GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Lab #11: Design with Fourier Series

Date: 12–18 Nov 2003

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time. You **MUST** complete the online Pre-Post-Lab exercise on Web-CT at the beginning of your scheduled lab session. You can use MATLAB and also consult your lab report or any notes you might have, but you cannot discuss the exercises with any other students. You will have approximately 20 minutes at the beginning of your lab session to complete the online Pre-Post-Lab exercise. The Pre-Post-Lab exercise for this lab includes some questions about concepts from the previous Lab report as well as questions on the Pre-Lab section of this lab.

The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. After completing the warm-up section, turn in the verification sheet to your TA.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports but the submitted work should be original and it should be your own work.

The lab report for this week will be an **Informal Lab Report.** It is only necessary to turn in Section 4 as this week's lab report. The report will be **due during the last week of the semester when your lab meets: 1–4 Dec**.

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1 Introduction & Objective

The goal of this laboratory project is to show that Fourier Series analysis and the frequency response $H(j\omega)$ are powerful methods for predicting the response of a LTI system when the input is a periodic signal. In this particular lab, we will use Fourier Series and the Fourier transform to analyze a power supply design problem in the frequency domain.

This lab uses two MATLAB GUIs: one for continuous-time frequency response, **`CLTIdemo`,** and one for Fourier Series analysis, **`FSeriesdemo`.** The **`CLTIdemo`** GUI provides a convenient way to visualize the sinusoidal response of LTI systems. When the input signal is an infinitely long sinusoid that extends over the range $-\infty < n < \infty$, the output is also a sinusoid. The frequency response tells us how the magnitude and phase of the output sinusoid can be calculated. The **`FSeriesdemo`** GUI shows the Fourier Series coefficients for common waveforms such as square waves, and rectified sine waves.

1.1 Background: Fourier Series Analysis and Synthesis

Recall the *analysis* integral and *synthesis* summation for the Fourier Series expansion of a periodic signal $x(t) = x(t + T_0)$. The Fourier synthesis equation for a periodic signal $x(t) = x(t + T_0)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \tag{1}$$

where $\omega_0 = 2\pi/T_0$ is the *fundamental* frequency. To determine the Fourier series coefficients $\{a_k\}$ from a periodic signal, we must evaluate the *analysis* integral for every integer value of k:

$$a_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt \tag{2}$$

where $T_0 = 2\pi/\omega_0$ is the *fundamental* period. If necessary, we can evaluate the analysis integral over any period; in (2) the choice was $[-\frac{1}{2}T_0, \frac{1}{2}T_0]$, but integrating over the interval $[0, T_0]$ would also give exactly the same answer.

The Fourier Series representation is extremely useful when studying the effects of an LTI filter, because the output signal is also periodic. The Fourier Series coefficients of the output signal $\{b_k\}$ are obtained by **multiplying** by the frequency response:

$$b_k = a_k H(j\omega_0 k) \tag{3}$$

where $H(j\omega_0 k)$ is the frequency response of the LTI system evaluated at the harmonics, $\omega = j\omega_0 k$.

2 Pre-Lab: Run the Frequency Response GUI

2.1 Sinusoidal Response (CLTIdemo)

In this demo, you can select an input signal that is a sinusoid, and see the change created by the frequency response. This demo reinforces the concept that "sinusoid in gives sinusoid out." Figure 1 shows the interface for the `CLTIdemo`.

