GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 30-May-03 COURSE: ECE 2025

NAME: Solutions (LAST, FIRST) STUDENT #: ____________

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-10:00am (D. Taylor)

L02:Tues-12:00am (T. Michaels)

L03:Tues-2:00pm (D. Taylor)

L04:Tues-4:00pm (D. Taylor)

L06:Mon-4:00pm (T. Michaels)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive any partial credit. Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	

Problem Q1.1:

Define $x(t)$ as

$$x(t) = 10\cos(50\pi t + 3\pi/4) - 15\cos(50\pi(t + 0.01))$$

(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

$$x(t) = \mathrm{Re}\left\{ \left(10 e^{j3\pi/4} - 15 e^{j0.5\pi}\right) e^{j50\pi t} \right\}$$

$$X_1 = 10 e^{j3\pi/4} = 10\cos(3\pi/4) + j10\sin(3\pi/4) \quad -5\sqrt{2} + j5\sqrt{2}$$

$$X_2 = -15 e^{j0.5\pi} = -15j$$

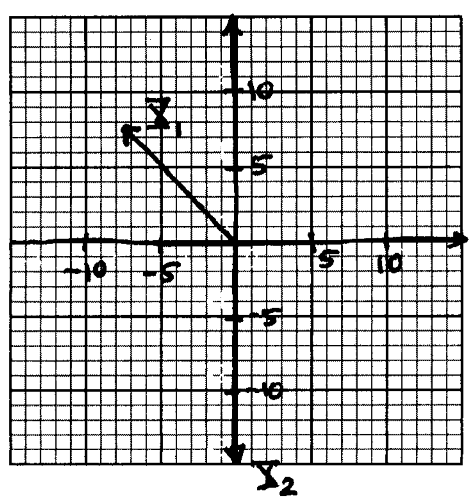
$$X_1 + X_2 = -5\sqrt{2} + (5\sqrt{2} - 15)j = -7.071 - j7.929$$

$$= 10.624 \cdot e^{-j0.73\pi}$$

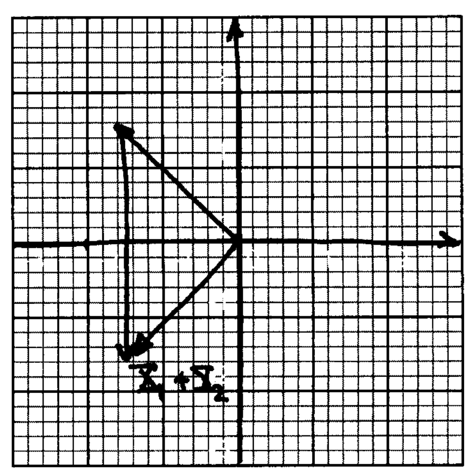
$\boxed{\omega_0 = 50\pi}$ $\boxed{A = 10.624}$ $\boxed{\phi = -0.73\pi}$

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show the two complex amplitudes being added; on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail).

Two Vectors Here



Head-to-tail Plot Here



Problem Q1.2:
Simplify the following complex-valued expressions. In each case, reduce the answers to a simple numerical form.

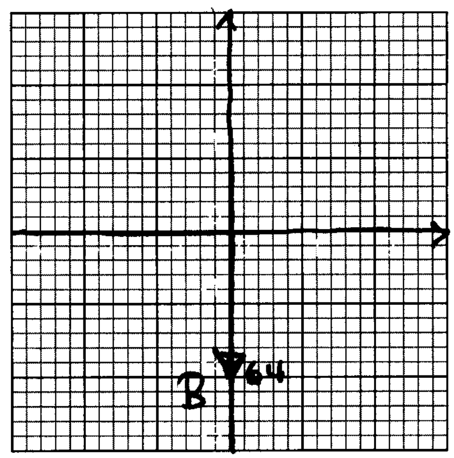
(a) Let $A = -2 + j2\sqrt{3}$. Express the complex number $B = (jA)^3$ in polar form, and plot the vector B in the graph provided below.

