

EE-2025

Fall-03

LECTURE #2

Phase & Time-Shift

Complex Exponentials

22-Aug-03

INFORMATION

- **MATLAB: 6pm Mon, Tues, Wed in VL-261**
- LABS start NEXT week (**MONDAY**)
 - Attend correct section (in Bunger-Henry 216)
 - ECE Computer acct: **gtxxxx**, password: **ID #**
 - Verification must be signed during Lab
 - **Pre-Lab & Post-Lab ON-LINE Questions**
- RECITATIONS
 - Attend your assigned time

Web-CT Info

- Check the Bulletin Board for msgs
 - **MAKE YOUR OWN POSTINGS**
- Web-CT Login: everyone should have acct
- PDF Files on WebCT
 - Lectures are being posted (4 per page)
 - Get PDF file of Lab #1 from WebCT
 - Hard copy of Instructor Verification Sheet
 - HW #1 was posted as PDF
 - HW #2 today or tomorrow
 - Adobe Acrobat Reader required

Homework Info

- HWs will be posted on Friday/Sat
 - Covered in Rec during the following Week
 - Due the week after that (9+ days later)
- Format info on WebCT
 - Cover page for Homework
- Prob Set #1 due **in RECITATION**
 - **At the beginning of class**
 - Solutions will be posted to WebCT after the last Recitation on Thursday which ends at 6pm

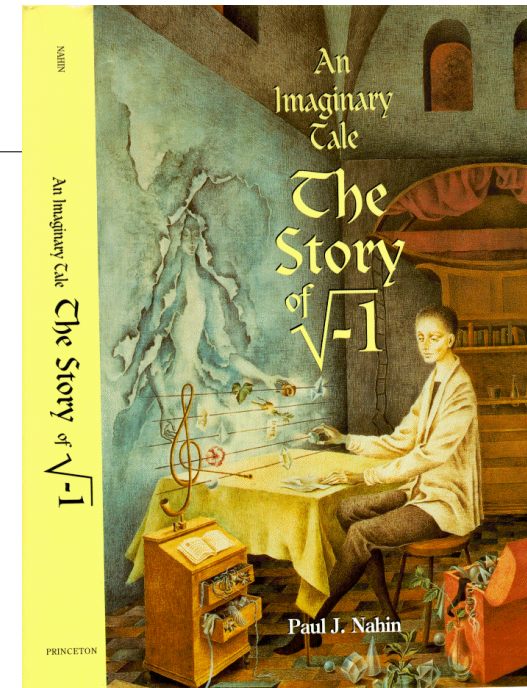
Homework Formatting

- Include a Cover page with
 - Name
 - Lab section, ie, L05, L20, etc.
 - Recitation Prof's name
 - **Download example from Web-CT**
- Write on **ONE** side only
 - Use Engineer's paper or plain paper
- STAPLE

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JUMP

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 2, Sects. 2-3 to 2-5
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2,
 - Section 2-6 to end

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LECTURE OBJECTIVES

- Define Sinusoid Formula from a plot
- Relate TIME-SHIFT to PHASE

Introduce an **ABSTRACTION**:
Complex Numbers **represent** Sinusoids
Complex Exponential Signal

$$z(t) = X e^{j\omega t}$$

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SINUSOIDAL SIGNAL

$$A \cos(\omega t + \varphi)$$

- **FREQUENCY** ω
 - Radians/sec
 - or, Hertz (cycles/sec)
$$\omega = (2\pi)f$$
- **PERIOD** (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$
- **AMPLITUDE** A
 - Magnitude
- **PHASE** φ