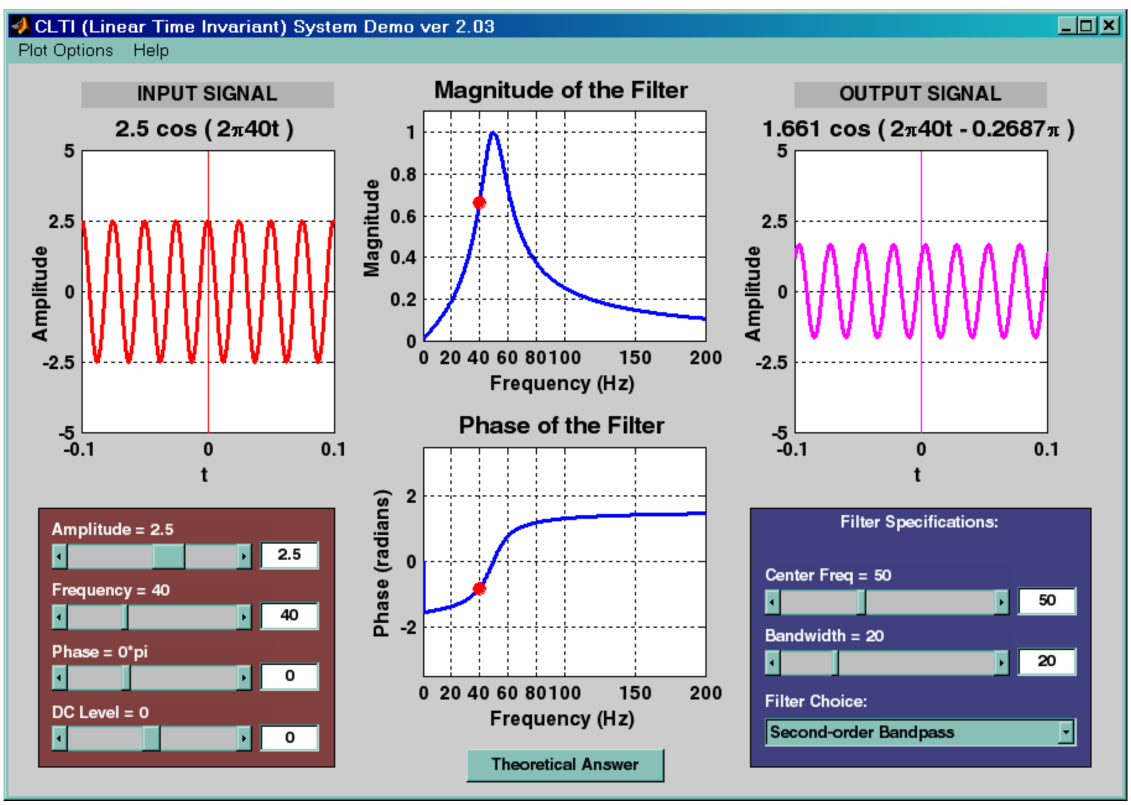

Figure 1: `CLTIdemo` interface for continuous-time frequency response.

We know that if the input to an LTI continuous-time system is a sinusoid of the form

$$x(t) = A + B\cos(\omega_0 t + \phi) \qquad -\infty < t < \infty \tag{4}$$

then the corresponding output is also a sinusoid:

$$y(t) = AH(j0) + B|H(j\omega_0)| \cos(\omega_0 t + \phi + \angle H(j\omega_0)) \qquad -\infty < t < \infty, \tag{5}$$

where

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} d\omega \tag{6}$$

is the *frequency response* of the continuous-time LTI system. The **`CLTIdemo`** GUI illustrates this for a variety of simple analog filters.

(a) Use the `CLTIdemo` GUI to find the output of a first-order lowpass filter by selecting "First-Order Lowpass" from the menu and setting the cutoff frequency to 30 Hz. Recall that the frequency response of this lowpass filter is

$$H(j\omega) = \frac{1}{j\omega + a} \tag{7}$$

where a is the cutoff frequency in rads/sec.

(b) Set the input to

$$x(t) = 1.0 + \cos(20\pi t).$$

Look at the output and compare its amplitude and phase to the input amplitude and phase. Click the box labeled "Theoretical Answer" to see a formula for the output $y(t)$.
Note: The GUI input frequencies are in hertz, which is $f = \omega/(2\pi)$; ω would have units of rad/s.

(c) Keeping the DC level and the amplitude of the cosine the same, use the slider to increase the input frequency and observe the change in the output. Keep increasing the slider until the frequency is $\omega = 80\pi$ rad/s (or $f = 40$ Hz). Compare the output in this case to the output at the original frequency of $\omega = 20\pi$. If you were to describe the output as having a "ripple", does the ripple increase or decrease as ω increases?

(d) Repeat the previous part with the filter set to "Ideal Lowpass" with a cutoff frequency of 30 Hz. Start with the input signal from part (a).

