ECE-2025 Fall-2003

Lecture 12
Frequency Response of FIR
3-Oct-03

Info: Web-CT, Lab, HW

- Lab #6: FIR Filtering of Images
  - Edge Detection
  - Due next week (7-9 Oct)
    except Monday Labs (after Fall break)
- Prob Set #6 due next week
- Quiz #2 on 24-Oct (Friday)
  - Coverage: HW #5, #6, #7 and #8

Lecture

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, Sections 6-1, 6-2, 6-3, 6-4, & 6-5
- Other Reading:
  - Recitation: Chapter 6
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chap. 6, Sects. 6-6, 6-7 & 6-8

LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG

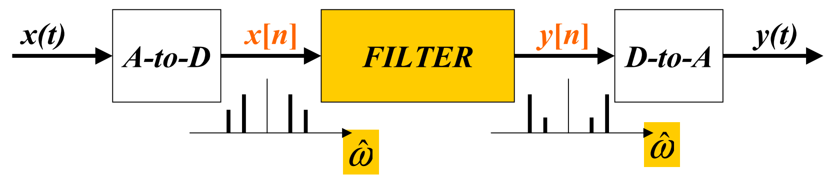
PHASE

$$H(e^{j\hat{\omega}}) = \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

DOMAINS: Time & Frequency

- **Time-Domain: "n" = time**
  - x[n] discrete-time signal
  - x(t) continuous-time signal
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs. f (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs. omega-hat
- Move back and forth QUICKLY

DIGITAL "FILTERING"



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

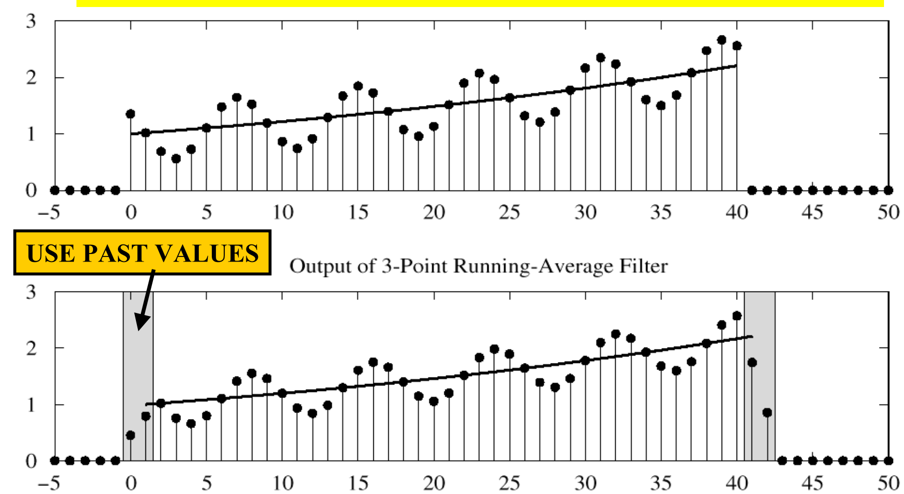
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

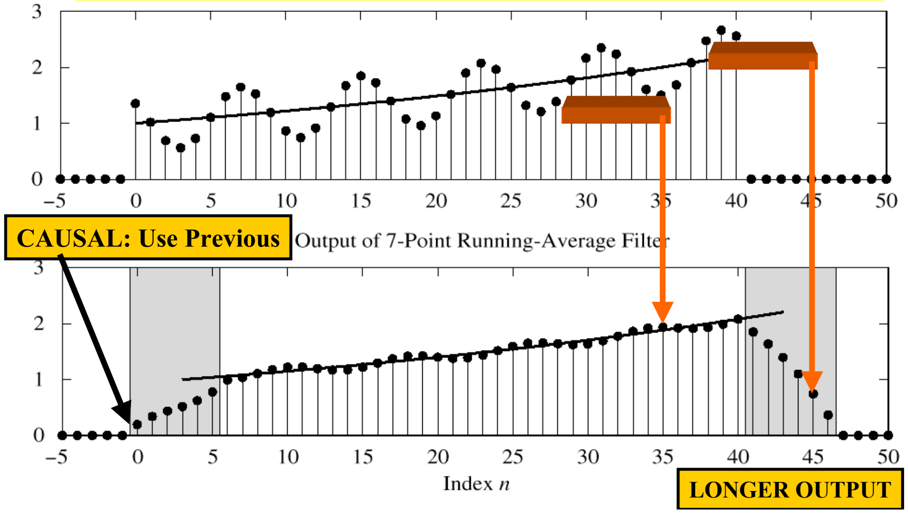
3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$



7-pt FIR EXAMPLE (AVG)

