}\right)$$

$$=2\cos\left(\frac{3\pi}{4}n-\frac{\pi}{4}\right)+11\cos\left(0.5\pi n+\frac{\pi}{3}\right)$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

$$\hat{\omega} \qquad\qquad\qquad \hat{\omega}$$

$H(z) = 1 + z^{-4} \Rightarrow H(e^{j\hat{\omega}}) = 1 + e^{-4j\hat{\omega}}$

When $\hat{\omega} = \frac{3\pi}{4}$, $H(e^{j\hat{\omega}}) = 0$

When $\hat{\omega} = 0.5\pi$, $H(e^{j\hat{\omega}}) = 2$

Therefore,

$y[n] = 2 \times 11 \cos(0.5\pi n + \frac{\pi}{3})$

$= 22 \cos(\underbrace{0.5\pi}_{\text{lies between } -\pi \text{ and } \pi} n + \frac{\pi}{3})$

Substituting $n = 8000t$ into $y[n]$:

$y(t) = 22 \cos(4000\pi t + \frac{\pi}{3})$

We see that the $\omega = 6000\pi$ rads/sec component in $x(t)$ is filtered out by the LTI system; the $\omega = 12000\pi$ rads/sec component is folded and appears as the 4000π rads/sec component.

Problem 9.5

(a) $h[n] = 8\delta[n] + 8\delta[n-4]$

$H(z) = 8 + 8z^{-4}$

(b) Frequency response: $H(e^{j\hat{\omega}}) = 8 + 8e^{-4j\hat{\omega}}$

(c) $x[n] = 2\cos(0.75\pi n - \frac{\pi}{4}) + 11\cos(1.5\pi n - \frac{\pi}{3})$

$= 2\cos(0.75\pi n - \frac{\pi}{4}) + 11\cos(0.5\pi n + \frac{\pi}{3})$

Similar to Problem 9.4, we find

$H(e^{j\hat{\omega}})\big|_{\hat{\omega}=0.75\pi} = 0$

$H(e^{j\hat{\omega}})\big|_{\hat{\omega}=0.5\pi} = 16$

Therefore, $y[n] = 176\cos(0.5\pi n + \frac{\pi}{3})$

$f_s = 8000$ Hz here, so substituting with $n = 8000t$, we have

$y(t) = 176\cos(4000\pi t + \frac{\pi}{3})$.