

GEORGIA INSTITUTE OF TECHNOLOGY  
SCHOOL OF ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 21-June-02

COURSE: ECE 2025

NAME: SOLUTIONS VER. 3  
LAST, FIRST

STUDENT #: \_\_\_\_\_

**Recitation Section:** Circle the day & time when your Recitation Section meets:

L05: Mon-4:00pm (Bordelon)    L01: Tues-10:00am (Hunt)    L02: Tues-12:00pm (Bordelon)

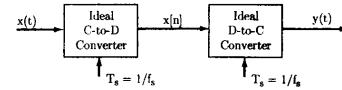
L03: Tues-2:00pm (Bordelon)    L04: Tues-4:00pm (Brown)    L06: Tues-6:00pm (Brown)

- Write your name on the front page ONLY. DO NOT unstaple the test.
- This exam is closed book. However, one page (8½" × 11") of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning CLEARLY to receive partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	

**Problem SUMMER-02-Q.1.1:**

Consider the following system.



- (a) Suppose that the discrete-time signal  $x[n]$  is given by the formula

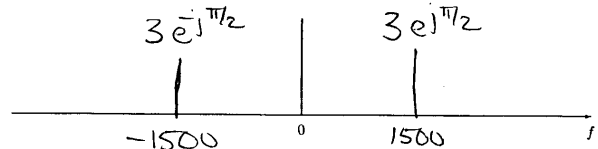
$$x[n] = 6 \cos(0.3\pi n + \pi/2)$$

If the sampling rate is  $f_s = 10,000$  samples/second, determine two *different* continuous-time signals  $x(t) = x_1(t)$  and  $x(t) = x_2(t)$  that could have been inputs to the above systems, i.e., find  $x_1(t)$  and  $x_2(t)$  such that  $x[n] = x_1(nT_s) = x_2(nT_s)$  if  $T_s = 100\mu\text{sec}$ . Both of these input signals should have a frequency less than 10,000 Hz. Give a formula for each signal.

$$x_1(t) = 6 \cos(3000\pi t + \pi/2)$$

$$x_2(t) = 6 \cos(17,000\pi t - \pi/2)$$

- (b) Given that  $f_s = 10,000$  samples/second, plot the frequency spectrum for  $y(t)$ .



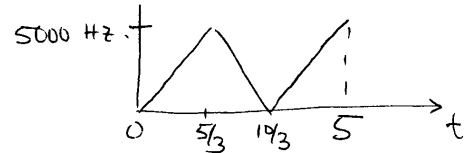
- (c) If the input  $x(t)$  is given by the chirp formula

$$x(t) = \cos(3000\pi t^2), \quad \text{for } 0 \leq t \leq 5$$

$$w_i(t) = 6000\pi t$$

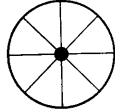
$$f_i(t) = 3000 t$$

determine the output signal when  $f_s = 10,000$  samples/sec. Give a plot of instantaneous frequency versus time for  $y(t)$ .



**Problem SUMMER-02-Q.1.2:**

In old TV movies, all of us have seen the phenomenon where a spoked wagon wheel appears to move backwards. This is due to the 30 frames/sec sampling rate used in transmitting TV images. In the figure to the right, an eight-spoked wheel is shown. Assume that the wheel is rotating *clockwise* at a constant speed. However, when seen on TV the spoke pattern of the wheel appears to make a full 360° counter-clockwise revolution once every 90 frames (i.e., 3 seconds). We will determine the rotation rate(s) that could have caused this illusion.



- (a) Write a rotating phasor formula for the observed movement of an individual spoke when the wheel is making  $f_0$  rotations per second. This should be a discrete-time signal formula that depends on  $n$  (the frame index).

$$x(t) = e^{-j 2\pi f_0 t + \phi}$$

$$x[n] = e^{-j 2\pi f_0 n / 30 + \phi} \quad \text{for 1 spoke.}$$

NOTE: For 8 spokes, we have

$$x_8[n] = e^{-j 2\pi (8) f_0 n / 30 + \phi}$$

(b) Assume an arbitrary rotation rate  $f_0$  that produces the observation.