$$A = -2 + j2\sqrt{3} = 4e^{j2\pi/3}$$

$$B = (jA)^3 = \left(4e^{j2\pi/3} \cdot e^{j\pi/2}\right)^3$$

$$= \left(4e^{j7\pi/6}\right)^3$$

$$= 64e^{j7\pi/2} = 64e^{-j\pi/2}$$



(b) Let $C = -1 + 2j$. Evaluate the following expression, and express your answer in cosine form:

$$Re\left\{j^3 C e^{j24\pi t}\right\}$$

$$C = -1 + 2j = \sqrt{5}\,e^{j0.647\pi}$$

$$Re\left\{j^3 C e^{j24\pi t}\right\} = Re\left\{e^{j3\pi/2}\sqrt{5}\,e^{j0.647\pi} \cdot e^{j24\pi t}\right\}$$

$$= Re\left\{\sqrt{5}\,e^{j(24\pi t + 2.147\pi)}\right\}$$

$$= \sqrt{5}\cos(24\pi t + 0.147\pi)$$

(c) Let $Y = 4 - 2j$ and $Z = e^{j\pi/3}$. Evaluate the following expression, and give your answer in rectangular form.

$$W = Y + Z$$

$$Z = e^{j\pi/3} = \frac{1}{2} + \frac{\sqrt{3}}{2} j$$

$$W = Y + Z = (4 - 2j) + \left(\frac{1}{2} + \frac{\sqrt{3}}{2} j\right)$$

$$= 4.5 + \left(\frac{\sqrt{3}}{2} - 2\right) j = 4.5 - 1.134 j$$

(d) Evaluate the following expression

$$|(1 + j)e^{j(25\pi t - \pi/3)}|^2$$

$$|(1+j)\, e^{j(25\pi t - \pi/3)}|^2 = |1+j|^2 \cdot \underbrace{|e^{j(25\pi t - \pi/3)}|^2}_{1} = 2$$

Problem Q1.3:
A signal $x(t)$ is given by the equation

$$x(t) = [2 + \cos(43\pi t + \pi/3)] \cos(330\pi t - \pi/2)$$

This signal can also be expressed as a sum of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k) \tag{1}$$

where each of the frequencies ω_k are different **(all frequencies are positive)**.

(a) Determine the number of cosine terms in $x(t)$ in Eq. (1), i.e., find the value for N.

$\boxed{N = 3}$

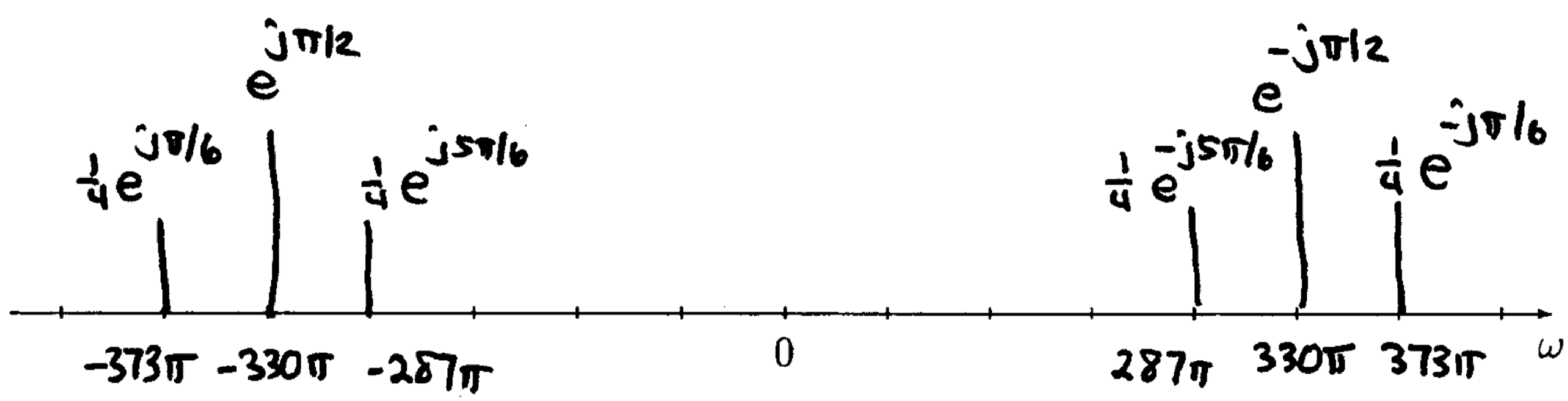
$$x(t) = 2\cos(330\pi t - \pi/2) + \cos(43\pi t + \pi/3)\cdot\cos(330\pi t - \pi/2)$$

