ECE-2025 Fall-2003

Lecture 18

Continuous-Time Signals and Systems

31-Oct-03

Info: Web-CT, Lab, HW

- Quiz #2: resolve grades by **7-Nov**
  - Graders were posted
- Quiz #3 will be 11-Nov (Friday)

- HW #9 due next week
- Lab #9 on DTMF decoding

2nd ORDER EXAMPLE

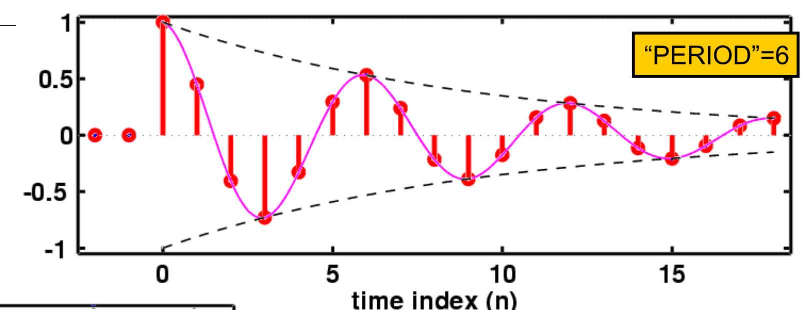
$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3} n) u[n] = (0.9)^n \tfrac{1}{2}(e^{j\pi n/3} + e^{-j\pi n/3}) u[n]$$

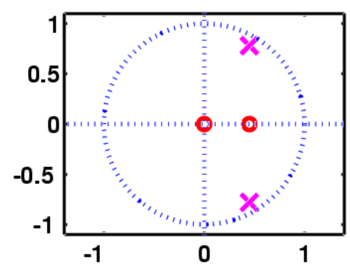
$$H(z) = \frac{0.5}{1 - 0.9 e^{j\pi/3} z^{-1}} + \frac{0.5}{1 - 0.9 e^{-j\pi/3} z^{-1}}$$

$$H(z) = \frac{1 - 0.9\cos(\frac{\pi}{3}) z^{-1}}{(1 - 0.9 e^{j\pi/3} z^{-1})(1 - 0.9 e^{-j\pi/3} z^{-1})}$$