(e) Set the frequency of the input back to $\omega = 20\pi$ and change the filter to "First-Order Highpass" with a cutoff frequency of 30 Hz. Observe the output as the frequency is increased. What is the DC component of the output? Does the amplitude of the output sinusoid get bigger or smaller as the frequency is increased?

(f) Convince yourself that the following frequency response is a first-order HPF:

$$H(j\omega) = \frac{j\omega}{j\omega + b}$$

where the parameter b is the cutoff frequency of the HPF in rad/s.

2.2 Spectrum from Fourier Series

Use the `FSeriesDemo` GUI to show the spectrum for a half-wave rectified sine wave. Notice that the GUI will also show the resynthesized signal for a finite number of coefficients. Figure 2 show the resynthesis for $N = 4$.

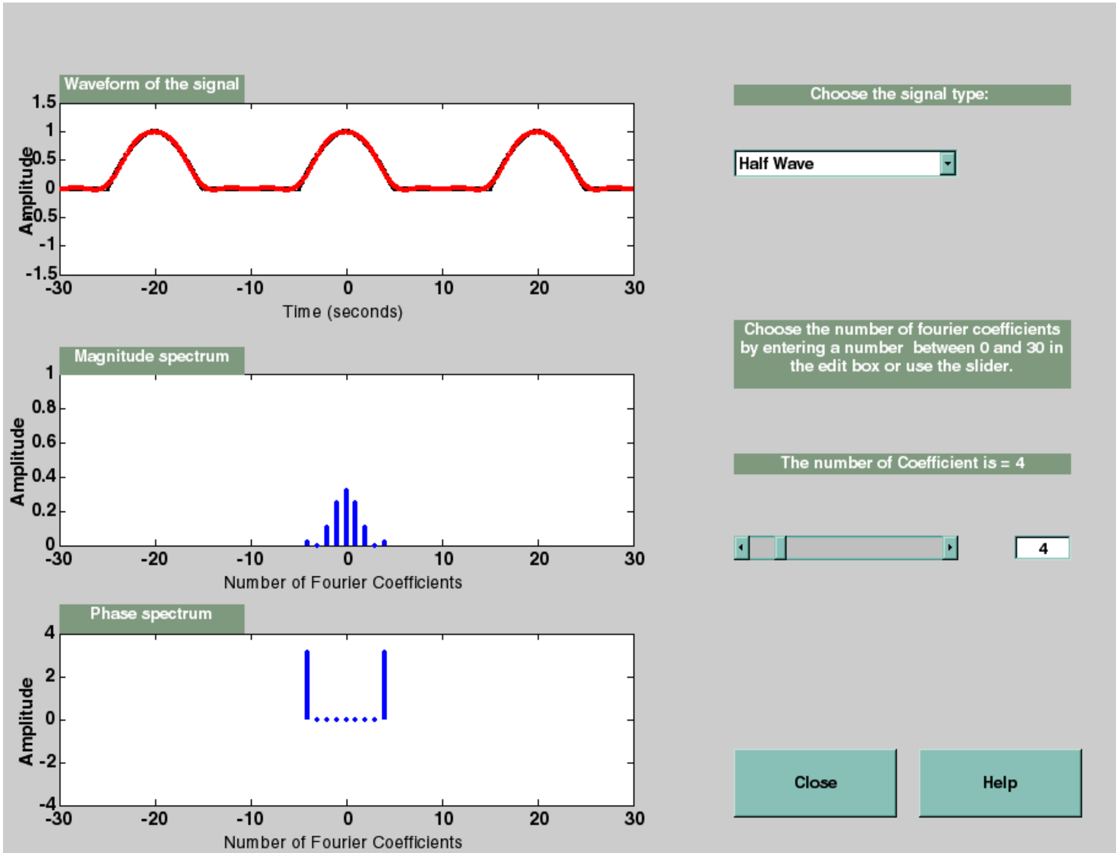

Figure 2: `FSeriesDemo` interface for Fourier Series synthesis.

