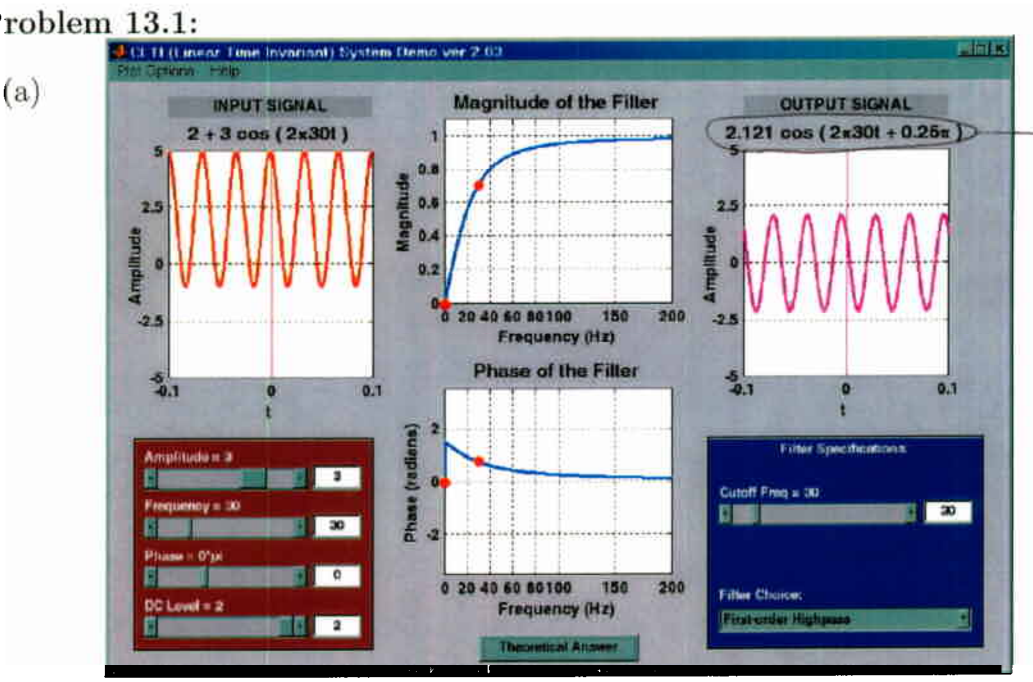
GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 2025

Spring Semester 2003
Solutions for Problem Set #13

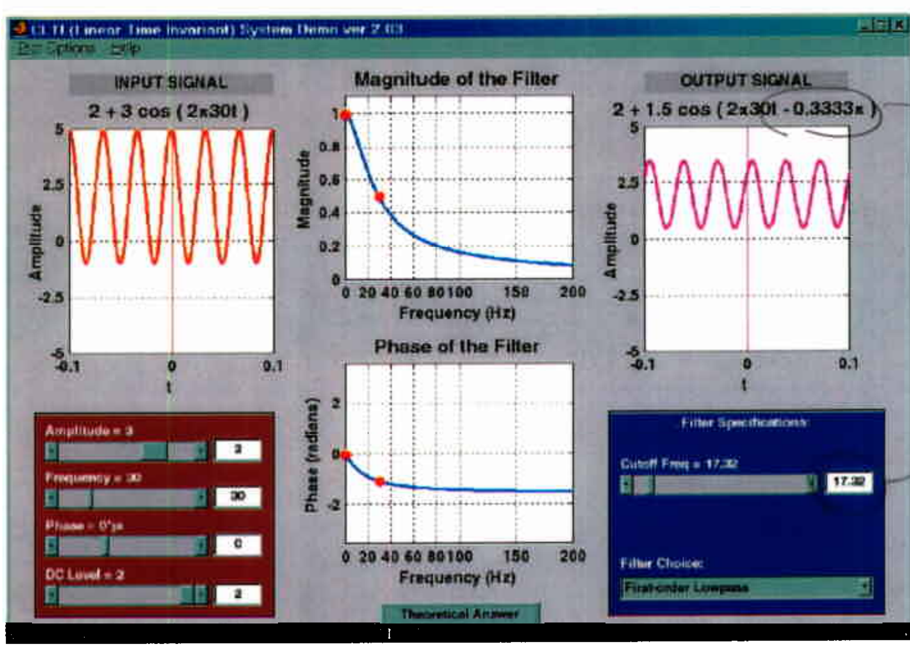
Problem 13.1:

(a)



$y(t) = 2.121 \cos(60\pi t + \pi/4)$

(b)



$\phi = -\pi/3$

$f_c = 17.32$ Hz

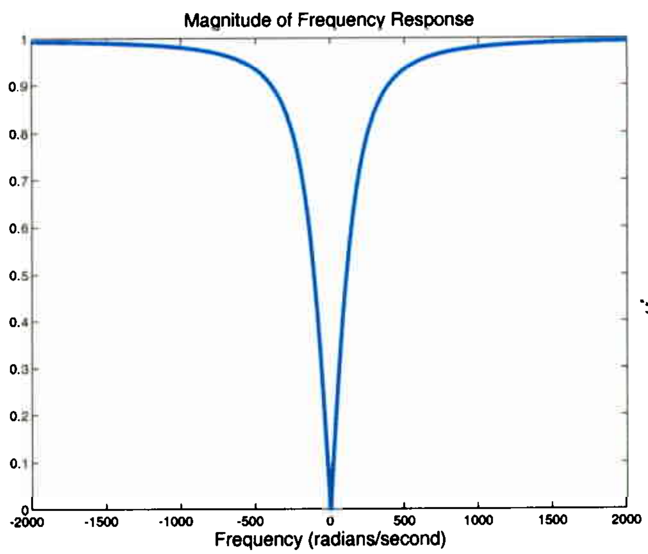
Problem 13.2:

(a) We know that $e^{-bt}u(t) \longleftrightarrow \frac{1}{b+j\omega}$ for $b>0$

and that $\frac{d}{dt}x(t) \longleftrightarrow j\omega X(j\omega)$.

$$\Rightarrow H(j\omega) = \frac{j\omega}{b+j\omega}$$

(b)



$$|H(j\omega)|^2 = \frac{j\omega}{b+j\omega} \cdot \frac{-j\omega}{b-j\omega}$$

$$\Rightarrow |H(j\omega)| = \left(\frac{\omega^2}{b^2+\omega^2}\right)^{1/2}$$

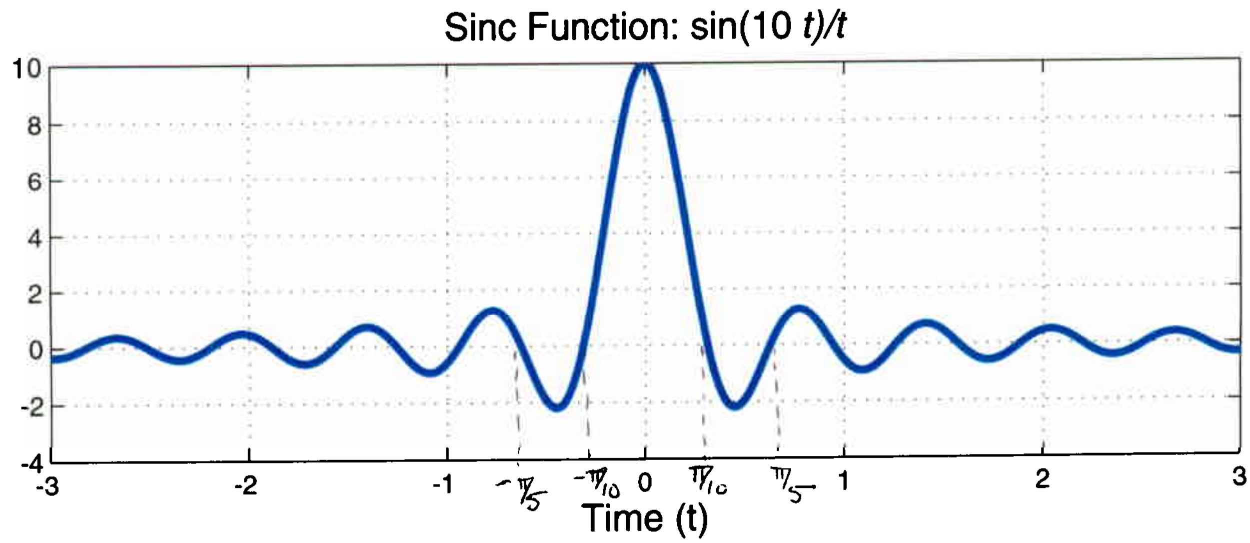
(c) highpass filter (HPF)