$$H(z) = \frac{1 - 0.45 z^{-1}}{1 - 0.9 z^{-1} + 0.81 z^{-2}}$$

h[n]: Decays & Oscillates

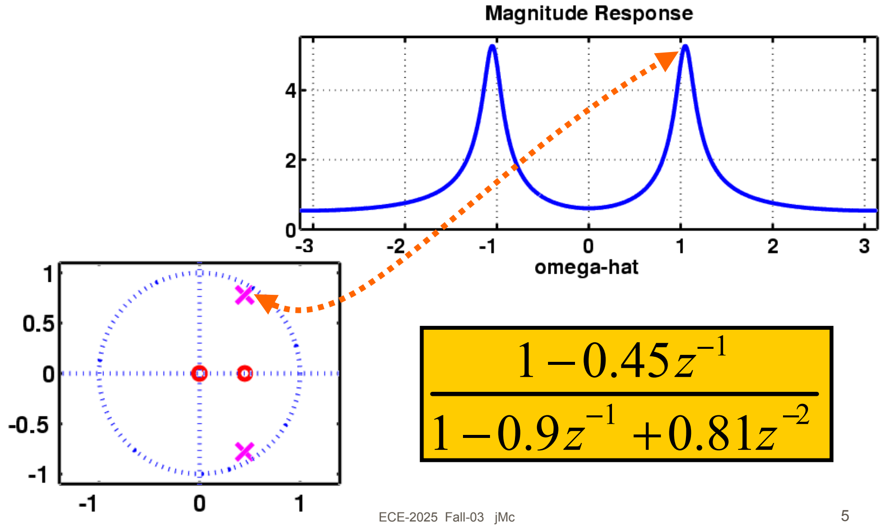




$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3} n) u[n]$$

$$\frac{1 - 0.45 z^{-1}}{1 - 0.9 z^{-1} + 0.81 z^{-2}}$$

Complex POLE-ZERO PLOT



$$\frac{1-0.45z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

2nd ORDER EX: n-Domain

$$\frac{1-0.45z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

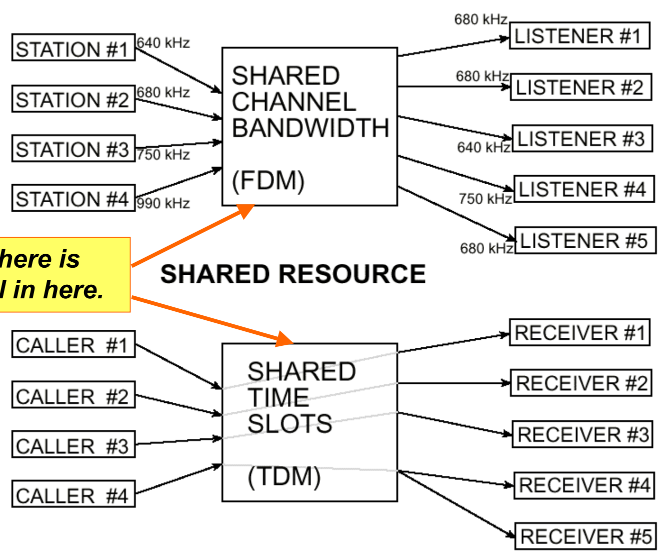
$$y[n]=0.9y[n-1]-0.81y[n-2]+x[n]-0.45x[n-1]$$

```
aa = [ 1, -0.9, 0.81 ];
bb = [ 1, -0.45 ];
nn = -2:19;
hh = filter( bb, aa, (nn==0) );
HH = freqz( bb, aa, [-pi,pi/100:pi] );
```


The way communication systems work

It's an analog world out here.

These days, there is A lot of digital in here.



Lecture

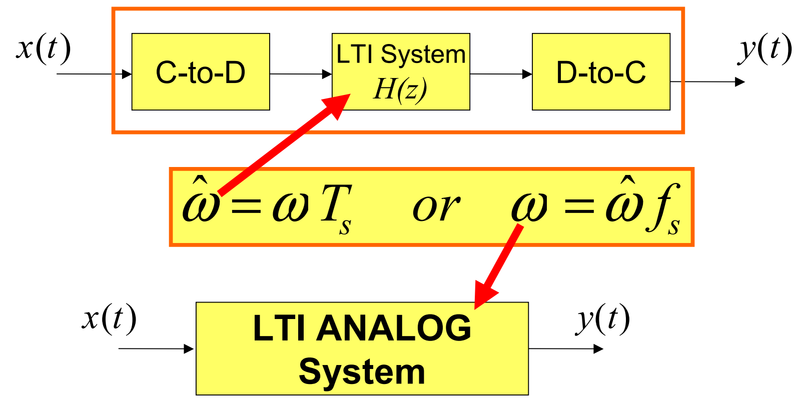
READING ASSIGNMENTS

- This Lecture:
  - Chapter 9, Sects 9-1 to 9-5
- Other Reading:
  - Recitation: Ch. 9, all
  - Next Lecture: Chapter 9, Sects 9-6 to 9-8

LECTURE OBJECTIVES

- Bye bye to D-T Systems for a while
- The UNIT IMPULSE signal
  - Definition
  - Properties
- Continuous-time signals and systems
  - Example systems
  - Review: Linearity and Time-Invariance
  - Convolution integral: impulse response

D-T Filtering of C-T Signals



$$\hat{\omega} = \omega T_s \quad or \quad \omega = \hat{\omega} f_s$$

ANALOG SIGNALS x(t)

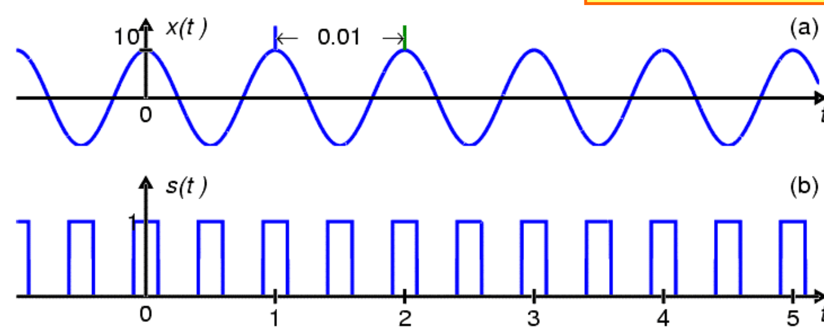
- INFINITE LENGTH
  - SINUSOIDS: (t = time in secs)
  - PERIODIC SIGNALS
  - ONE-SIDED, e.g., for t>0
    - UNIT STEP: u(t)
- FINITE LENGTH
  - SQUARE PULSE
- IMPULSE SIGNAL: δ(t)

