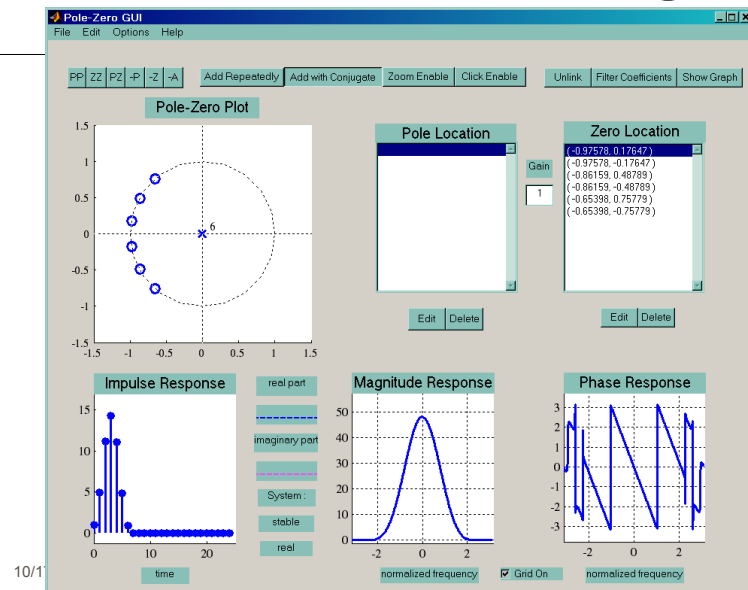


Lecture 16
IIR Filters: Feedback & H(z)
20-Oct-03

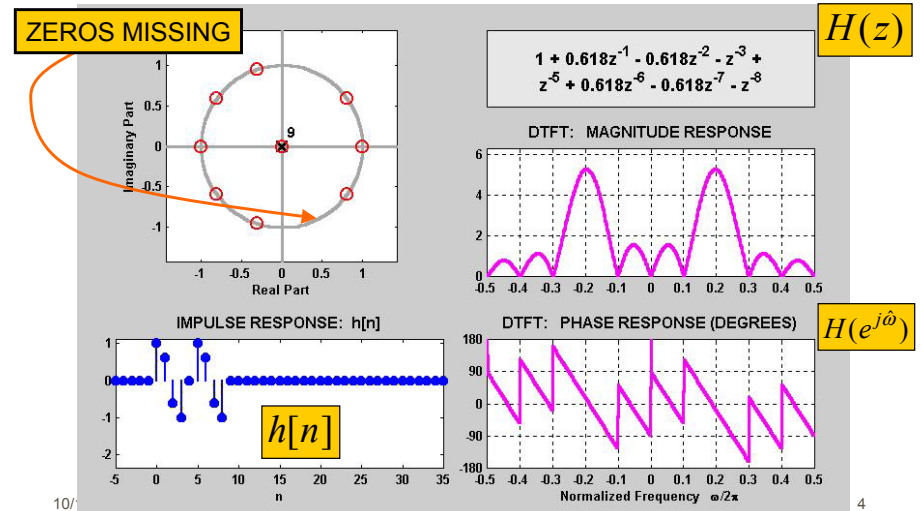
Info: Web-CT, Lab, HW

- Quiz #2 on 24-Oct (Friday)
 - Coverage: Homeworks #5, #6, and #7
 - One page of Hand-written notes (8.5 by 11 in.)
 - Calculator OK
- **Quiz Review: Wednesday, 22-Oct, 6:30 pm in Physics, Lecture Hall #2**
- Lab #8 on Bandpass Filter Design is posted
- HW #8 (posted) not due until week of 27-Oct

PeZ Demo: Zero Placing



3 DOMAINS MOVIE: FIR



READING ASSIGNMENTS

- This Lecture:
 - Chapter 8, Sects. 8-1, 8-2 & 8-3
- Other Reading:
 - Recitation: Ch. 8, Sects 8-1 thru 8-4
 - POLES & ZEROS
 - Next Lecture: Chapter 8, Sects. 8-4 8-5 & 8-6