3 Warm-up: Periodic Signal Filtered by an Analog Bandpass Filter

3.1 Sinusoidal Synthesis from Fourier Series Coefficients

One activity during this Warm-up section is to recall information about Fourier Series, and also to show that you can adapt your existing MATLAB functions to a new situation quickly.

In the lab project, you will use the Fourier Series coefficients to predict the response of a LTI system. It will be necessary to have two functions: one to calculate the Fourier Series coefficients, and another to synthesize a signal from a given set of Fourier Series coefficients. We will restrict our attention to the case of half-wave rectified sine wave signals whose definition over one period (T_0) is:

$$x(t) = \begin{cases} |\sin(\omega_0 t)| & \text{for } 0 \le t \le \frac{1}{2}T_0 \\ 0 & \text{for } \frac{1}{2}T_0 < t < T_0 \end{cases} \tag{8}$$

In this case, you can derive a general expression for the Fourier Series coefficients as

$$a_k = \begin{cases} -0.25j & \text{for } k = 1 \\ 0.25j & \text{for } k = -1 \\ \dfrac{1 + e^{-j\pi k}}{2\pi(1-k^2)} & \text{for } k \neq \pm 1 \end{cases} \tag{9}$$

If you are mathematically inclined, you might want to work out the Fourier Series integral for $x(t)$ in Eq. (8) by hand to show that the $\{a_k\}$ values in (9) are correct.

(a) Write a MATLAB function that will evaluate the $\{a_k\}$ coefficients for a half-wave rectified sine using (9). The function call should look like:

```
ak = ak4rectsine( krange, T0 )
```

The argument `krange` should be a vector that gives the set of k indices for evaluating the $\{a_k\}$ Fourier Series coefficients, e.g., `krange=[-5:5]`.

(b) For synthesis, you could modify a function that you have already developed in Lab #2 (`addcos.m`). Recall that the Fourier Series synthesis using $2N + 1$ coefficients would be

$$x_N(t) = \sum_{k=-N}^{N} a_k e^{j2\pi k f_0 t}$$

and $x_N(t)$ would be real-valued if the coefficients satisfy the conjugate-symmetry property, $a_{-k} = a_k^*$. Write a function called `ak2sig` that will produce a time signal from a given set of Fourier Series coefficients. Here is the format for the function call:

```
[xx,tt] = ak2sig( ak, krange, Tperiod, tstart, dur, fs )
```

where the vectors `ak` and `krange` must have exactly the same length because `ak` will be the set of $\{a_k\}$ coefficients at the indices given in `krange`. The parameters `tstart` and `dur` define the starting time and duration of the time interval over which the signal will be synthesized, `Tperiod` is the period of the synthesized signal, and `fs` the sampling rate.

(c) Demonstrate that your MATLAB functions written in the previous two parts work correctly by generating $x_4(t)$ for the half-wave rectified sine wave with period equal to 0.05 sec. Show that you can generate a signal like the waveform in Fig. 2.
Note: The time interval for the plot should be three periods of the signal from $t = -T_0$ to $t = 2T_0$. The sampling rate should be quite a bit higher than the Nyquist rate which is dictated by the highest harmonic frequency. Over-sampling will be needed to "see" convergence.

Instructor Verification (separate page)

(d) Explain how you are getting convergence as N increases. Where does the approximation error seem to be largest?

3.2 Rectified Sine Spectrum

In this part of the warm-up, the objective is to make a plot of the spectrum versus frequency for the half-wave rectified sine wave defined above.

(a) The first step is to get the Fourier Series coefficients. Utilize the M-file `ak4rectsine` to evaluate the $\{a_k\}$ coefficients for the half-wave rectified sine wave over the range of indices $k = -N, \ldots, -1, 0, 1, 2, \ldots, N$. This M-file returns the $\{a_k\}$ coefficients as a vector containing $2N+1$ elements.

Evaluate the Fourier Series coefficients $\{a_k\}$ given in Eq. (9) for $x(t)$ defined in Eq. (8) for the case where the parameters of the half-wave rectified sine wave are $T_0 = 0.05$ secs. Determine the $\{a_k\}$ coefficients for $N = 8$ and make a stem plot of the magnitude of the coefficients versus k.

