GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #3

DATE: 07-12-2002 COURSE: ECE 2025

NAME: SOLUTIONS, Ver. 3 STUDENT #: ________
LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L05:Mon-4:00pm (Bordelon) L01:Tues-10:00am (Hunt) L02:Tues-12:00pm (Bordelon)

L03:Tues-2:00pm (Bordelon) L04:Tues-4:00pm (Brown) L06:Tues-6:00pm (Brown)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning Clearly to receive partial credit. Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem SUMMER-02-Q.3.1:

A discrete-time system (FIR filter) is defined by the following z-transform transfer function:

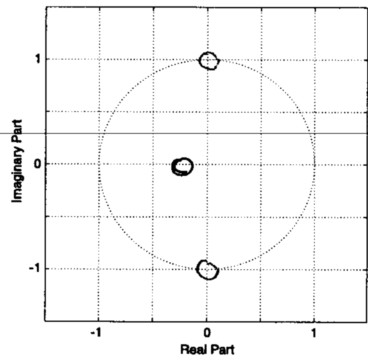
$$H(z) = (1 + 0.25z^{-1})(1 - e^{-j\pi/2}z^{-1})(1 - e^{j\pi/2}z^{-1})$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$ of the system. Give the numerical values of all filter coefficients.

$$H(z) = (1 + \tfrac{1}{4}z^{-1})(1 + z^{-2})$$
$$= 1 + \tfrac{1}{4}z^{-1} + z^{-2} + \tfrac{1}{4}z^{-3}$$

$$y[n] = x[n] + \tfrac{1}{4}x[n-1] + x[n-2] + \tfrac{1}{4}x[n-3]$$

(b) Determine *all* the zeros of $H(z)$ and plot them in the z-plane.



(c) If the input is of the form $x[n] = 5\cos(\hat{\omega}_o n - \pi/3)$, for what value of frequency $\hat{\omega}_o$ (in the range $0 \le \hat{\omega}_o \le \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\pi/2} = 0 \implies \hat{\omega}_o = \pi/2$$

Problem SUMMER-02-Q.3.2:

In each of the following cases, simplify the expression as much as possible using the properties of the continuous-time unit impulse signal. Provide some $\boxed{\text{explanation}}$ or intermediate steps for each answer.

(a) $[2\cos(3\pi t - \pi/2) + e^{-3t}u(t-2)]\delta(t-4) = \left[2\cos(12\pi - \pi/2) + e^{-12}u(6)\right]\delta(t-4)$

$$= e^{-12}\delta(t-4)$$

(b) $[2\cos(3\pi t - \pi/2) + e^{-3t}u(t-2)] * \delta(t-4) = \left[2\cos(3\pi(t-4) - \pi/2) + e^{-3(t-4)}u(t-6)\right]$

$$= 2\cos(3\pi t - \pi/2) + e^{-3(t-4)}u(t-6)$$

(c) $\frac{d}{dt}\{e^{-3t}u(t-2)\} = -3e^{-3t}u(t-2) + e^{-3t}\delta(t-2)$

$$= -3e^{-3t}u(t-2) + e^{-6}\delta(t-2)$$

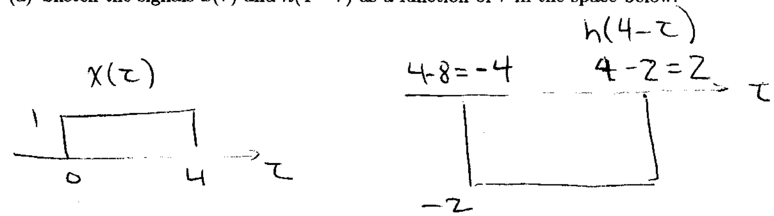
(d) $\int_{-\infty}^{\infty} e^{-3t}u(t-2)\delta(t-4)dt = \int_{-\infty}^{\infty} e^{-12}(1)\,\delta(t-4)\,dt$

$$= e^{-12}$$

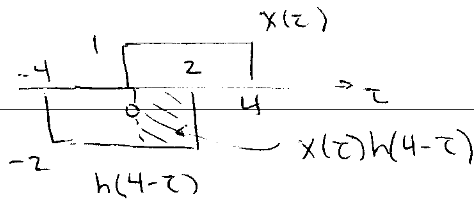
Problem SUMMER-02-Q.3.3:

Consider the signal $x(t) = u(t) - u(t-4)$ as the input to a continuous-time LTI system whose impulse response is $h(t) = -2u(t-2) + 2u(t-8)$.