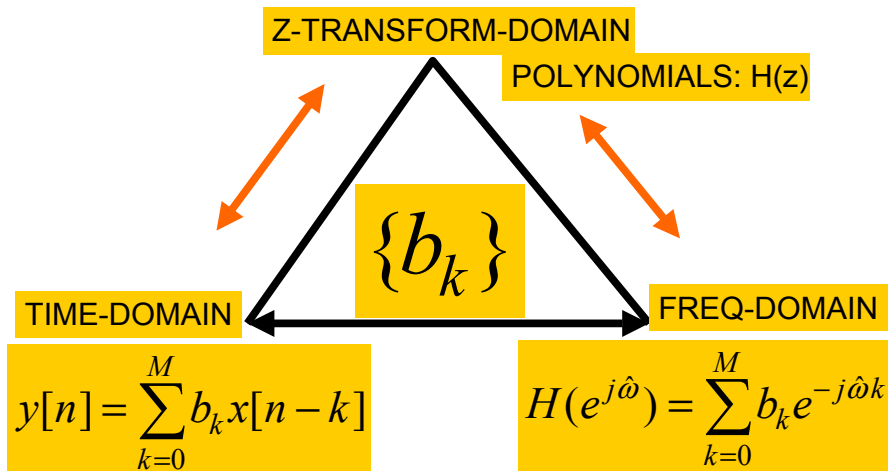
LECTURE OBJECTIVES

- INFINITE IMPULSE RESPONSE FILTERS
 - Define **IIR** DIGITAL Filters
 - Have **FEEDBACK**: use PREVIOUS OUTPUTS

$$y[n] = \sum_{\ell=1}^N a_{\ell} y[n-\ell] + \sum_{k=0}^M b_k x[n-k]$$

- Show how to compute the output $y[n]$
 - FIRST-ORDER CASE ($N=1$)
 - Z-transform: Impulse Response $h[n] \leftrightarrow H(z)$

THREE DOMAINS



Quick Review: Delay by n_d

$$y[n] = x[n - n_d]$$

IMPULSE RESPONSE $h[n] = \delta[n - n_d]$

SYSTEM FUNCTION $H(z) = z^{-n_d}$

FREQUENCY RESPONSE $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}n_d}$

LOGICAL THREAD

- FIND the IMPULSE RESPONSE, $h[n]$

- INFINITELY LONG
- IIR Filters

$$H(z) = \sum_{n=0}^{\infty} h[n]z^{-n}$$

- EXPLOIT THREE DOMAINS:

- Show Relationship for IIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

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ONE FEEDBACK TERM

- ADD PREVIOUS OUTPUTS

$$y[n] = a_1y[n-1] + b_0x[n] + b_1x[n-1]$$

PREVIOUS
FEEDBACK

FIR PART of the FILTER
FEED-FORWARD

- CAUSALITY

- NOT USING FUTURE OUTPUTS or INPUTS

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FILTER COEFFICIENTS

- ADD PREVIOUS OUTPUTS

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

FEEDBACK COEFFICIENT

SIGN CHANGE

- MATLAB

- `yy = filter([3,-2],[1,-0.8],xx)`

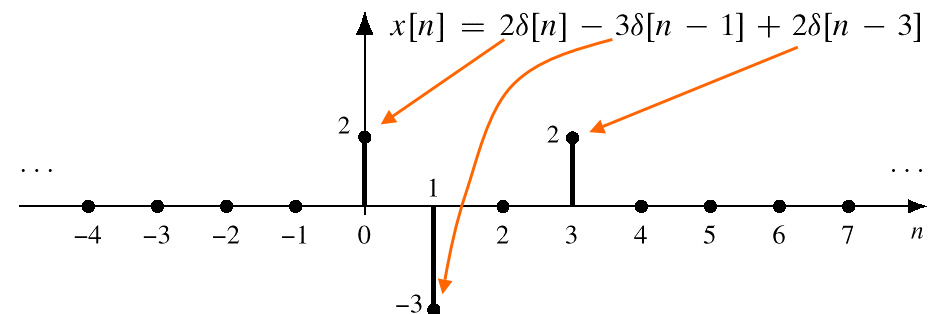
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COMPUTE OUTPUT

$$y[n] = 0.8y[n-1] + 5x[n]$$



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COMPUTE $y[n]$

- FEEDBACK DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 5x[n]$$

- NEED $y[-1]$ to get started

$$y[0] = 0.8y[-1] + 5x[0]$$

AT REST CONDITION

- $y[n] = 0$, for $n < 0$
- BECAUSE $x[n] = 0$, for $n < 0$

INITIAL REST CONDITIONS

- The input must be assumed to be zero prior to some starting time n_0 , i.e., $x[n] = 0$ for $n < n_0$. We say that such inputs are *suddenly applied*.
- The output is likewise assumed to be zero prior to the starting time of the signal, i.e., $y[n] = 0$ for $n < n_0$. We say that the system is *initially at rest* if its output is zero prior to the application of a suddenly applied input.

