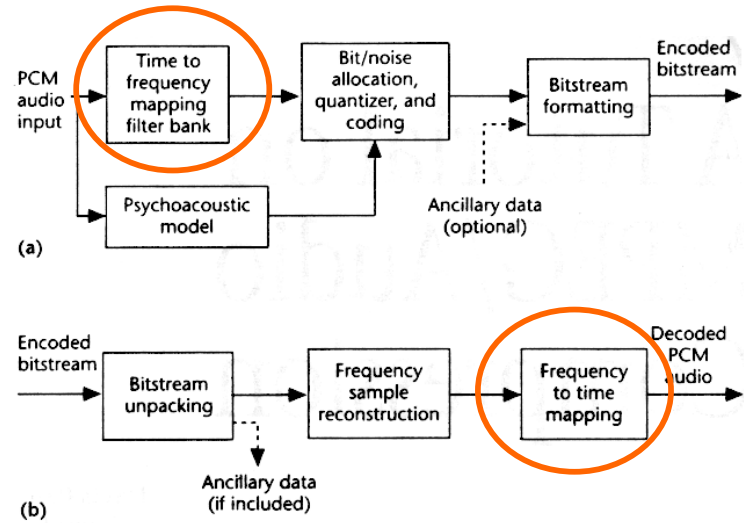


**Lecture 8**  
**Sampling & Aliasing**  
**15-Sept-03**

**MP-3 Block Diagram**



**Quiz #1 Info**

- **Quiz #1 on 19-Sept-03 (Friday)**
  - Coverage: HW #1, #2, #3 and #4
  - Allowed one page of notes (Handwritten)
  - Review Session Planned:
    - Thursday, 18-Sept, in ECE Auditorium
  - Check WebCT for other related announcements
- Old Quizzes & Problems are linked via WebCT:
  - **“Word from Previous Semesters”**

**READING ASSIGNMENTS**

- This Lecture:
  - **Chap 4, Sections 4-1 and 4-2**
    - Replaces Ch 4 in DSP First, pp. 83-94
- Other Reading:
  - Recitation: Strobe Demo (Sect 4-3)
  - Next Lecture: **Chap. 4 Sects. 4-4 and 4-5**

# LECTURE OBJECTIVES

- SAMPLING can cause ALIASING
  - **Sampling Theorem**
  - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals,  $x[n]$ 
  - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi \ell$$

**↑**  
**ALIASING**

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# SYSTEMS Process Signals



- PROCESSING GOALS:
  - Change  $x(t)$  into  $y(t)$ 
    - For example, more BASS
  - Improve  $x(t)$ , e.g., image deblurring
  - Extract Information from  $x(t)$

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# System IMPLEMENTATION

- ANALOG/ELECTRONIC:
  - Circuits: resistors, capacitors, op-amps



- DIGITAL/MICROPROCESSOR
  - Convert  $x(t)$  to **numbers** stored in memory



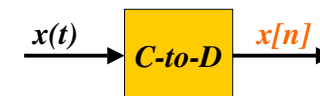
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# SAMPLING $x(t)$

- SAMPLING PROCESS
  - Convert  $x(t)$  to **numbers**  $x[n]$
  - “n” is an integer;  $x[n]$  is a sequence of values
  - Think of “n” as the storage address in memory
- UNIFORM SAMPLING at  $t = nT_s$ 
  - IDEAL:  $x[n] = x(nT_s)$



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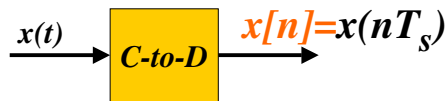
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# SAMPLING RATE, $f_s$