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PLOTTING COSINE SIGNAL from the FORMULA

$$5 \cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3$$

- Determine a **peak** location by solving

$$(\omega t + \varphi) = 0$$

- **Peak at t=-4**

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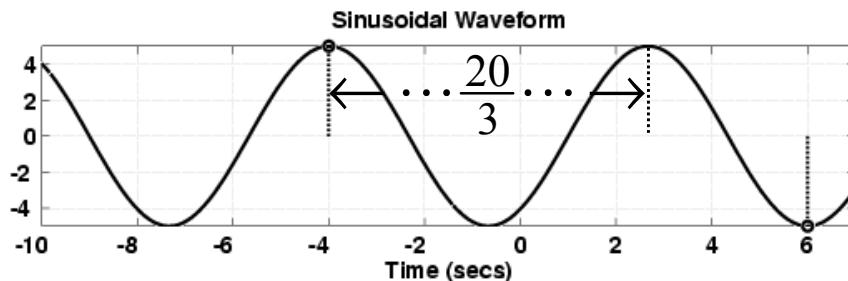
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ANSWER for the PLOT

$$5 \cos(0.3\pi t + 1.2\pi)$$

- Use $T=20/3$ and the peak location at $t=-4$



TIME-SHIFT

- In a mathematical formula we can replace t with $t-t_m$

$$x(t - t_m) = A \cos(\omega(t - t_m))$$

- Then the $t=0$ point moves to $t=t_m$
- Peak value of $\cos(\omega(t-t_m))$ is now at $t=t_m$

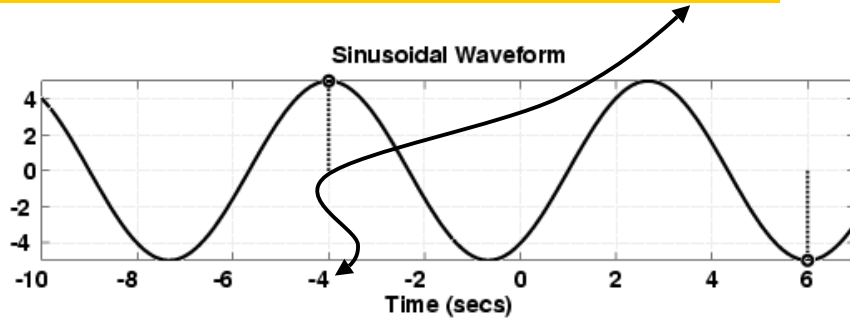
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TIME-SHIFTED SINUSOID

$$x(t+4) = 5 \cos(0.3\pi(t+4)) = 5 \cos(0.3\pi(t - (-4)))$$



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PHASE <--> TIME-SHIFT

- Equating the formulas:

$$A \cos(\omega(t - t_m)) = A \cos(\omega t + \phi)$$

- and we obtain:

$$-\omega t_m = \phi$$

- or,

$$t_m = -\frac{\phi}{\omega}$$

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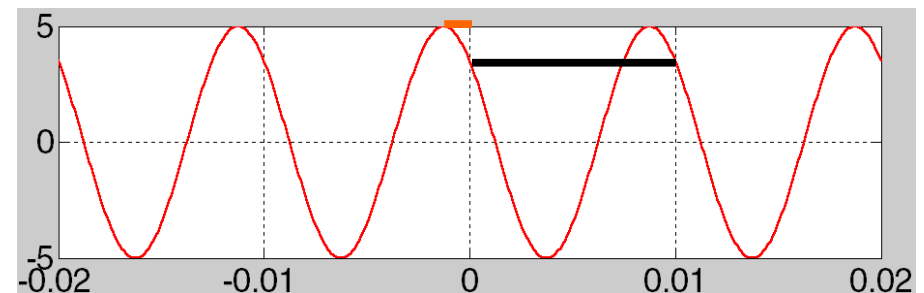
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SINUSOID from a PLOT

- Measure the period, T
 - Between peaks or zero crossings
- Compute frequency: $\omega = 2\pi/T$
- Measure time of a peak: t_m
 - Compute phase: $\phi = -\omega t_m$
- Measure height of positive peak: A