(b) Make the same stem plot of the magnitude of the Fourier Series coefficients as in part (a), but convert the horizontal axis into frequency, so that it becomes a plot of the spectrum of the half-wave rectified sine wave.

Instructor Verification (separate page)

3.3 Frequency Response of an Analog Filter

In the lab project, you will use a continuous-time LTI system for filtering. In this section of the warm-up, we will investigate the following frequency response:

$$H(j\omega) = \frac{b}{a + j\omega} \tag{10}$$

where a controls the bandwidth of the filter.

(a) Make a plot of the magnitude and phase of $H(j\omega)$ versus ω in rad/s. Pick the parameters of the frequency response to be $a = 40\pi$, and $b = 20\pi$. In order to get values for the plot, you should evaluate the $H(j\omega)$ formula directly for a dense grid of frequencies. Use a range of frequencies that extends from -500 rad/s to $+500$ rad/s.[1] From the plot of $|H(j\omega)|$ versus ω, determine what kind of filter $H(j\omega)$ is.

Instructor Verification (separate page)

(b) Determine the peak value of the magnitude (frequency) response and the location of the peak. Use the algebraic form of the frequency response formula $H(j\omega)$ to explain that the peak value is correct.

3.4 Sinusoidal Response of LPF

The **`CLTIdemo`** GUI can implement the LPF defined by (10) if you choose the filter named "First-Order Lowpass."

(a) Use the `CLTIdemo` GUI to create a first-order lowpass filter by selecting "First-Order Lowpass" from the menu and setting the cutoff frequency to 20 Hz.[2] This should be the same frequency response as in Section 3.3.

(b) Set the input signal to

$$x(t) = 1.0 + \cos(40\pi t)$$

Look at the output and compare its amplitudes and phases to the input amplitudes and phases. Click the box labeled "Theoretical Answer" and record the result.

(c) Now change the input signal to $x(t) = \cos(80\pi t)$, and record the numerical values of the output signal's amplitude and phase. Repeat for $x(t) = \cos(120\pi t)$, again recording the amplitude and phase of the output signal.

(d) Now consider the case where the input signal $x(t)$ is the half-wave rectified sine wave whose spectrum was plotted in Section 3.2. Use the information from the previous part (along with the values of $\{a_k\}$) to write the output $y(t)$ as a sum of cosines.

$$y(t) = B_0 + B_1\cos(\omega_0 t + \psi_1) + B_2\cos(2\omega_0 t + \psi_2) + B_3\cos(3\omega_0 t + \psi_3) + \ldots$$

Use the values of the frequency response and the $\{a_k\}$ coefficients from Section 3.2 to determine the numerical values for ω_0, B_1, B_2, B_3, and ψ_1, ψ_2, and ψ_3.

Instructor Verification (separate page)

The calculation above amounts to an analysis of how you can "filter" the periodic input signal (the half-wave rectified sine wave from Section 3.2) through a continuous-time LTI system whose frequency response is given in Section 3.3. Since this is an analog system, we cannot do the actual filtering in MATLAB; instead, we can only calculate what the output signal would be by finding the Fourier Series of the output.

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[1] You can plot the frequency response versus frequency in hertz or radians/sec. Either way is acceptable, but make sure that you label the horizontal axis.

[2] The `CLTIdemo` GUI will convert the frequencies from hertz to rad/s.

4 Analysis of a Power Supply Circuit

A common electrical design problem is that of converting an AC voltage to a DC voltage; no doubt you have in your possession many of these little "power packs" for modems, calculators, phones, etc. An alternating current (AC) voltage waveform is a sinusoid at 60 Hz (in the US); a direct current (DC) voltage is a constant, or zero frequency waveform. Figure 3 depicts a circuit for creating a DC voltage from an AC voltage. The diode bridge circuit implements what is called a "full-wave rectifier," and the RC circuit provides lowpass filtering to produce a (nearly) constant DC output.