## SAMPLING RATE ( $f_s$ )

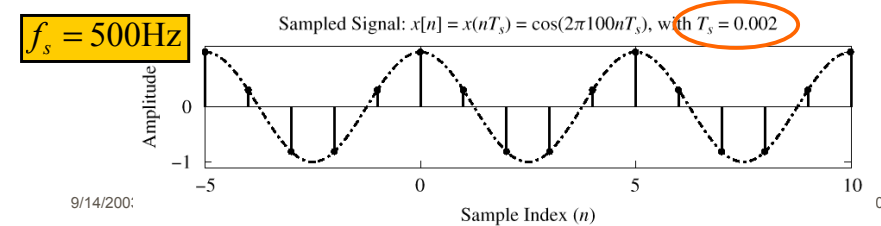
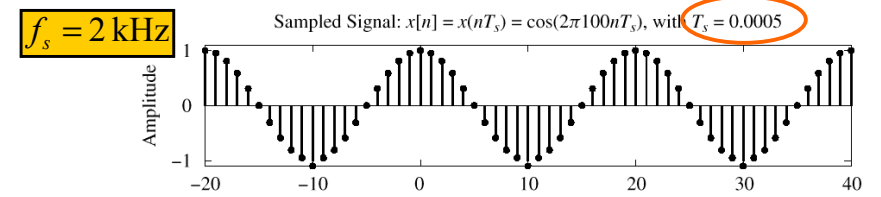
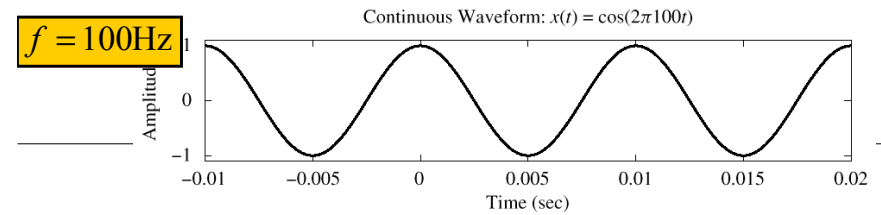
- $f_s = 1/T_s$ 
  - NUMBER of SAMPLES PER SECOND
- $T_s = 125 \text{ microsec} \rightarrow f_s = 8000 \text{ samples/sec}$ 
  - UNITS ARE HERTZ: 8000 Hz
- UNIFORM SAMPLING at  $t = nT_s = n/f_s$ 
  - IDEAL:  $x[n] = x(nT_s) = x(n/f_s)$



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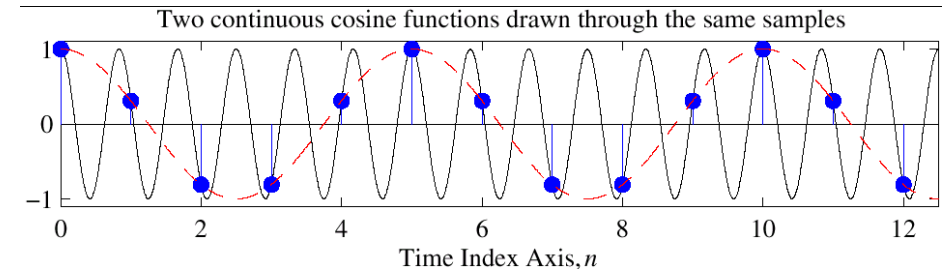
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# Reconstruction? Which One?

Given the samples, draw a sinusoid through the values



$$x[n] = \cos(0.4\pi n)$$

When  $n$  is an integer  
 $\cos(0.4\pi n) = \cos(2.4\pi n)$

## HOW OFTEN ?

- DEPENDS on FREQUENCY of SINUSOID
- ANSWERED by SHANNON/NYQUIST Theorem
- ALSO DEPENDS on **RECONSTRUCTION**

### Shannon Sampling Theorem

A continuous-time signal  $x(t)$  with frequencies no higher than  $f_{\max}$  can be reconstructed exactly from its samples  $x[n] = x(nT_s)$ , if the samples are taken at a rate  $f_s = 1/T_s$  that is greater than  $2f_{\max}$ .

