GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #2

DATE: 21-June-02 COURSE: ECE 2025

NAME: SOLUTIONS VER. 1 STUDENT #: ____________
LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

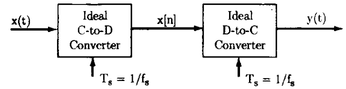
L05:Mon-4:00pm (Bordelon) L01:Tues-10:00am (Hunt) L02:Tues-12:00pm (Bordelon)

L03:Tues-2:00pm (Bordelon) L04:Tues-4:00pm (Brown) L06:Tues-6:00pm (Brown)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning clearly to receive partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem SUMMER-02-Q.1.1:
Consider the following system.



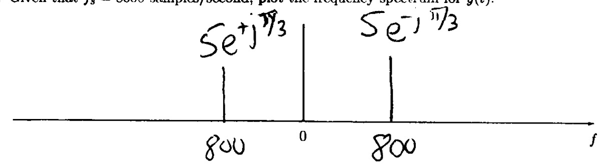
(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 10\cos(0.2\pi n - \pi/3)$$

If the sampling rate is $f_s = 8000$ samples/second, determine two *different* continuous-time signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have been inputs to the above systems, i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 125\mu$sec. Both of these input signals should have a frequency less than 8000 Hz. Give a formula for each signal.

$$x_1(t) = 10\cos(1600\pi t - \pi/3)$$
$$x_2(t) = 10\cos(14{,}400\pi t + \pi/3)$$

(b) Given that $f_s = 8000$ samples/second, plot the frequency spectrum for $y(t)$.

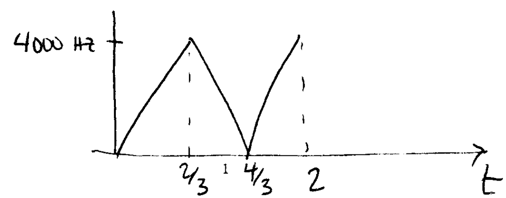


(c) If the input $x(t)$ is given by the chirp formula

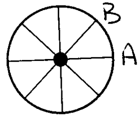
$$x(t) = \cos(6000\pi t^2), \qquad \text{for } 0 \le t \le 2$$

$\omega_i(t) = 12{,}000\pi t$
$f_i(t) = 6000t$

determine the output signal when $f_s = 8000$ samples/sec. Give a plot of instantaneous frequency versus time for $y(t)$.



Problem SUMMER-02-Q.1.2:
In old TV movies, all of us have seen the phenomenon where a spoked wagon wheel appears to move backwards. This is due to the 30 frames/sec sampling rate used in transmitting TV images. In the figure to the right, an eight-spoked wheel is shown. Assume that the wheel is rotating *clockwise at a constant speed*. However, when seen on TV the spoke pattern of the wheel appears to make a full 360° counter-clockwise revolution once every 120 frames (i.e., 4 seconds). We will determine the rotation rate(s) that could have caused this illusion.



(a) Write a rotating phasor formula for the observed movement of an individual spoke when the wheel is making f_o rotations per second. This should be a discrete-time signal formula that depends on n (the frame index).

$$x(t) = e^{-j2\pi f_o t} + \phi$$

$$x[n] = e^{-j\,2\pi f_o n/30} + \phi$$

for one spoke.

(b) Determine the $\boxed{\text{slowest}}$ rotation rate f_o that produces the observation.

For $\phi = 0$, $y[n] = e^{j2\pi n/120}$ is the observed signal.

For $n=0$, spoke A is at 0 radians.

For $n=1$, spoke B rotates $\frac{\pi}{4} - \frac{2\pi}{120} = \frac{28\pi}{120}$ radians counter-clockwise in $\frac{1}{30}$ second.

$$\Rightarrow y(t) = e^{-j\frac{28\pi}{120}\cdot 30t} = e^{-j7\pi t}$$

$$\Rightarrow f_o = 3.5 \text{ rotations/sec.}$$

(see next quiz version for a different approach.)

Problem SUMMER-02-Q.1.3:
For each of the following frequency responses, pick *one* of the representations below that define *exactly* the same LTI system. Write your answer (i.e., S_1, S_2, S_3, S_4, S_5, or S_6) in the box next to each frequency response. In addition, evaluate the frequency response for the specified $\hat{\omega}$, *simplify* your answer to polar form, and write it in the space provided.