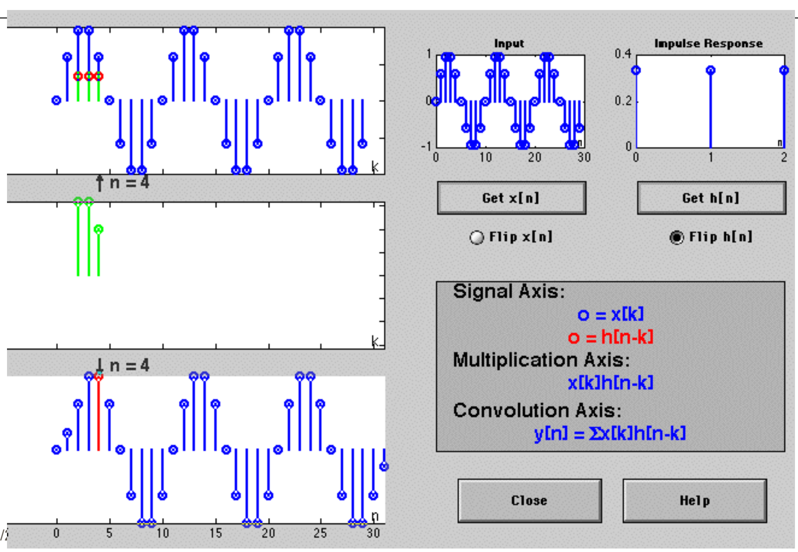
$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



SINUSOIDAL RESPONSE

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - **SAME** Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the **FREQUENCY RESPONSE**

DCONVDEMO: MATLAB GUI



COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]=\sum_{k=0}^{M} b_k A e^{j\varphi} e^{j\hat{\omega}(n-k)}$$

$$=\left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)}\right) A e^{j\varphi} e^{j\hat{\omega}n}$$

$$=H(\hat{\omega}) A e^{j\varphi} e^{j\hat{\omega}n}$$

FREQUENCY REPONSE

- At each frequency, we can DEFINE

$$H(e^{j\hat{\omega}})=\sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

← **FREQUENCY RESPONSE**

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$

EXAMPLE 6.1

$\{b_k\}=\{1,\ 2,\ 1\}$

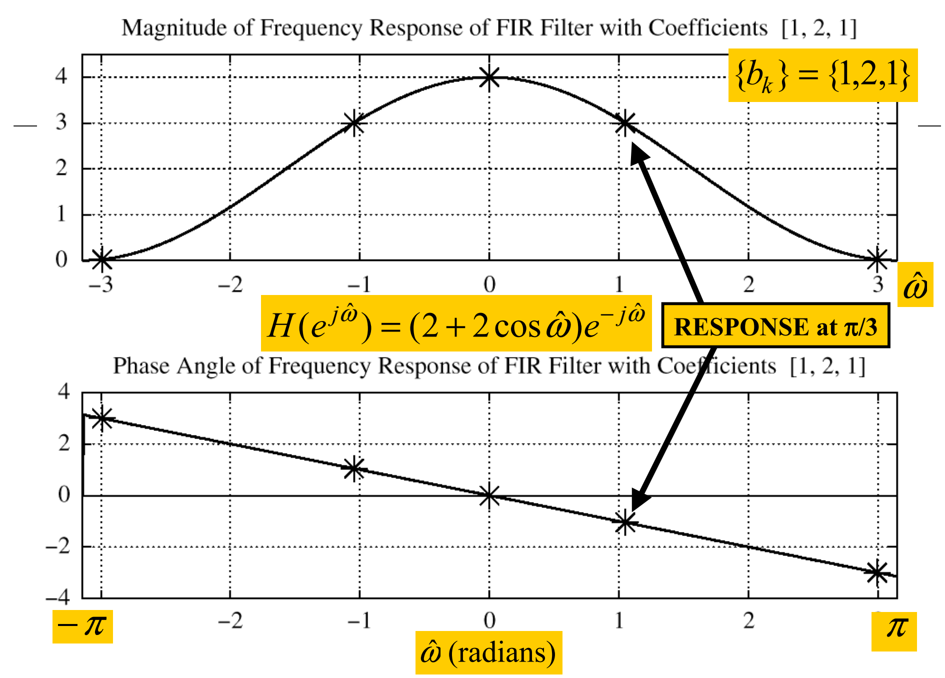
$$H(e^{j\hat{\omega}})=1+2e^{-j\hat{\omega}}+e^{-j2\hat{\omega}}$$
$$=e^{-j\hat{\omega}}(e^{j\hat{\omega}}+2+e^{-j\hat{\omega}})$$
$$=e^{-j\hat{\omega}}(2+2\cos\hat{\omega})$$

EXPLOIT SYMMETRY

Since $(2+2\cos\hat{\omega})\geq 0$

Magnitude is $|H(e^{j\hat{\omega}})|=(2+2\cos\hat{\omega})$

and Phase is $\angle H(e^{j\hat{\omega}})=-\hat{\omega}$



EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

EX: COSINE INPUT

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use Linearity

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

MATLAB: FREQUENCY RESPONSE

- **`HH = freqz(bb,1,ww)`**
  - VECTOR **`bb`** contains Filter Coefficients
  - DSP-First: `HH = freekz(bb,1,ww)`
- FILTER COEFFICIENTS $\{b_k\}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

LTI SYSTEMS

- LTI: Linear & Time-Invariant

- COMPLETELY CHARACTERIZED by:
  - FREQUENCY RESPONSE, or
  - IMPULSE RESPONSE h[n]