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# STORING DIGITAL SOUND

- $x[n]$  is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- EXAMPLE: audio CD
- CD rate is 44,100 samples per second
  - 16-bit samples
  - Stereo uses 2 channels
- Number of bytes for 1 minute is
  - $2 \times (16/8) \times 60 \times 44100 = 10.584$  Mbytes

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# DISCRETE-TIME SINUSOID

- Change  $x(t)$  into  $x[n]$  **DERIVATION**

$$x(t) = A \cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A \cos(\omega nT_s + \varphi)$$

$$x[n] = A \cos((\omega T_s)n + \varphi)$$

$$x[n] = A \cos(\hat{\omega}n + \varphi)$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s} \quad \text{DEFINE DIGITAL FREQUENCY}$$

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# DIGITAL FREQUENCY $\hat{\omega}$

- $\hat{\omega}$  VARIES from **0** to  **$2\pi$** , as  $f$  varies from 0 to the sampling frequency
- UNITS are radians, **not** rad/sec
  - DIGITAL FREQUENCY is NORMALIZED

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

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# SPECTRUM (DIGITAL)

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$$f_s = 1 \text{ kHz}$$

$$\frac{1}{2} X^*$$

$$-0.2\pi$$

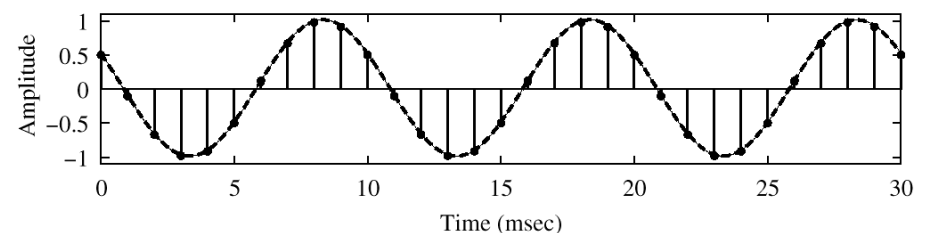
$$\frac{1}{2} X$$

$$2\pi(0.1)$$

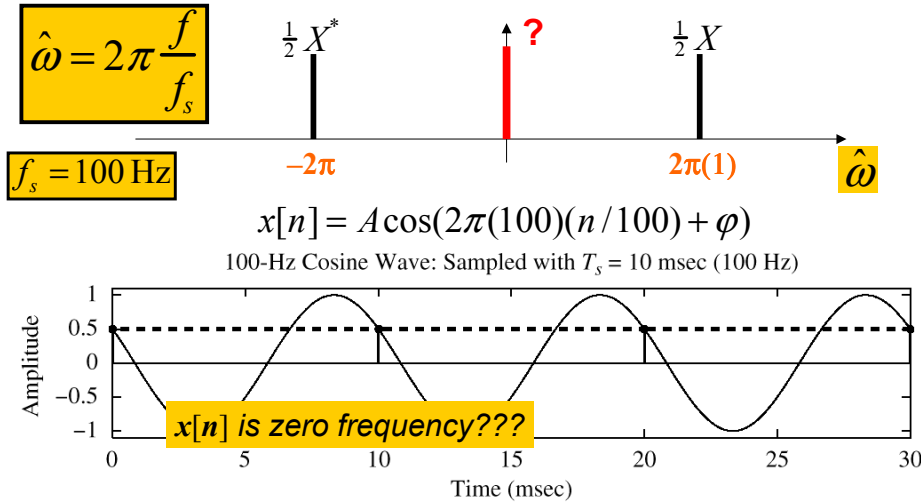
$$\hat{\omega}$$

$$x[n] = A \cos(2\pi(100)(n/1000) + \varphi)$$

100-Hz Cosine Wave: Sampled with  $T_s = 1$  msec (1000 Hz)