(a) $\mathcal{H}_a(\hat{\omega}) = 4e^{-j2\hat{\omega}}\cos(\hat{\omega}) = 2e^{-j\hat{\omega}} + 2e^{-3j\hat{\omega}} \Rightarrow \{0,2,0,2\}$

$\boxed{\text{Ans} = 4}$ $\boxed{\mathcal{H}_a(\pi/2) = 0}$ $e^{-j\pi}\cos(\pi/2) = 0$

(b) $\mathcal{H}_b(\hat{\omega}) = 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} \Rightarrow \{0,2,2\}$

$\boxed{\text{Ans} = 2}$ $\boxed{\mathcal{H}_b(-\pi) = 0}$ $2e^{j\pi} + 2e^{-j2\pi} = 0$

(c) $\mathcal{H}_c(\hat{\omega}) = 4e^{-j5\hat{\omega}/2}\cos(\hat{\omega}/2) = 2e^{-j2\hat{\omega}} + 2e^{-j3\hat{\omega}} \Rightarrow \{0,0,2,2\}$

$\boxed{\text{Ans} = 6}$ $\boxed{\mathcal{H}_c(0) = 4}$

(d) $\mathcal{H}_d(\hat{\omega}) = 2 + 2e^{-j3\hat{\omega}} \Rightarrow \{2,0,0,2\}$

$\boxed{\text{Ans} = 3}$ $\boxed{\mathcal{H}_d(-\pi/2) = 2\sqrt{2}\cdot e^{-j\pi/4}}$ $2 + 2e^{+j3\pi/2} = 2 - 2j$

POSSIBLE ANSWERS: (impulse response, filter coefficients, or difference equation)

S_1: $b_k = \{2, 0, 2\}$

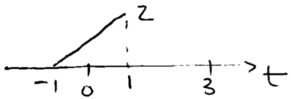
S_2: $h[n] = 2\delta[n-1] + 2\delta[n-2]$ $\{0,2,2\}$

S_3: $h[n] = 2\delta[n] + 2\delta[n-3]$ $\{2,0,0,2\}$

S_4: $y[n] = 2x[n-1] + 2x[n-3]$ $\{0,2,0,2\}$

S_5: $b_k = \{2, 2\}$

S_6: $y[n] = 2x[n-2] + 2x[n-3]$ $\{0,0,2,2\}$



Problem SUMMER-02-Q.1.4:
Suppose that a periodic signal $x(t)$ is defined over *one period* as:

$$x(t) = \begin{cases} 1+t, & -1 < t \leq 1 \\ 0, & 1 < t \leq 3 \end{cases}$$

(a) Is $x(t)$ $\boxed{\text{bandlimited}}$? If so, give the maximum frequency, If not, explain why.

No. Has discontinuities.

(b) Does $x(t)$ have a $\boxed{\text{fundamental frequency}}$? If so, give the frequency. If not, explain why.

Yes. $f_0 = \frac{1}{T_0} = \frac{1}{4}$ Hz.

(c) Determine the $\boxed{\text{DC value}}$ of $x(t)$.

$$a_0 = \frac{1}{4} \cdot 2 = \frac{1}{2}$$

(d) Write an integral expression for the Fourier series coefficient a_2 in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integral (e.g., the limits of integration), but do not evaluate the integral. All parameters in the integral should have numeric values.*

$$a_2 = \frac{1}{4} \int_{-1}^{1} (1+t)\, e^{-j4\pi t/4}\, dt$$

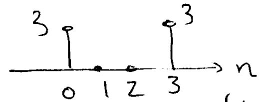
$$= \frac{1}{4} \int_{-1}^{1} (1+t)\, e^{-j\pi t}\, dt$$

Problem SUMMER-02-Q.1.5:
An FIR filter is described by the difference equation:

$$y[n] = 3x[n] + 3x[n-3]$$

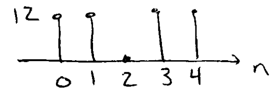
(a) Find its impulse response $h[n]$ and plot versus n.

$$h[n] = 3\delta[n] + 3\delta[n-3]$$



(b) Find the output when the input signal is $x[n] = \begin{cases} 4, & n = 0, 1 \\ 0, & \text{elsewhere} \end{cases}$. $= 4\delta[n] + 4\delta[n-1]$

$$y[n] = 3x[n] + 3x[n-3]$$
$$= 12\delta[n] + 12\delta[n-1]$$
$$+ 12\delta[n-3] + 12\delta[n-4]$$



(c) For an *unknown* linear, time-invariant system we are given that when the input signal is $x[n] = \cos(\pi n/4)$, then the output signal is $y[n] = 4\cos(\pi n/4 - \pi/2)$. Use linearity and time-invariance to find the output when the input $x[n]$ is

$$x[n] = \sqrt{2}\cos(\pi(n-4)/4)$$

$$y[n] = 4\sqrt{2}\cos\left(\frac{\pi(n-4)}{4} - \pi/2\right)$$
$$= 4\sqrt{2}\cos\left(\frac{\pi n}{4} - 3\pi/2\right)$$