$$= 2\cos(330\pi t - \pi/2) + \tfrac{1}{2}\cos(373\pi t - \pi/6) + \tfrac{1}{2}\cos(287\pi t - 5\pi/6)$$

(b) What are the largest and smallest frequencies of all the sinusoids in the sum form given in Eq. (1)?

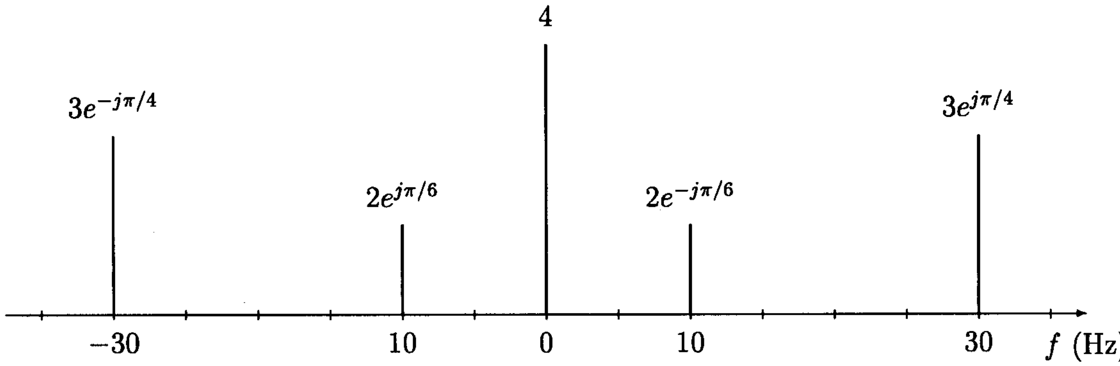
$\boxed{\text{LARGEST } \omega_k = 373\pi}$ $\boxed{\text{SMALLEST } \omega_k = 287\pi}$

(c) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency in radians/sec and their complex amplitude in polar form.



Problem Q1.4:

The spectrum of a signal $x(t)$ is shown in the following figure:



Note that the frequency axis is cyclic frequency (f) in Hertz.

(a) Write an equation for $x(t)$ in terms of cosine functions.

$\boxed{x(t) = 4 + 4\cos(20\pi t - \pi/6) + 6\cos(60\pi t + \pi/4)}$

(b) This signal is periodic. What is the fundamental frequency and the corresponding period of $x(t)$?

$f_0 = 10$ Hz $\Rightarrow$ $T = 1/f_0 = 0.1$ sec

$\omega_0 = 2\pi f_0 = 20\pi$

$\boxed{\omega_0 = 20\pi \text{ rad/sec}}$ $\boxed{T = 0.1 \text{ sec}}$

(c) A new signal is defined as $y(t) = x(t) + \cos(\alpha t + \pi)$. It is known that $y(t)$ is periodic with period 1 sec. Find **two** different **non-negative** values for the frequency α that will satisfy this condition.