# SPECTRUM (DIGITAL) ???



# The REST of the STORY

- Spectrum of  $x[n]$  has more than one line for each complex exponential
  - Called **ALIASING**
  - **MANY SPECTRAL LINES**
- SPECTRUM is PERIODIC with period =  $2\pi$ 
  - Because

$$A \cos(\hat{\omega}n + \varphi) = A \cos((\hat{\omega} + 2\pi)n + \varphi)$$

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# ALIASING DERIVATION

- Other Frequencies give the same  $\hat{\omega}$ 
    - $x_1(t) = \cos(400\pi t)$  sampled at  $f_s = 1000 \text{ Hz}$
    - $x_1[n] = \cos(400\pi \frac{n}{1000}) = \cos(0.4\pi n)$
    - $x_2(t) = \cos(2400\pi t)$  sampled at  $f_s = 1000 \text{ Hz}$
    - $x_2[n] = \cos(2400\pi \frac{n}{1000}) = \cos(2.4\pi n)$
    - $x_2[n] = \cos(2.4\pi n) = \cos(0.4\pi n + 2\pi n) = \cos(0.4\pi n)$
    - $\Rightarrow x_2[n] = x_1[n]$
- $2400\pi - 400\pi = 2\pi(1000)$

# ALIASING DERIVATION-2

- Other Frequencies give the same  $\hat{\omega}$
- If  $x(t) = A \cos(2\pi(f + lf_s)t + \varphi)$   $t \leftarrow \frac{n}{f_s}$
- and we want:  $x[n] = A \cos(\hat{\omega}n + \varphi)$
- then:  $\hat{\omega} = \frac{2\pi(f + lf_s)}{f_s} = \frac{2\pi f}{f_s} + \frac{2\pi lf_s}{f_s}$
- $$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi l$$

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## ALIASING CONCLUSIONS

- ADDING  $f_s$  or  $2f_s$  or  $-f_s$  to the FREQ of  $x(t)$  gives exactly the same  $x[n]$ 
  - The samples,  $x[n] = x(n/f_s)$  are EXACTLY THE SAME VALUES
- GIVEN  $x[n]$ , WE CAN'T DISTINGUISH  $f_0$  FROM  $(f_0 + f_s)$  or  $(f_0 + 2f_s)$

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## NORMALIZED FREQUENCY

- DIGITAL FREQUENCY

*Normalized Radian Frequency*

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi l$$

*Normalized Cyclic Frequency*

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

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## SPECTRUM for $x[n]$

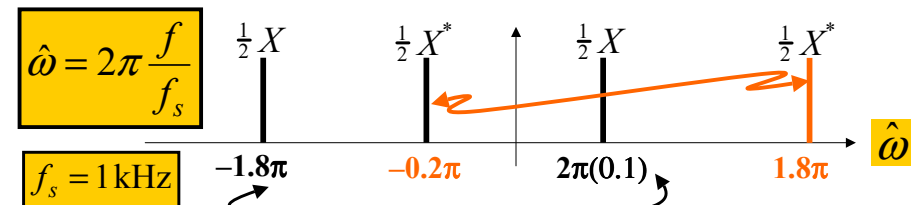
- PLOT versus NORMALIZED FREQUENCY
- INCLUDE **ALL** SPECTRUM LINES
  - ALIASES
    - ADD MULTIPLES of  $2\pi$
    - SUBTRACT MULTIPLES of  $2\pi$
  - FOLDED ALIASES
    - (to be discussed later)
    - ALIASES of NEGATIVE FREQS

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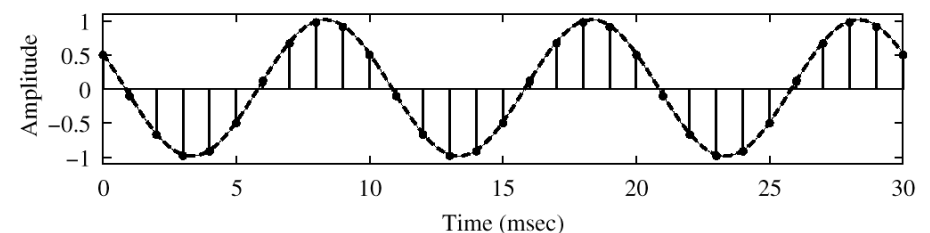
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## SPECTRUM (MORE LINES)