- Sinusoid IN -----> Sinusoid OUT
  - At the SAME Frequency

Time & Frequency Relation

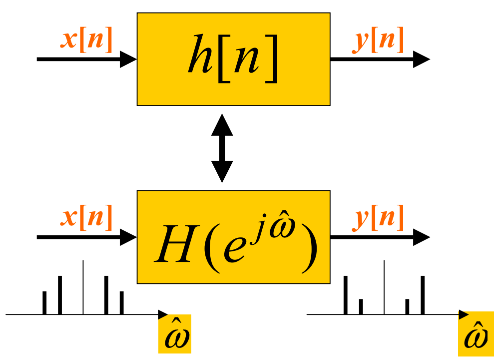
- Get Frequency Response from h[n]
  - Here is the FIR case:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

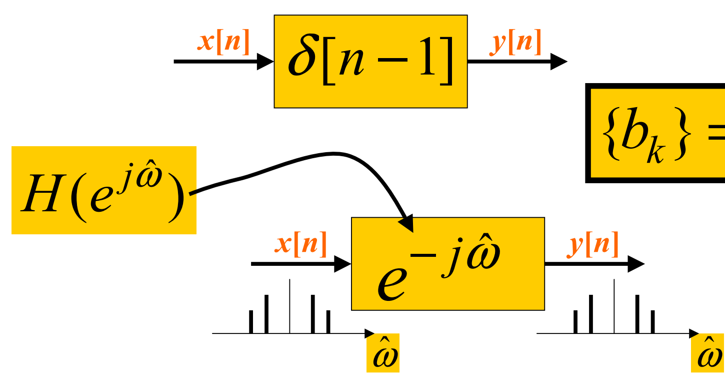
BLOCK DIAGRAMS

- Equivalent Representations



UNIT-DELAY SYSTEM

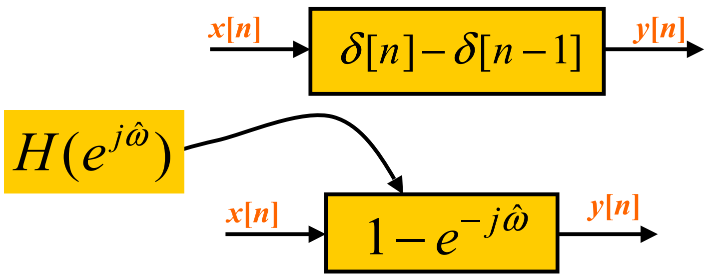
Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-1]$



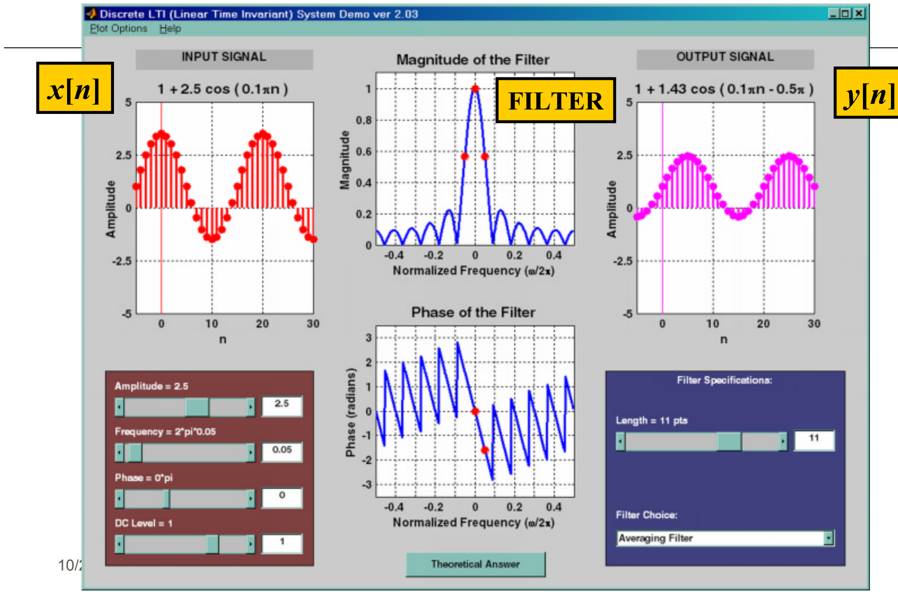
$$\{b_k\} = \{\,0,\ 1\,\}$$

FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for the Difference Equation: $y[n] = x[n] - x[n-1]$



DLTI Demo with Sinusoids



CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?

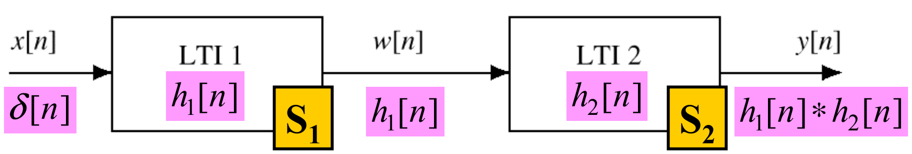


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- MULTIPLY the Frequency Responses