3 steps

(A, ω , ϕ) from a PLOT



$$T = \frac{0.01 \text{ sec}}{1 \text{ period}} = \frac{1}{100}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$$

$$t_m = -0.00125 \text{ sec}$$

$$\phi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$$

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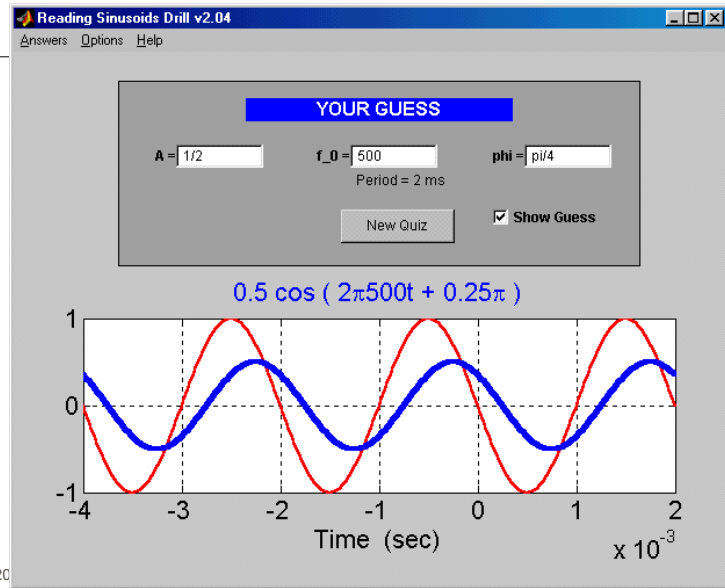
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SINE DRILL (MATLAB GUI)



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PHASE is AMBIGUOUS

- The cosine signal is periodic

- Period is 2π

$$A \cos(\omega t + \varphi + 2\pi) = A \cos(\omega t + \varphi)$$

- Thus adding any multiple of 2π leaves $x(t)$ unchanged

if $t_m = \frac{-\varphi}{\omega}$, then

$$t_{m_2} = \frac{-(\varphi + 2\pi)}{\omega} = \frac{-\varphi}{\omega} - \frac{2\pi}{\omega} = t_m - T$$

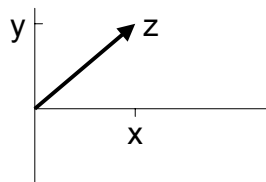
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COMPLEX NUMBERS

- To solve: $z^2 = -1$
 - $z = j$
 - Math and Physics use $z = i$
- Complex number: $z = x + jy$



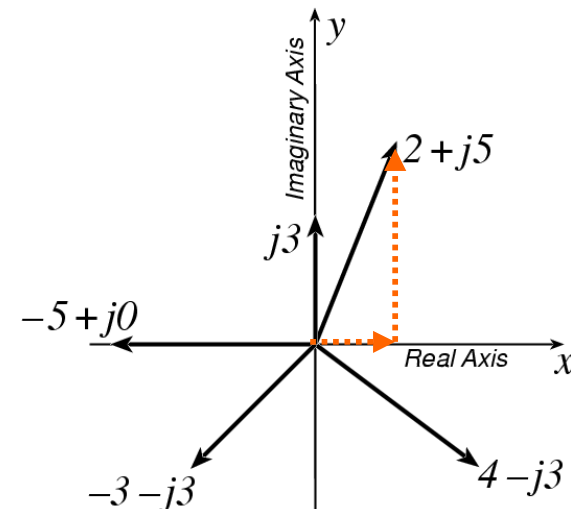
Cartesian coordinate system

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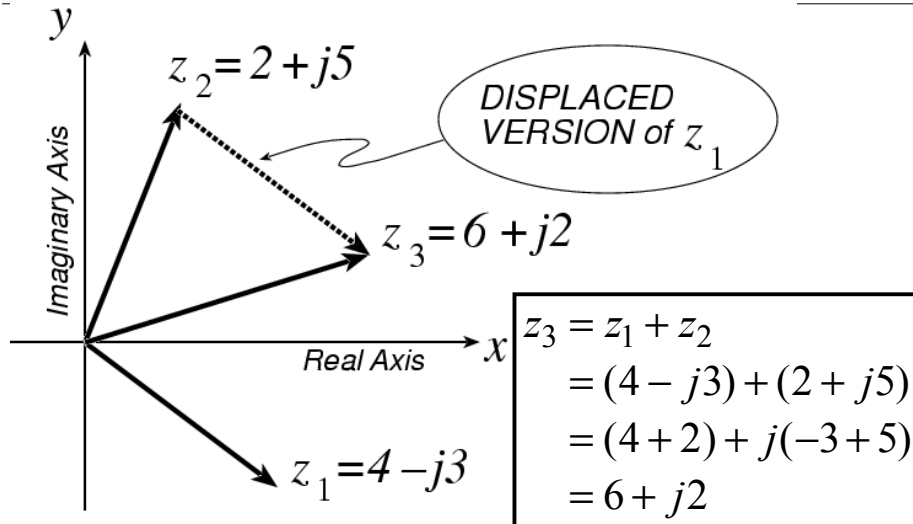
PLOT COMPLEX NUMBERS