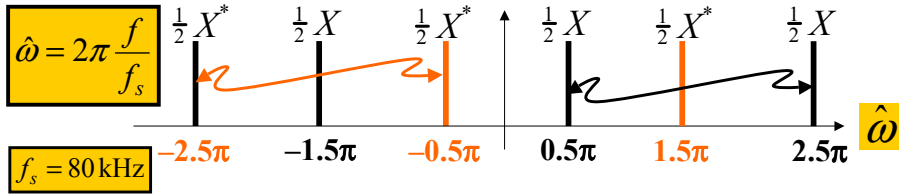


$$x[n] = A \cos(2\pi(100)(n/1000) + \varphi)$$

100-Hz Cosine Wave: Sampled with  $T_s = 1$  msec (1000 Hz)

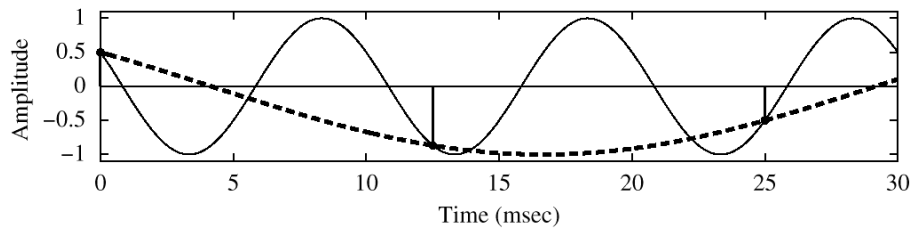


# SPECTRUM (ALIASING CASE)

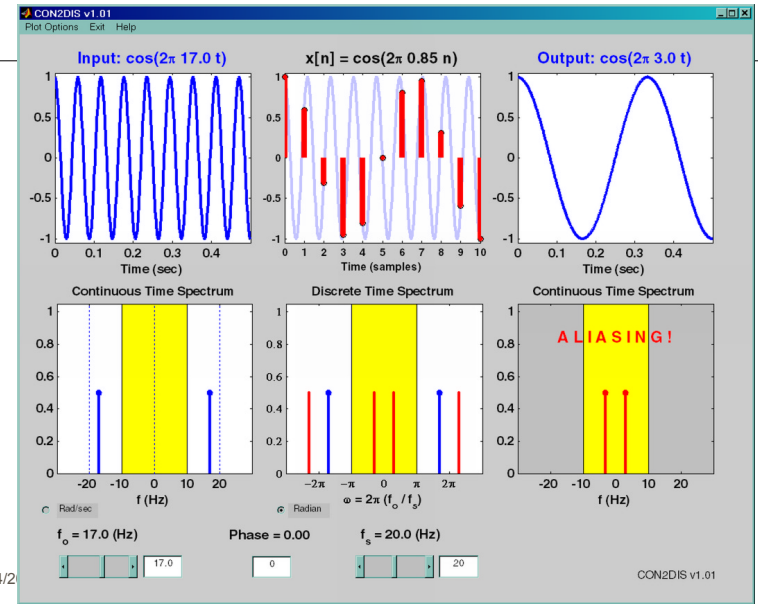


$$x[n] = A \cos(2\pi(100)(n/80) + \varphi)$$

100-Hz Cosine Wave: Sampled with  $T_s = 12.5 \text{ msec}$  (80 Hz)



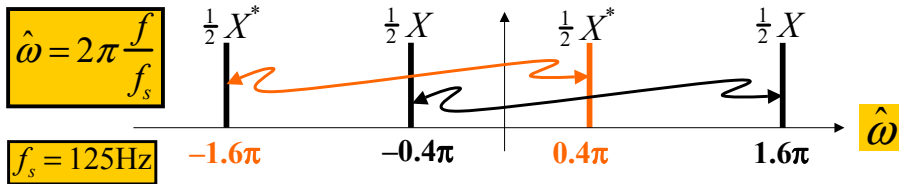
# SAMPLING GUI (con2dis)



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# SPECTRUM (FOLDING CASE)



$$x[n] = A \cos(2\pi(100)(n/125) + \varphi)$$

100-Hz Cosine Wave: Sampled with  $T_s = 8 \text{ msec}$  (125 Hz)