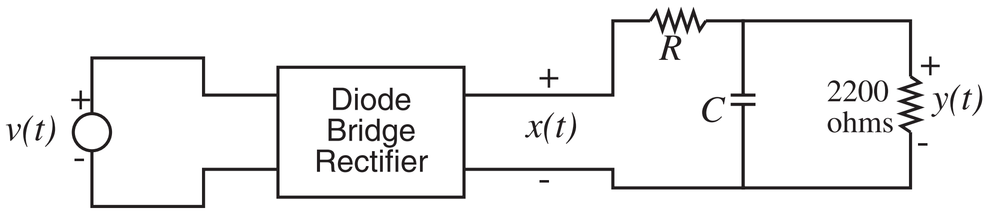

Figure 3: Power Supply circuit: AC to DC voltage converter

In this lab exercise, you will analyze a power supply circuit (Fig. 3) and design its parameters by using methods in the frequency domain. The strategy consists of finding the Fourier Series for the signal $x(t)$; multiplying by the frequency response of the RC circuit (a lowpass filter); and then evaluating the size of the Fourier coefficients for the output signal. The diode-bridge rectifier is a full-wave rectifier defined by the equation

$$x(t) = |v(t)| \tag{11}$$

where $v(t)$ is the input and $x(t)$ is the output. The output of the rectifier, $x(t)$, is the input to the RC circuit and $y(t)$ is the output of that circuit. Once you have learned circuits, it would be easy to write the differential equation that describes the R-C circuit in Fig. 3. The result would be the following differential equation which gives the relationship between the input and output voltages of the circuit:

$$\frac{d}{dt}y(t) + \left(\frac{2200 + R}{2200RC}\right)y(t) = \frac{1}{RC}x(t) \tag{12}$$

In a real power supply, the signal $v(t)$ would be the 60-Hz powerline AC voltage, which would be represented mathematically as $v(t) = 120\sqrt{2}\cos(120\pi t)$. The units of $v(t)$ are *volts.*

If you take the Fourier transform of (12) and solve for the frequency response $H(j\omega)$ the result is

$$H(j\omega) = \frac{\beta}{j\omega + \alpha}$$

where the parameters α and β can be written in terms of R and C.

The block diagram shown in Fig. 4 represents the operations performed by the different parts of the circuit. The input-output equation for the full-wave rectifier is defined by $x(t) = |v(t)|$. The purpose of the rectifier is to generate a periodic signal with a non-zero DC component. The purpose of the lowpass filter is

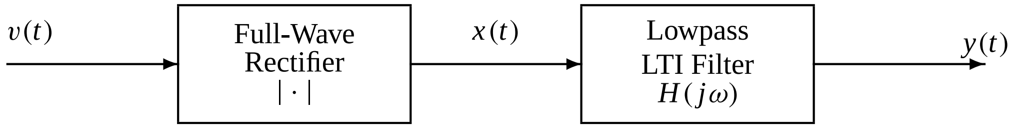

Figure 4: Block diagram representation of power supply.

to remove most of the high frequencies in the output of the rectifier, leaving the DC component.

4.1 Fourier Analysis of the Full-Wave Rectifier Output

Assume that the input to the rectifier is a power line voltage $v(t) = 120\sqrt{2}\cos(120\pi t)$.

(a) It is not too hard to verify that the true impulse response of the R-C circuit is a one-sided exponential:

$$h(t) = \frac{1}{RC}e^{-\alpha t}u(t)$$

where $\alpha = (2200 + R)/2200RC$. You just substitute $h(t)$ into the differential equation (12) and show that both sides match. It involves taking the first derivative and adding together the two terms on the left-hand side of the differential equation (12). Remember that the derivative of the unit-step signal, $u(t)$, is the unit impulse signal, $\delta(t)$. You should include this derivation in a lab report.

(b) To find the frequency response of the circuit, we can "take the Fourier transform" of $h(t)$ and get the frequency response of the lowpass filter in the following form:

$$H(j\omega) = \frac{\beta}{j\omega + \alpha}$$

Notice that the values of α and β can be written in terms of R and C; you will need this later on when evaluating the frequency response $H(j\omega)$.[3]

Write a MATLAB function that will plot the magnitude and phase of $H(j\omega)$ over the range $-2\pi(400) \le \omega \le 2\pi(400)$. For the input arguments, use R and C. Write this code from scratch using `plot()` or `fplot()`; don't use MATLAB's built-in frequency response function.[4] Test your function by making the the plot for some typical values of R and C (e.g., $R = 22,000$ ohms and $C = 6 \times 10^{-6}$ farads).

