GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #1

DATE: 31-May-02 COURSE: ECE 2025

NAME: Solutions, Ver. 3 STUDENT #: ____________
LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L05:Mon-4:00pm (Bordelon) L01:Tues-10:00am (Hunt) L02:Tues-12:00pm (Bordelon)

L03:Tues-2:00pm (Bordelon) L04:Tues-4:00pm (Brown) L06:Tues-6:00pm (Brown)

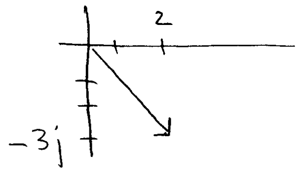
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning CLEARLY to receive partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem SUMMER-02-Q.1.1:
Simplify the following complex-valued expressions. Reduce the answers to a $\boxed{\text{simple}}$ numerical form.

(a) For $z = 2 - 3j$, express z in $\boxed{\text{polar form}}$. In addition, $\boxed{\text{plot}}$ z as a vector in the complex plane.

$$z = 3.606\, e^{-j\,0.983}$$
$$= 3.606\, e^{-j\,0.313\pi}$$



(b) For $z = 2 - 3j$, evaluate $\Im m\{z e^{j\pi/2}\}$.

$$2$$

(c) For $z = 2 - 3j$, evaluate $\frac{z}{z^*}$. Give your answer in $\boxed{\text{polar form}}$.

$$e^{-j\,1.966} = e^{-j\,0.626\pi}$$

(d) For $z = 2 - 3j$, evaluate $\Re e\{z e^{\cancel{j\,10\pi t}}\}$ ($-j10\pi t$). Reduce the answer to a $\boxed{\text{sinusoidal}}$ formula.

$$x(t) = 3.606 \cos(10\pi t + 0.313\pi)$$

Problem SUMMER-02-Q.1.2:
Four different sinusoidal signals are defined by the following representations:

(a) $x_a(t) = \Re e\left\{\frac{1}{2}e^{j2\pi/3}e^{j100\pi t}\right\}$

(b) $x_b(t) = \cos(100\pi t - 2\pi/3)$

(c) $x_c(t) = \frac{1}{2}e^{-j\pi/3}e^{j100\pi t} + \frac{1}{2}e^{j\pi/3}e^{-j100\pi t}$

(d) $x_d(t) = \Re e\left\{\frac{1}{2}(1 + j\sqrt{3})e^{j100\pi t}\right\}$

For each of the following signals, pick one of the representations above that defines an identical signal. Indicate your match ((a), (b), (c), or (d)) in the answer box next to each signal. In addition, write the complex amplitude (i.e., phasor X_k) of the sinusoid for each case in the space provided.

1. $x_1(t) = \cos(100\pi t + 4\pi/3)$

   Ans = b $\quad$ $X_1 = e^{j\frac{4\pi}{3}}$

2. $x_2(t) = \sin(100\pi t - \pi/6)$

   Ans = b $\quad$ $X_2 = e^{-j\frac{2\pi}{3}}$

3. $x_3(t) = \frac{1}{4}e^{j2\pi/3}e^{j100\pi t} + \frac{1}{4}e^{-j2\pi/3}e^{-j100\pi t}$

   Ans = a $\quad$ $X_3 = \frac{1}{2}e^{j\frac{2\pi}{3}}$

4. $x_4(t) = \Re e\left\{\frac{1}{2}(1 - j\sqrt{3})e^{j100\pi t}\right\}$

   Ans = c $\quad$ $X_4 = e^{-j\pi/3}$

5. $x_5(t) = \frac{1}{2}e^{-j5\pi/3}e^{j100\pi t} + \frac{1}{2}e^{j5\pi/3}e^{-j100\pi t}$

   Ans = d $\quad$ $X_5 = e^{-j\frac{5\pi}{3}}$

Problem SUMMER-02-Q.1.3:
Define $x(t)$ as

$$x(t) = 2\cos(31\pi t + 5\pi) + 2\cos(31\pi t + 3\pi/4) - 3\cos(31\pi t + 5\pi/2)$$

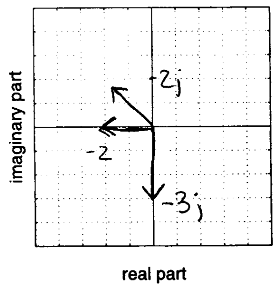
(a) Express $x(t)$ in the form $x(t) = A\cos(\omega_o t + \phi)$ by finding the numerical values of A, ω_o, and ϕ.