Assume  $\phi = 0$ . We observe the 8-spoke signal

$$y[n] = e^{j 2\pi (8) n / 90}$$

$$\Rightarrow \text{Observed freq.} = 8/90. \quad \text{True freq.} = \frac{-8f_0}{30}$$

$$\text{Aliasing: } \frac{-8f_0}{30} + k = 8/90, \quad k=1 \Rightarrow \text{least amount of aliasing.}$$

$$\frac{-8f_0}{30} + 1 = 8/90$$

$$\Rightarrow f_0 = 4 1/12 \text{ rotations/sec.}$$

**Problem SUMMER-02-Q.1.3:**

For each of the following frequency responses, pick *one* of the representations below that define *exactly* the same LTI system. Write your answer (i.e.,  $S_1, S_2, S_3, S_4, S_5,$  or  $S_6$ ) in the box next to each frequency response. In addition, evaluate the frequency response for the specified  $\hat{\omega}$ , *simplify* your answer to polynomial form, and write it in the space provided.

(a)  $\mathcal{H}_a(\hat{\omega}) = 4e^{-j5\hat{\omega}/2} \cos(\hat{\omega}/2)$

Ans = 3       $\mathcal{H}_a(0) = 4$

(b)  $\mathcal{H}_b(\hat{\omega}) = 2 + 2e^{-j3\hat{\omega}}$

Ans = 2       $\mathcal{H}_b(-\pi/2) = 2\sqrt{2} e^{j\pi/4}$

(c)  $\mathcal{H}_c(\hat{\omega}) = 4e^{-j2\hat{\omega}} \cos(\hat{\omega})$

Ans = 5       $\mathcal{H}_c(-\pi) = 4e^{j\pi}$

(d)  $\mathcal{H}_d(\hat{\omega}) = 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}$

Ans = 1       $\mathcal{H}_d(\pi/2) = 2\sqrt{2} e^{-j3\pi/4}$

**POSSIBLE ANSWERS:** (impulse response, filter coefficients, or difference equation)

$S_1: h[n] = 2\delta[n-1] + 2\delta[n-2]$


$S_2: h[n] = 2\delta[n-3] + 2\delta[n]$

$S_3: y[n] = 2x[n-2] + 2x[n-3]$

$S_4: b_k = \{2, 0, 2\}$

$S_5: y[n] = 2x[n-1] + 2x[n-3]$

$S_6: b_k = \{2, 2\}$

Problem SUMMER-02-Q.1.4: 

$$x(t) = \begin{cases} -1+t, & -1 < t \leq 1 \\ 0, & 1 < t \leq 4 \end{cases}$$

(a) Is  $x(t)$  bandlimited? If so, give the maximum frequency. If not, explain why.

No. Has discontinuities.

(b) Does  $x(t)$  have a fundamental frequency? If so, give the frequency. If not, explain why.

Yes.  $f_0 = \frac{1}{T_0} = \frac{1}{5} \text{ Hz}$ .

(c) Determine the DC value of  $x(t)$ .

$$a_0 = \frac{1}{5} \cdot (-2) = -\frac{2}{5}$$

(d) Write an integral expression for the Fourier series coefficient  $a_4$  in terms of the specific signal  $x(t)$  defined above. Set up all the specifics of the integral (e.g., the limits of integration), but do not evaluate the integral. All parameters in the integral should have numeric values.

$$a_4 = \frac{1}{5} \int_{-1}^1 (-1+t) e^{-j8\pi t/5} dt$$

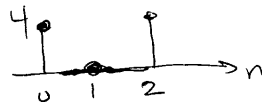
Problem SUMMER-02-Q.1.5:

An FIR filter is described by the difference equation:

$$y[n] = 4x[n] + 4x[n-2]$$

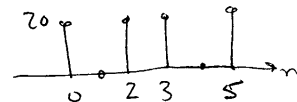
(a) Find its impulse response  $h[n]$  and plot versus  $n$ .

$$h[n] = 4\delta[n] + 4\delta[n-2]$$



(b) Find the output when the input signal is  $x[n] = \begin{cases} 5, & n=0,3 \\ 0, & \text{elsewhere} \end{cases} = 5\delta[n] + 5\delta[n-3]$

$$\begin{aligned} y[n] &= 4x[n] + 4x[n-2] \\ &= 20\delta[n] + 20\delta[n-3] \\ &\quad + 20\delta[n-5] + 20\delta[n-2] \end{aligned}$$



(c) For an unknown linear, time-invariant system we are given that when the input signal is  $x[n] = \cos(\pi n/5)$ , then the output signal is  $y[n] = 5 \cos(\pi n/5 + \pi/4)$ . Use linearity and time-invariance to find the output when the input  $x[n]$  is

$$x[n] = \sqrt{3} \cos(\pi(n-2)/5)$$

$$\begin{aligned} y[n] &= 5\sqrt{3} \cos\left(\frac{\pi(n-2)}{5} + \frac{\pi}{4}\right) \\ &= 5\sqrt{3} \cos\left(\frac{\pi n}{5} - \frac{3\pi}{20}\right) \end{aligned}$$