- DISCRETE-TIME: x[n] is list of numbers

CT Signals: PERIODIC

$x(t) = 10\cos(200\pi t)$

Sinusoidal signal



INFINITE DURATION

Square Wave

CT Signals: ONE-SIDED

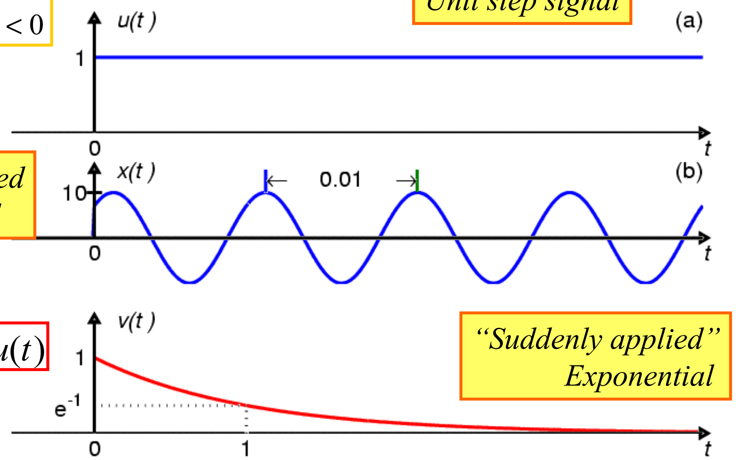
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Unit step signal

One-Sided Sinusoid

$$v(t) = e^{-t}u(t)$$

"Suddenly applied" Exponential

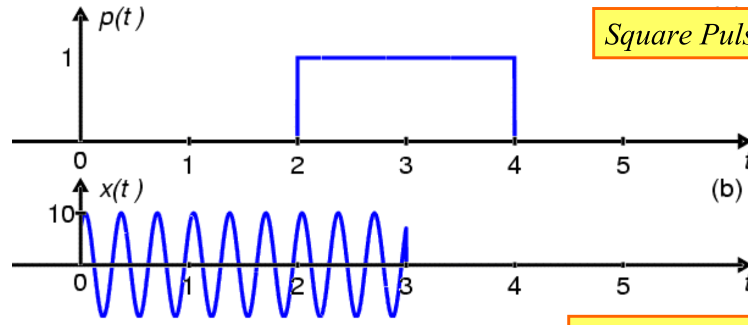


CT Signals: FINITE LENGTH

$$p(t) = u(t-2) - u(t-4)$$

Square Pulse signal

Sinusoid multiplied by a square pulse

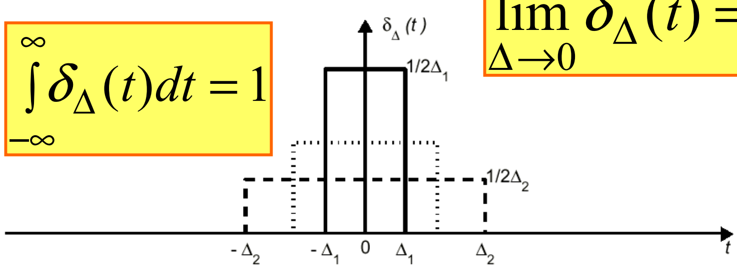


What is an Impulse?