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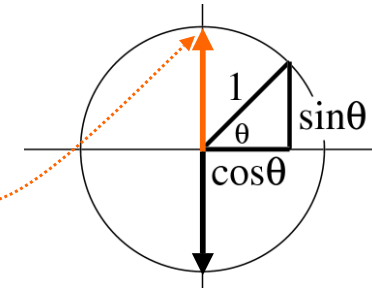
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COMPLEX ADDITION = VECTOR Addition



*** POLAR FORM ***

- Vector Form
 - Length = 1
 - Angle = θ
- Common Values
 - j has angle of 0.5π
 - -1 has angle of π
 - $-j$ has angle of 1.5π
 - also, angle of $-j$ could be $-0.5\pi = 1.5\pi - 2\pi$
 - because the PHASE is **AMBIGUOUS**

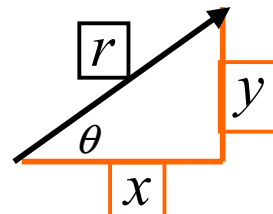


POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)

$$r^2 = x^2 + y^2$$

$$\theta = \text{Tan}^{-1}\left(\frac{y}{x}\right)$$



$$x = r \cos \theta$$

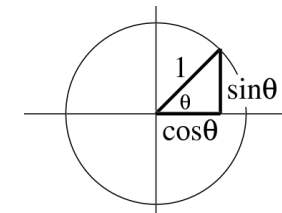
$$y = r \sin \theta$$

Most calculators do Polar-Rectangular

Need a notation for POLAR FORM

Euler's FORMULA

- Complex Exponential**
 - Real part is cosine
 - Imaginary part is sine
 - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

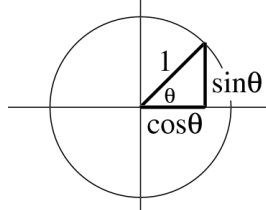
$$re^{j\theta} = r \cos(\theta) + jr \sin(\theta)$$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

- Interpret this as a **Rotating Vector**

- $\theta = \omega t$
- Angle changes vs. time
- ex: $\omega = 20\pi$ rad/s
- Rotates 0.2π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

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cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A \cos(\omega t + \varphi)$$

So,

$$\begin{aligned} A \cos(\omega t + \varphi) &= \Re\{A e^{j(\omega t + \varphi)}\} \\ &= \Re\{A e^{j\varphi} e^{j\omega t}\} \end{aligned}$$

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REAL PART EXAMPLE

$$A \cos(\omega t + \varphi) = \Re\{A e^{j\varphi} e^{j\omega t}\}$$

Evaluate:

$$x(t) = \Re\{-3j e^{j\omega t}\}$$

Answer:

$$\begin{aligned} x(t) &= \Re\{(-3j) e^{j\omega t}\} \\ &= \Re\{3e^{-j0.5\pi} e^{j\omega t}\} = 3 \cos(\omega t - 0.5\pi) \end{aligned}$$

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COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A \cos(\omega t + \varphi) = \Re\{A e^{j\varphi} e^{j\omega t}\}$$

Complex AMPLITUDE = X

$$z(t) = X e^{j\omega t} \quad X = A e^{j\varphi}$$

Then, any Sinusoid = REAL PART of $X e^{j\omega t}$

$$x(t) = \Re\{X e^{j\omega t}\} = \Re\{A e^{j\varphi} e^{j\omega t}\}$$

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