(c) Define MATLAB expressions for the functions $v(t)$ and $x(t)$. The output of the full-wave rectifier is a periodic signal such that $x(t) = x(t + T_0)$. Determine the *fundamental* period T_0 of the rectified signal $x(t)$. Make a plot showing both $v(t)$ and $x(t)$ over the range $0 \le t \le 3T_0$.

(d) Since the output of the full-wave rectifier, $x(t)$, is a periodic signal, it has a Fourier Series. Derive the $\{a_k\}$ coefficients for the full-wave rectified sine wave. Then write a MATLAB function, like `ak4rectsine`, to evaluate the Fourier coefficients of $x(t)$.

(e) Since $x(t)$ is periodic, it can be represented approximately by a truncated Fourier series. Use your formula for the Fourier coefficients $\{a_k\}$ for $k = 0, \pm 1, \pm 2, \ldots, \pm N$ to synthesize an approximation to $x(t)$; use $N = 10$ and call the resulting signal $x_{10}(t)$. This can be done by using the function `ak2sig` to so the synthesis.

Make a plot over the range $0 \le t \le 3T_0$ that compares the exact $x(t)$ to the approximate one, $x_{10}(t)$. At which times is $x_{10}(t)$ most different from the exact $x(t)$?

4.2 Find the Output Signal

The Fourier coefficients of the output signal are $b_k = a_k H(jk\omega_0)$, because the theory of the frequency response tells us how to determine the exact output of the lowpass filter by tracking each sinusoidal component through the filter: Using our $2N+1$ term approximation for the input, the approximate output is

$$y_N(t) = \sum_{k=-N}^{N} b_k e^{jk\omega_0 t} = \sum_{k=-N}^{N} a_k H(jk\omega_0) e^{jk\omega_0 t} \tag{13}$$

where the a_k are the Fourier coefficients of $x(t)$.

[3] Alternate approach: We can "take the Fourier transform" of the differential equation; the derivative term becomes a multiplication by $j\omega$ in the frequency domain. and $H(j\omega)$ is found by dividing $Y(j\omega)/X(j\omega)$.

[4] You cannot use `freqz()` because this is not a digital filter; MATLAB has a function called `freqs()`, but you shouldn't use this function in this lab.

(a) *Frequency Domain:* Make a three-panel plot showing the spectrum of $x_N(t)$ in the top for $N = 3$; the magnitude of $H(j\omega)$ in the middle (use a frequency range that lines up with the top plot); and the **spectrum** of $y_N(t)$ in the bottom plot (for $N = 3$ also). To do this, you must have specific values for R and C to determine α and β in the formula for $H(j\omega)$, so let $R = 22,000$ ohms and $C = 6 \times 10^{-6}$ farads in this part and the next.

(b) *Time Domain:* Next, you should make a plot of the output signal in the time domain for $N = 10$, i.e., plot $y_{10}(t)$ versus t over the range $0 \le t \le 11T_0$.

This requires that you evaluate the b_k Fourier coefficients numerically and use `ak2sig` to create $y_N(t)$. In this approach, use the Fourier coefficients a_k that were evaluated numerically, and then evaluate the frequency response $H(j\omega)$ at the appropriate frequencies. Then the product would be the Fourier coefficients of the output, as given by Eq. (3).

4.3 Design the Power Supply in the Frequency Domain

The power supply circuit could be solved in the time-domain to get the output waveform. If you had the signal $y(t)$, then you could measure the DC component of the output voltage, and you would also notice that the output signal contains an oscillating component called the *ripple.* The objective of the design is to control the DC level by choosing R, and then find the amount of ripple when $C = 6 \times 10^{-6}$ farads.

(a) Refer to your Fourier Series formula above, and determine the DC value of the input $x(t)$. You should have already made a plot of $x(t)$, so mark the DC value of $x(t)$ on that plot.