- A signal that is concentrated at one point.

$$\int_{-\infty}^{\infty} \delta_\Delta(t)dt = 1$$

$$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$$



Defining the Impulse

- Assume the properties apply to the limit:

$$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$$

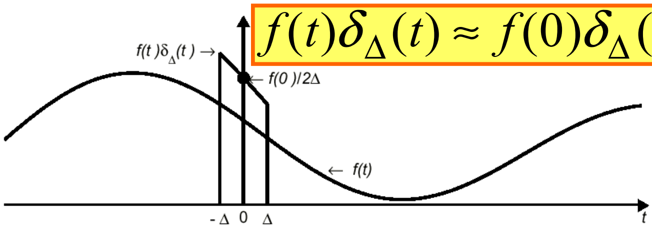
- One "**INTUITIVE**" definition is:

$$\delta(t) = 0, \quad t \neq 0$$ *Concentrated at t=0*

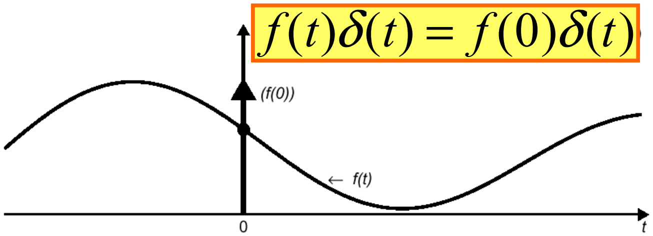
$$\int_{-\infty}^{\infty} \delta(\tau)d\tau = 1$$ *Unit area*

Sampling Property

$$f(t)\delta_\Delta(t) \approx f(0)\delta_\Delta(t)$$

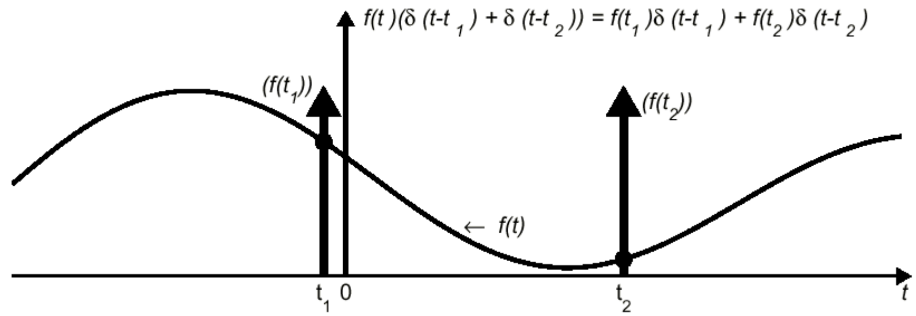


$$f(t)\delta(t) = f(0)\delta(t)$$



General Sampling Property

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$



Properties of the Impulse

$\delta(t-t_0) = 0, \quad t \neq t_0$ *Concentrated at one time*

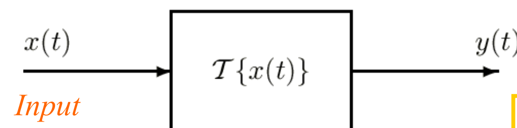
$\int_{-\infty}^{\infty} \delta(t-t_0)dt = 1$ *Unit area*

$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$ *Sampling Property*

$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$ *Extract one value of f(t)*

$\frac{du(t)}{dt} = \delta(t)$ *Derivative of unit step*

Continuous-Time Systems



Output

$$x(t) \mapsto y(t)$$

- Examples:
  - Delay $y(t) = x(t-t_d)$
  - Modulator $y(t) = [A + x(t)]\cos\omega_c t$
  - Integrator $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

CT BUILDING BLOCKS

- INTEGRATOR (CIRCUITS)
- DIFFERENTIATOR
- DELAY by t_o
- MODULATOR (e.g., AM Radio)
- MULTIPLIER & ADDER

Ideal Delay:

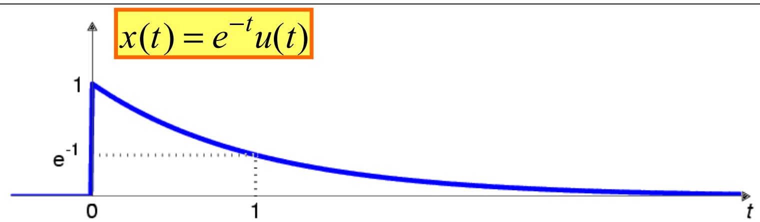
- Mathematical Definition:

$$y(t) = x(t - t_d)$$

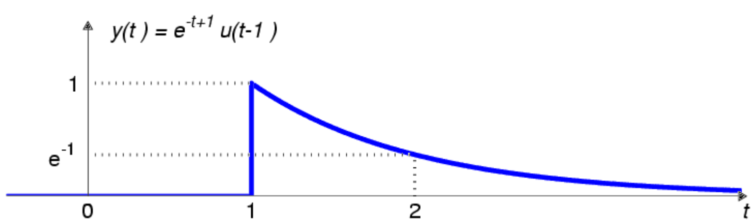
- To find the IMPULSE RESPONSE, *h(t)*, let *x(t)* be an impulse, so

$$h(t) = \delta(t - t_d)$$

Output of Ideal Delay of 1 sec



$$y(t) = x(t-1) = e^{-(t-1)}u(t-1)$$



Integrator:

- Mathematical Definition:

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

Running Integral

- To find the IMPULSE RESPONSE, *h(t)*, let *x(t)* be an impulse, so

$$h(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

Integrator:

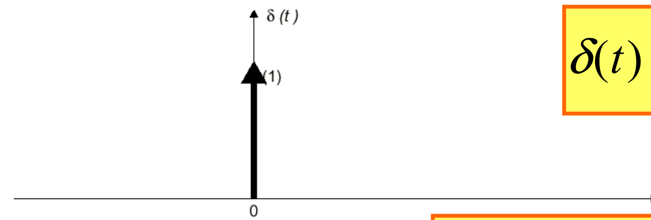
$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

- Integrate the impulse

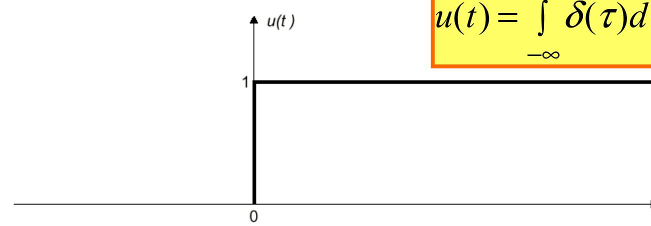
$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

- IF t<0, we get zero
- IF t>0, we get one
  - Thus we have h(t) = u(t) for the integrator

Graphical Representation



$$\delta(t) = \frac{du(t)}{dt}$$

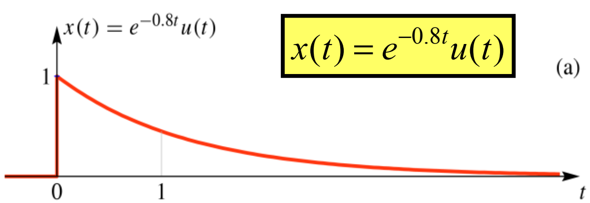


$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

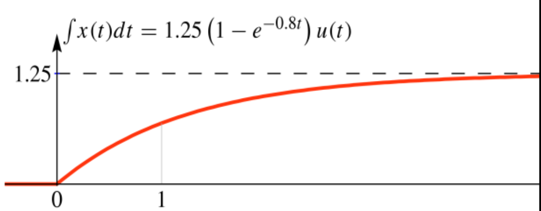
Output of Integrator

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$
$$= x(t) * u(t)$$

$$x(t) = e^{-0.8t}u(t)$$



(a)



$$y(t) = \int_{-\infty}^{t} e^{-0.8\tau}u(\tau)d\tau$$
$$= \begin{cases} 0 & t<0 \\ \int_{-\infty}^{t} e^{-0.8\tau}u(\tau)d\tau & t\geq 0 \end{cases}$$
$$= 1.25(1-e^{-0.8t})u(t)$$

Differentiator:

- Mathematical Definition:

$$y(t) = \frac{dx(t)}{dt}$$

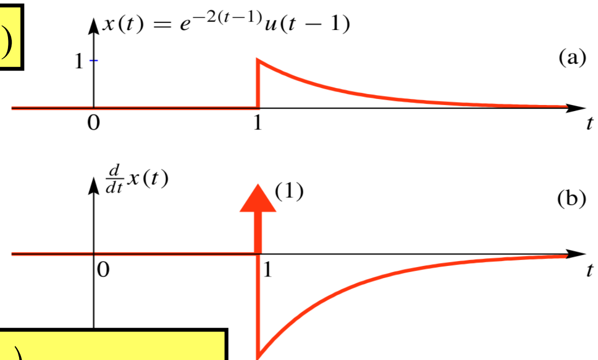
- To find *h(t),* let *x(t)* be an impulse, so