(d) $H(j0) = 0$, $H(j60\pi) = \frac{j60\pi}{60\pi + j60\pi} = \frac{1}{2} + j\frac{1}{2} = \frac{1}{\sqrt{2}} e^{j\pi/4}$

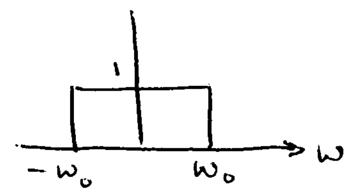
$$\Rightarrow y(t) = \frac{3}{\sqrt{2}} \cos\left(60\pi t + \pi/4\right)$$

Problem 13.3:

(a)



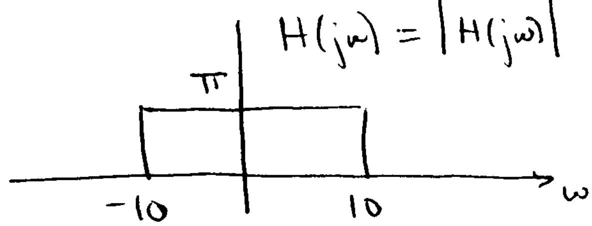
(b) We know that $\dfrac{\sin(\omega_0 t)}{\pi t} \longleftrightarrow$



Using linearity and choosing $\omega_0 = 10$, we have

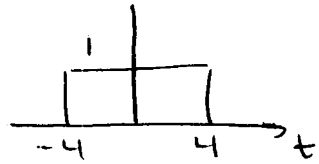
$$H(j\omega) = \begin{cases} \pi, & |\omega| < 10 \\ 0, & \text{else} \end{cases}$$

$H(j\omega) = |H(j\omega)|$



Problem 13.4:

(a) We know that

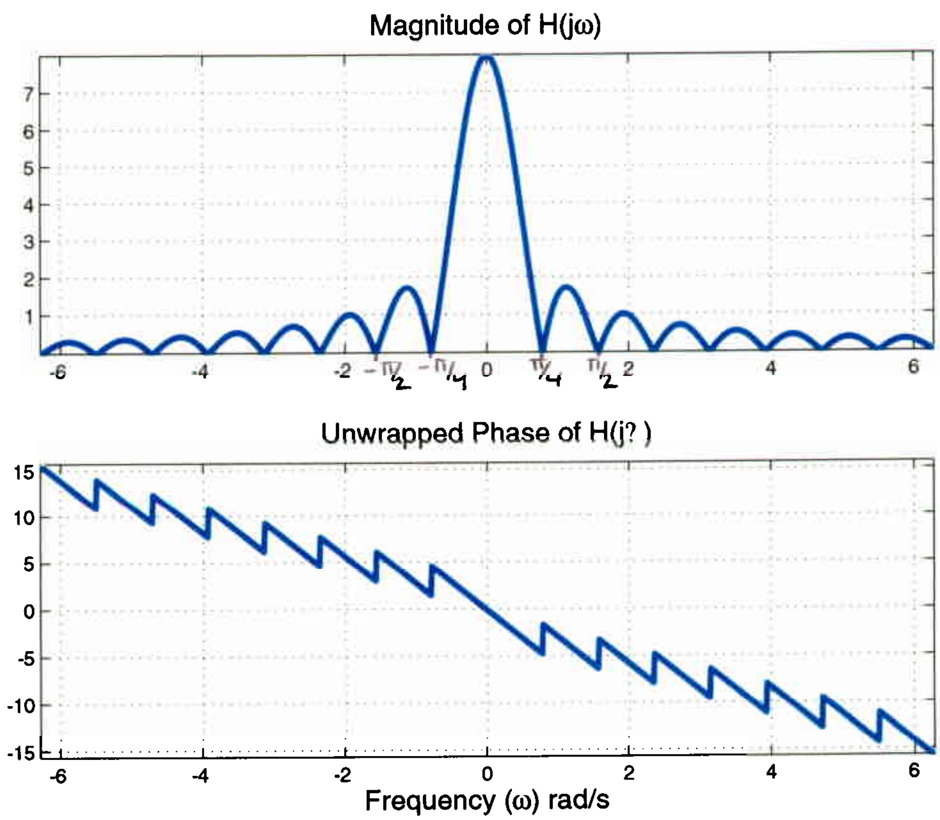


$$\longleftrightarrow \frac{\sin(4\omega)}{\omega/2}$$

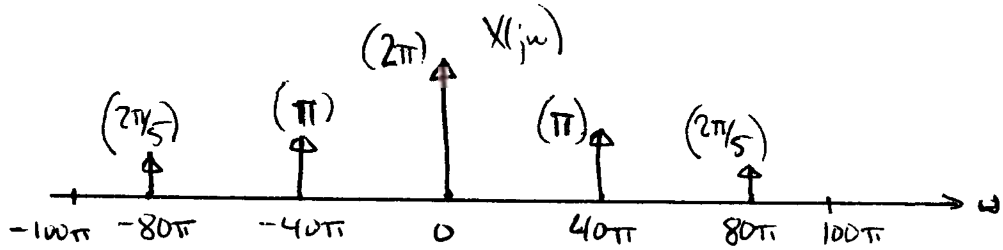
and that $x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(j\omega)$

$$\Rightarrow H(j\omega) = \frac{e^{-j6\omega}\sin(4\omega)}{\omega/2}$$

(b)



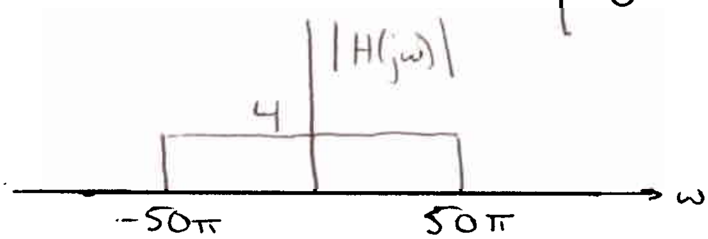
13.5 (a) $X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$ for $\omega_0 = \frac{2\pi}{T_0} = 40\pi$

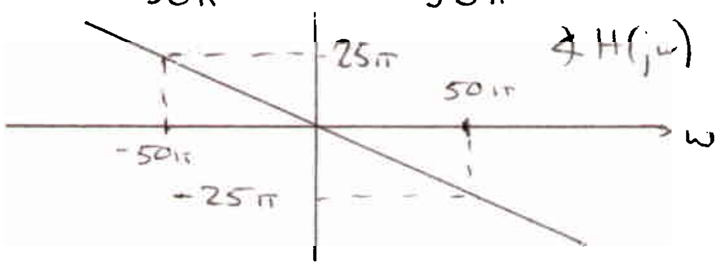


(b) $\frac{\sin(\omega_c t)}{\pi t} \longleftrightarrow$ 

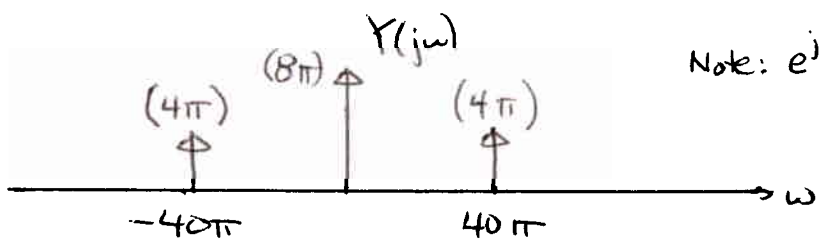
and $x(t-\tfrac{1}{2}) \longleftrightarrow e^{-j\omega/2} X(j\omega)$

Combined with linearity gives $H(j\omega) = \begin{cases} 4e^{-j\omega/2}, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$





(c)



Note: $e^{j20\pi} = e^{-j20\pi} = 1$

(d) $y(t) = 4 + 2e^{j40\pi t} + 2e^{-j40\pi t}$

$= 4 + 4\cos(40\pi t)$