(b) In this part, let the value of R be an unknown parameter, but fix the value of C as 6×10^{-6} farads. Use your knowledge of the input Fourier series and $H(j\omega)$ to write a mathematical formula for the DC component of the output (in terms of R and C). Hint: Use the frequency response $H(j\omega)$ to find the DC value of the first-order RC filter in terms of R and C. As a sanity check, when you finally get the correct formula for the output DC term, it should not depend on C.

(c) When you made the plot of $y_{10}(t)$, you should have observed a "ripple" in the time-domain signal. The ripple in the output is due to all the non-DC terms in the input Fourier Series, but it is mostly due to the terms for $k = \pm 1$.

$$y(t) = b_0 + \sum_{k=1}^{\infty} \left(b_k e^{jk\omega_0 t} + b_{-k} e^{-jk\omega_0 t} \right)$$

Therefore, the output can be well approximated by considering the signal $y_1(t)$ that contains only one sinusoidal term plus DC. You should derive the mathematical formula for $y_1(t)$ in terms of the parameters R and C. However, you only need the mathematical formula for the *magnitude* of the first sinusoidal term in $y_1(t)$ (the phase is less important). In the process of doing this derivation you will have to determine the period of the ripple, so give the value of the period in secs.

(d) Now we can complete a general design of the power supply for any specification on the output DC voltage (V_{out}). Also we can find the ripple voltage (V_r). Suppose that our design specification is to have the DC component of the output be $V_{\text{out}} = 9$ volts. The design amounts to finding a value for R. In previous parts, we have written equations for the DC component and the first harmonic, so we can solve for R, given that $C = 6 \times 10^{-6}$ farads.

4.4 Power Supply Output Waveform: Ripple

(a) Verify your formulas in the previous section by using the value of R needed to get $V_{\text{out}} = 9$ volts.

(b) Make plots of the output signal to confirm that your design in the previous part is correct.
To verify your R value from the previous part, calculate the values of the output Fourier Series coefficients $\{b_k\}$, and make a plot of the output voltage signal when using $N = 1$ and $N = 10$ coefficients, i.e., plot $y_1(t)$ and $y_{10}(t)$. The reason for plotting $y_{10}(t)$ is that it should represent the true output, while $y_1(t)$ is an approximation because it uses only one Fourier coefficient.

(c) Measure the ripple on the output waveform. Explain the validity of using one Fourier coefficient for the design. How much error is introduced by ignoring $\{b_k\}$ for $k > 1$.

4.5 Power Supply Output Waveform: Half-Wave Rectifier

Now change the rectifier from "full-wave" to "half-wave".

(a) Determine the Fourier Series coefficients for the half-wave rectified sine wave.

(b) Using $C = 6 \times 10^{-6}$ farads, determine the value of R to get a DC output of 9 volts.

(c) Synthesize the output waveform, and then measure the ripple on the output waveform.

(d) Compare the ripple between the "full-wave" and " half-wave" cases.

Lab #11
ECE-2025
Fall-2003
INSTRUCTOR VERIFICATION PAGE

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ______________________________ Date of Lab: __________

Part 3.1 Illustrate Fourier synthesis from the $\{a_k\}$ for a half-wave rectified sine wave. Make some plots with different numbers of coefficients. Explain convergence.

Verified:____________________ Date/Time:__________

Part 3.2 Determine the numerical values of the Fourier coefficients $\{a_k\}$ for a half-wave rectified sine wave. Plot the spectrum versus frequency.[5]

Verified:____________________ Date/Time:__________

Part 3.3 Plot the magnitude and phase of the frequency response of the continuous-time filter, $H(j\omega)$, defined in Eq. (10).

Verified:____________________ Date/Time:__________

Part 3.4 Find the $\{b_k\}$ Fourier Series coefficients of the output signal. Plot the spectrum versus frequency for the output signal. List the values of ω_0, B_k and ψ_k in the table below.

	$\omega_0 =$	
$k = 0$	$B_0 =$	
$k = 1$	$B_1 =$	$\psi_1 =$
$k = 2$	$B_2 =$	$\psi_2 =$
$k = 3$	$B_3 =$	$\psi_3 =$

Verified:____________________ Date/Time:__________

[5] It might be natural to plot a_k versus k, but when you show the spectrum the horizontal axis must be frequency (in Hz or rad/s).