$$h(t) = \frac{d\delta(t)}{dt} = \delta^{(1)}(t)$$

Doublet

Differentiator Output: $y(t) = \frac{dx(t)}{dt}$

$x(t) = e^{-2(t-1)}u(t-1)$



$$y(t) = \frac{d}{dt}\left(e^{-2(t-1)}u(t-1)\right)$$
$$= -2e^{-2(t-1)}u(t-1) + e^{-2(t-1)}\delta(t-1)$$
$$= -2e^{-2(t-1)}u(t-1) + 1\delta(t-1)$$

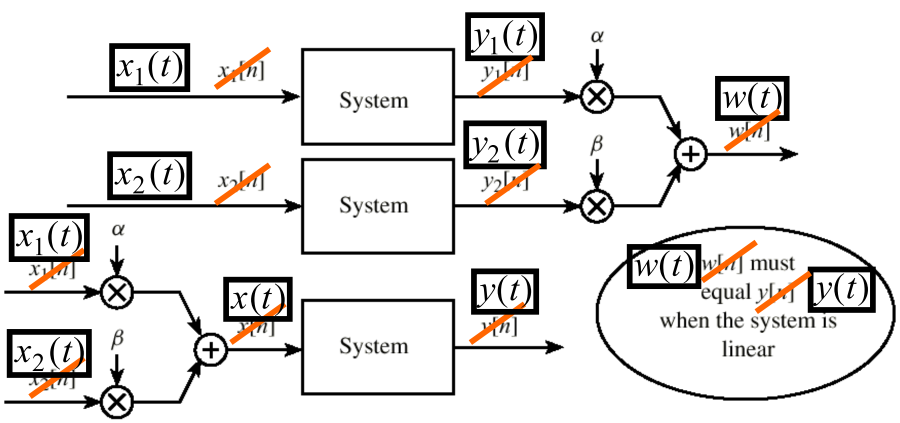
Linear and Time-Invariant (LTI) Systems

- If a continuous-time system is both linear and time-invariant, then the output *y(t)* is related to the input *x(t)* by a ***convolution integral***

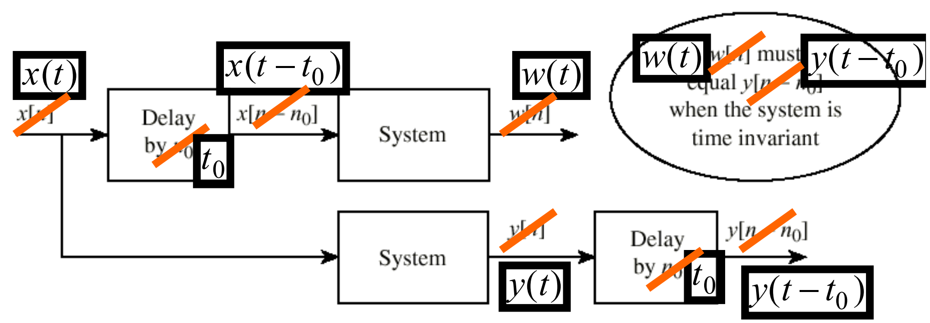
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

where *h(t)* is the ***impulse response*** of the system.

Testing for Linearity



Testing Time-Invariance



Integrator: $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

- Linear

$$\int_{-\infty}^{t} [a x_1(\tau) + b x_2(\tau)] d\tau = a y_1(t) + b y_2(t)$$

- And Time-Invariant

$$w(t) = \int_{-\infty}^{t} x(\tau - t_0) d\tau \qquad \text{let } \sigma = \tau - t_0$$

$$\Rightarrow \quad w(t) = \int_{-\infty}^{t - t_0} x(\sigma) d\sigma = y(t - t_0)$$

Modulator: $y(t) = [A + x(t)] \cos \omega_c t$

- ***Not*** linear--obvious because

$$[A + a x_1(t) + b x_2(t)] \neq$$

$$[A + a x_1(t)] + [A + b x_2(t)]$$

- ***Not*** time-invariant

$$w(t) = [A + x(t - t_0)] \cos \omega_c t \neq y(t - t_0)$$