COMPUTE $y[0]$

- THIS STARTS THE RECURSION:

With the initial rest assumption, $y[n] = 0$ for $n < 0$,
 $y[0] = 0.8y[-1] + 5(2) = 0.8(0) + 5(2) = 10$

- SAME with MORE FEEDBACK TERMS

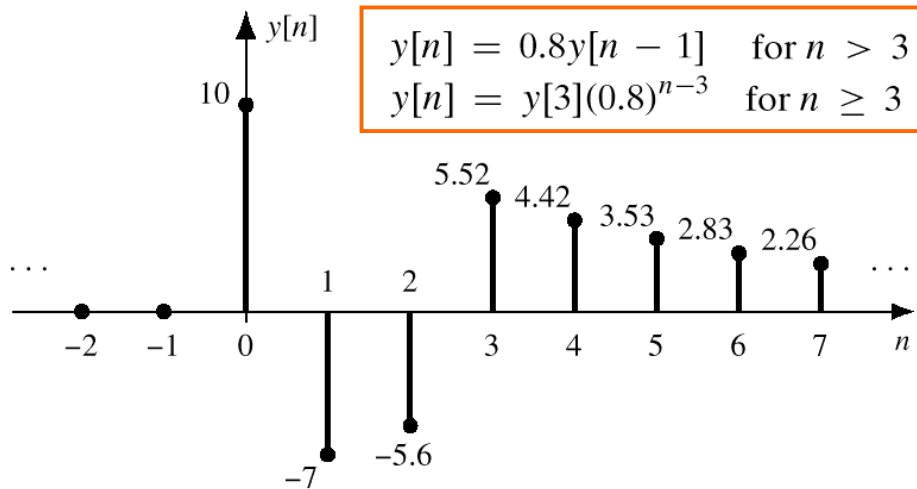
$$y[n] = a_1y[n-1] + a_2y[n-2] + \sum_{k=0}^2 b_k x[n-k]$$

COMPUTE MORE $y[n]$

- CONTINUE THE RECURSION:

$$\begin{aligned} y[1] &= 0.8y[0] + 5x[1] = 0.8(10) + 5(-3) = -7 \\ y[2] &= 0.8y[1] + 5x[2] = 0.8(-7) + 5(0) = -5.6 \\ y[3] &= 0.8y[2] + 5x[3] = 0.8(-5.6) + 5(2) = 5.52 \\ y[4] &= 0.8y[3] + 5x[4] = 0.8(5.52) + 5(0) = 4.416 \\ y[5] &= 0.8y[4] + 5x[5] = 0.8(4.416) + 5(0) = 3.5328 \\ y[6] &= 0.8y[5] + 5x[6] = 0.8(3.5328) + 5(0) = 2.8262 \end{aligned}$$

PLOT $y[n]$



IMPULSE RESPONSE

$$h[n] = a_1 h[n - 1] + b_0 \delta[n]$$

n	$n < 0$	0	1	2	3	4
$\delta[n]$	0	1	0	0	0	0
$h[n - 1]$	0	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$
$h[n]$	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$	$b_0(a_1)^4$

From this table it is obvious that the general formula is

$$h[n] = \begin{cases} b_0(a_1)^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$h[n] = b_0(a_1)^n u[n]$$

$$u[n] = 1, \text{ for } n \geq 0$$

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IMPULSE RESPONSE

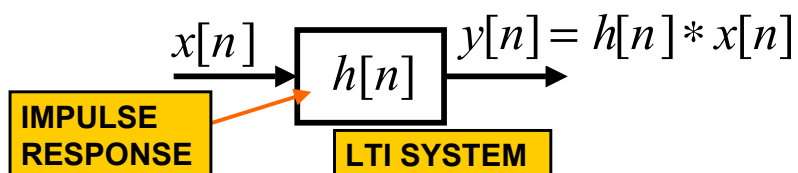
- DIFFERENCE EQUATION:

$$y[n] = 0.8y[n - 1] + 3x[n]$$

- Find $h[n]$

$$h[n] = 3(0.8)^n u[n]$$

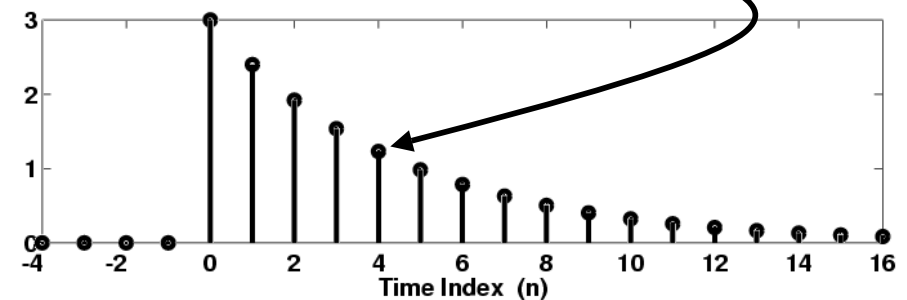
- CONVOLUTION** in TIME-DOMAIN



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PLOT IMPULSE RESPONSE

$$h[n] = b_0(a_1)^n u[n] = 3(0.8)^n u[n]$$



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Infinite-Length Signal: $h[n]$

- POLYNOMIAL Representation

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

APPLIES to
Any SIGNAL

- SIMPLIFY the SUMMATION

$$H(z) = \sum_{n=-\infty}^{\infty} b_0(a_1)^n u[n]z^{-n} = b_0 \sum_{n=0}^{\infty} a_1^n z^{-n}$$

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Derivation of $H(z)$

- Recall Sum of Geometric Sequence:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

- Yields a COMPACT FORM

$$\begin{aligned} H(z) &= b_0 \sum_{n=0}^{\infty} a_1^n z^{-n} = b_0 \sum_{n=0}^{\infty} (a_1 z^{-1})^n \\ &= \frac{b_0}{1 - a_1 z^{-1}} \quad \text{if } |z| > |a_1| \end{aligned}$$

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$H(z) = z$ -Transform{ $h[n]$ }

- FIRST-ORDER IIR FILTER:

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$h[n] = b_0 (a_1)^n u[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

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$H(z) = z$ -Transform{ $h[n]$ }

- ANOTHER FIRST-ORDER IIR FILTER:

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

$$h[n] = b_0 (a_1)^n u[n] + b_1 (a_1)^{n-1} u[n-1]$$

z^{-1} is a shift

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}} + \frac{b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

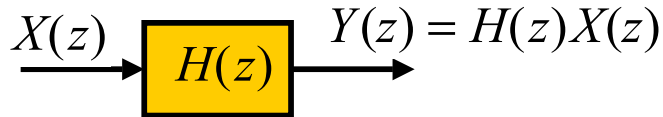
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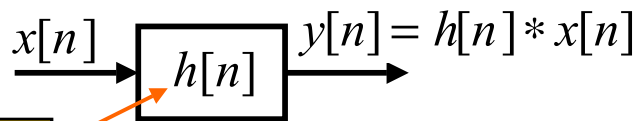
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CONVOLUTION PROPERTY

- MULTIPLICATION of z-TRANSFORMS



- CONVOLUTION in TIME-DOMAIN



IMPULSE RESPONSE

STEP RESPONSE: $x[n]=u[n]$

$$y[n] = a_1 y[n - 1] + b_0 x[n]$$

n	$x[n]$	$y[n]$
$n < 0$	0	0
0	1	b_0
1	1	$b_0 + b_0(a_1)$
2	1	$b_0 + b_0(a_1) + b_0(a_1)^2$
3	1	$b_0(1 + a_1 + a_1^2 + a_1^3)$
4	1	$b_0(1 + a_1 + a_1^2 + a_1^3 + a_1^4)$
.	.	.

$u[n] = 1, \text{ for } n \geq 0$

DERIVE STEP RESPONSE

$$y[n] = b_0(1 + a_1 + a_1^2 + \dots + a_1^n) = b_0 \sum_{k=0}^n a_1^k$$

$$\sum_{k=0}^L r^k = \begin{cases} \frac{1 - r^{L+1}}{1 - r} & r \neq 1 \\ L + 1 & r = 1 \end{cases}$$

$$y[n] = b_0 \frac{1 - a_1^{n+1}}{1 - a_1} \quad \text{for } n \geq 0, \quad \text{if } a_1 \neq 1$$

PLOT STEP RESPONSE