We want $T = 1$ sec $\Rightarrow$ $f_0 = 1$ Hz or $\omega_0 = 2\pi$

There are many solutions. Two are given below

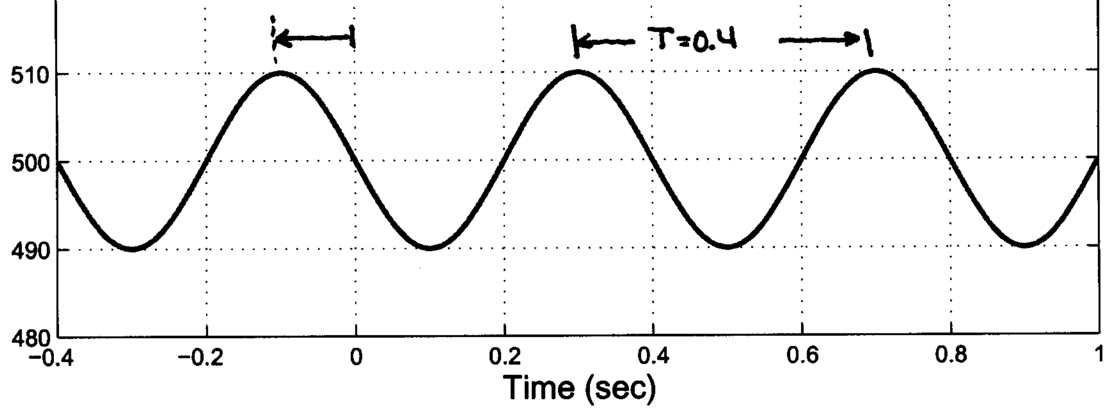
$\boxed{\alpha_1 = 2\pi(1)}$ $\boxed{\alpha_2 = 2\pi(3)}$

Problem Q1.5:

(a) Shown in the following figure is a plot of a sinusoidal signal of the form

$$\alpha(t) = A + B\cos(\omega_0 t + \phi)$$

$T_d = -0.1$



From this plot, find the numerical values for A, B, ω_0, and ϕ, with $-\pi < \phi \leq \pi$.

$T = 0.4 \Rightarrow f_0 = 5/2$ and $\omega_0 = 2\pi f_0 = 5\pi$

$\phi = -\omega_0 T_d = -5\pi(-0.1) = 0.5\pi$

$\omega_0 = 5\pi$	$A = 500$	$B = 10$	$\phi = \pi/2$

(b) If $\alpha(t)$ is the <u>instantaneous</u> frequency of a frequency modulated cosine of the form,

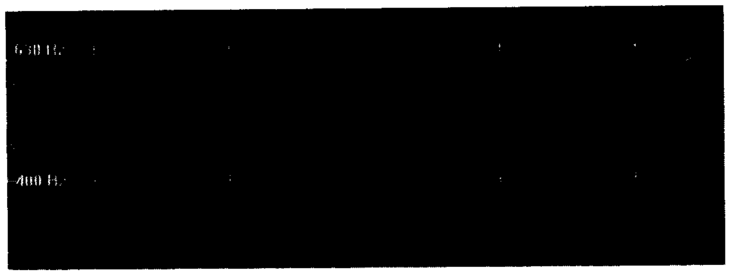
$$x(t) = \cos(\psi(t))$$

give an expression for the signal $x(t)$, i.e., find $\psi(t)$. **Note:** Assume that the vertical axis in the plot of $\alpha(t)$ is in **Hertz**.

$\psi'(t) = 500 + 10\cos(5\pi t + \pi/2)$

$\Rightarrow \psi(t) = \left[500t + \frac{10}{5\pi}\sin(5\pi t + \pi/2) + \phi\right] \cdot 2\pi$

(c) Shown in the figure below is the spectrogram of a signal, with time plotted along the horizontal axis, and frequency along the vertical.



Based on what you have learned about spectrograms and the examples that you have seen in class, which of the following type of signal does this spectrogram represent? (There is only one answer).

(a) The chirping noise made by a bird.

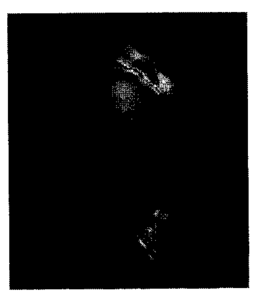
(b) A flute.

(c) Model airplanes

(d) A vowel spoken by a male speaker.

(e) The vibrato made by a male singer.

(d) What is the last name of the person shown in the figure below?



NAME: Euler