(a) Sketch the signals $x(\tau)$ and $h(4-\tau)$ as a function of τ in the space below.



(b) Determine the value of the output of this LTI system, $y(t)$, at $t = 4$; that is, determine $y(4)$. It is not necessary to evaluate $y(t)$ for all t, only for $t = 4$. Note: This problem may be answered without performing any integration.



$$y(4) = -4$$

(c) The output signal $y(t)$ reaches its minimum value over some range $T_1 \le t \le T_2$. Find the minimum value, y_{min}, and also the values for T_1 and T_2.

$y_{min} = -8$

$T_1 = 6$ $\quad t-2=4 \Rightarrow t=6$

$T_2 = 8$ $\quad t-8=0 \Rightarrow t=8$

Problem SUMMER-02-Q.3.4:

Assume that $x(t)$ is the periodic function given by

$$x(t) = 5 \sum_{k=-\infty}^{\infty} \delta(t-5k) = \sum_{k=-\infty}^{\infty} A e^{j\omega_o kt}$$

(a) Determine the numerical values of the constant A and the fundamental frequency ω_o.

$A = 1$
$\omega_o = 2\pi/5$

$$A = \frac{1}{5}\int_{-2}^{3} 5\, e^{-jk\omega_o t}\,\delta(t)\,dt = 1$$

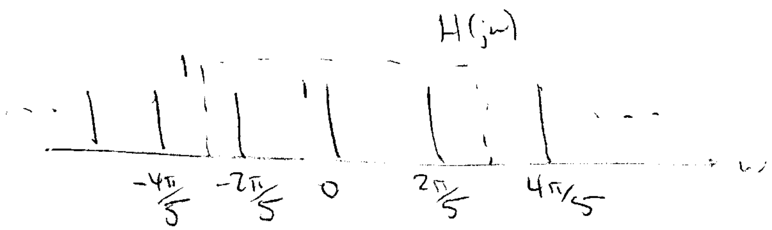
$$\omega_o = \frac{2\pi}{5}$$

(b) Suppose that $x(t)$ is the input to an LTI system with the frequency response

$$H(j\omega) = \begin{cases} 1, & |\omega| \le 0.6\pi \\ 0, & |\omega| > 0.6\pi \end{cases}.$$

Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. Your answer should be expressed in terms of only real quantities. (Hint: Plot the spectrum of $x(t)$ against a plot of the system's frequency response.)

$y(t) = 1 + 2\cos\left(\frac{2\pi}{5}t\right)$



Problem SUMMER-02-Q.3.5:

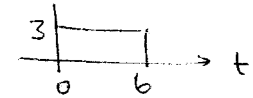
In each of the following cases, determine the Fourier transform. Give your answer as a $\boxed{\text{simple}}$ formula. $\boxed{\text{Explain}}$ each answer by stating which property and tranform pair you used.

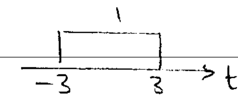
(a) $h(t) = e^{-4t}u(t-4) = e^{-16}\, e^{-4(t-4)}u(t-4)$

$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$$

Linearity & Time Shift : $H(j\omega) = \dfrac{e^{-16}e^{-j4\omega}}{4+j\omega}$

(b) $x(t) = 3u(t) - 3u(t-6)$





$$\longleftrightarrow \frac{\sin(3\omega)}{\omega/2}$$

Linearity & Time Shift : $X(j\omega) = \dfrac{6e^{-j3\omega}\sin(3\omega)}{\omega}$

(c) $s(t) = 6\delta(t+4) + \delta(t) + 6\delta(t-4)$

$$\delta(t-t_o) \longleftrightarrow e^{-j\omega t_o}$$

Linearity: $S(j\omega) = 6e^{j4\omega} + 1 + 6e^{-j4\omega}$

$$= 1 + 12\cos(4\omega)$$