$$X = 2e^{j5\pi} + 2e^{j3\pi/4} + 3e^{j3\pi/2}$$

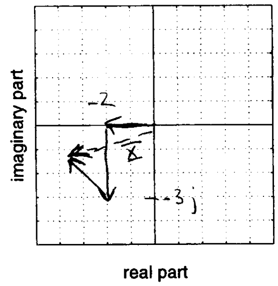
$$= 3.765\,e^{-j2.707} = 3.765\,e^{-j0.862\pi}$$

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). In the first plot, show only the phasors that were added to solve part (a); in the second plot, show your solution as a vector and the addition of the complex amplitudes as vectors (head-to-tail). *Use appropriate scales on the grids below and indicate the scales used.*

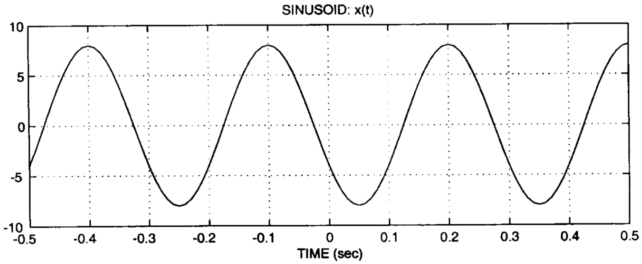
Three vectors here.



Head-to-tail plot here.



Problem SUMMER-02-Q.1.4:



(a) The figure above is a plot of a sinusoidal wave. From the plot, determine values for the amplitude (A), phase (ϕ), and frequency (ω_o) needed in the formula:

$$x(t) = A\cos(\omega_o t + \phi)$$

Give the answer as numerical values *including the units* where applicable.

$A = 8$

$T_o = 0.3 \Rightarrow \omega_o = 2\pi \cdot \frac{10}{3}$ rad/s

$t_d = -0.1 \Rightarrow \phi = -2\pi \cdot \frac{10}{3}\left(-\frac{1}{10}\right) = \frac{2\pi}{3}$ rad.

(b) By a suitable choice of delay t_d, we can shift $x(t)$ to obtain the new signal

$$y(t) = x(t - t_d) = A\cos(\omega_o t + \pi/4). \qquad (1)$$

There are an infinite number of values of t_d that satisfy Equation (1). Determine at least *two* different values of t_d that satisfy Equation (1), or give a general formula for all the possible values.

$\omega_o t + \pi/4 = \omega_o(t - t_d) + \frac{2\pi}{3} + 2\pi k$

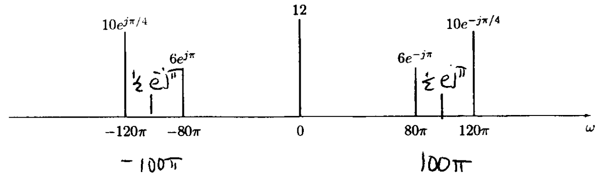
$\omega_o t_d = +\frac{5\pi}{12} + 2\pi k$

$t_d = \frac{5\pi}{12\omega_o} + \frac{2\pi k}{\omega_o} = \frac{1}{16} + \frac{3k}{10}$ sec.

$k = 0, \pm 1, \pm 2, \cdots$

Problem SUMMER-02-Q.1.5:

A signal $x(t)$ has the two-sided spectrum representation shown below:



(a) Write an equation for $x(t)$. Make sure to express $x(t)$ as a real-valued signal.

$x(t) = 12 + 12\cos(80\pi t - \pi) + 20\cos(120\pi t - \pi/4)$

(b) Is $x(t)$ a periodic signal? **You must explain this answer. Why or why not?** If it is periodic, what is its fundamental period?

Yes, frequencies are harmonics.

$f_o = 40\pi / 2\pi = 20$ Hz.

$T_o = 1/20$ sec.

(c) A new signal is defined as $y(t) = \cos(\beta t + \pi) + x(t)$, where $80\pi < \beta < 120\pi$. Choose the frequency β so that the fundamental frequency of $y(t)$ is *half* the fundamental frequency of $x(t)$.

$\beta = 100\pi$ rad/s

and $f_o = \frac{20\pi}{2\pi} = 10$ Hz

(d) Using the frequency β found in (c), modify the spectrum plot above so that it becomes the spectrum of $y(t)$. *Label the complex amplitude as well as the frequency and mark your changes directly on the above plot.*