

ECE-2025

Fall-2003

Lecture 20
Frequency Response of
Continuous-Time Systems
7-Nov-03

Tuesday ONLY Lab Change

- No Lab on 4-Nov for the **Tuesday** sections
 - All other sections stay on same schedule
- Lab #10 will start on 11-Nov
 - Due on 18-Nov (for **Tues** sections)
- Lab #11 will start on 18-Nov
 - There will be lab on 25-Nov (for **Tues** sections)
- Lab #9 should be turned in on 11-Nov

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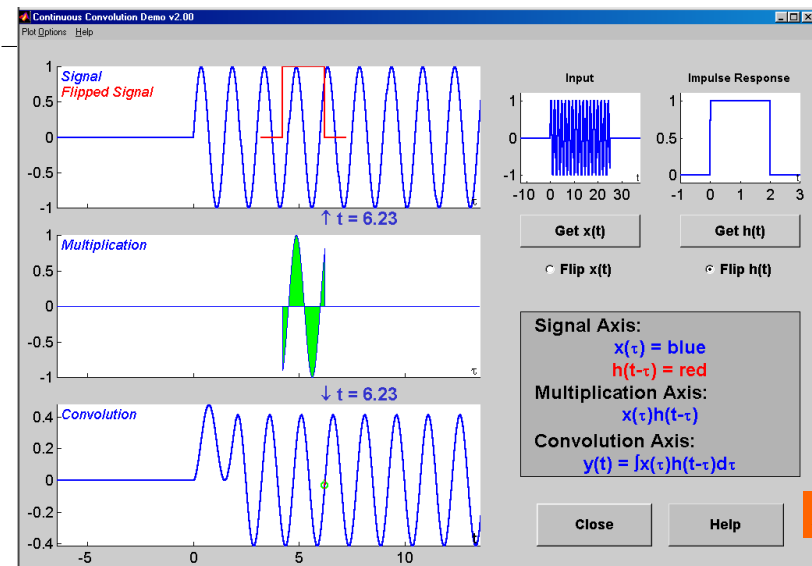
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Info: Web-CT, Lab, HW

- Quiz #2: resolve grades by **Friday (7-Nov)**
- **CHECK YOUR GRADES !!!**
 - Web-CT is the OFFICIAL gradebook
- **Quiz #3 will be 21-Nov (Friday)**
 - Coverage: HW #8, 9, 10, 11
 - Chapters 7, 8, 9, 10, and part of 11
 - Review Session, 20-Nov, Thurs @ 7:30pm

Convolution GUI: Sinusoid



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Lecture

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 10, all
- Other Reading:
 - Recitation: Ch. 10 all, start Ch 11
 - Next Lecture: Chapter 11

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LECTURE OBJECTIVES

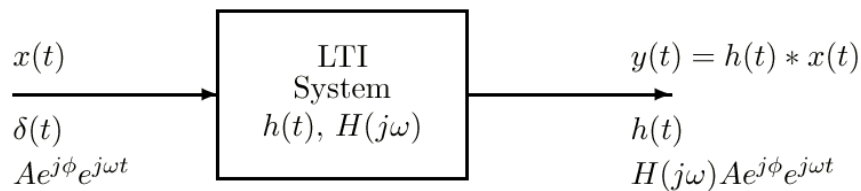
- Review of convolution
 - **THE** operation for **LTI** Systems
- Complex exponential input signals
 - **Frequency Response**
 - Cosine signals
 - Real part of complex exponential
- Fourier Series thru $H(j\omega)$
 - These are Analog Filters

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LTI Systems



- Convolution defines an LTI system

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

- Response to a complex exponential gives frequency response $H(j\omega)$

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Thought Process #1

- **SUPERPOSITION (Linearity)**
 - Make $x(t)$ a weighted sum of signals
 - Then $y(t)$ is also a sum—different weights
 - DIFFERENT OUTPUT SIGNALS usually
- Use **SINUSOIDS**
 - “SINUSOID IN GIVES SINUSOID OUT”
 - Make $x(t)$ a weighted sum of sinusoids
 - Then $y(t)$ is also a sum of sinusoids
 - Different Magnitudes and Phase
- **LTI SYSTEMS: Sinusoidal Response**

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Thought Process #2

- **SUPERPOSITION (Linearity)**
 - Make $x(t)$ a weighted sum of signals
- **Use SINUSOIDS**
 - **Any $x(t)$ = weighted sum of sinusoids**
 - **HOW? Use FOURIER ANALYSIS INTEGRAL**
 - **To find the weights from $x(t)$**
- **LTI SYSTEMS:**
 - Frequency Response changes each sinusoidal component

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Complex Exponential Input

$$x(t) = Ae^{j\varphi} e^{j\omega t} \mapsto y(t) = H(j\omega) Ae^{j\varphi} e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) Ae^{j\varphi} e^{j\omega(t-\tau)} d\tau$$

$$y(t) = \left(\int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \right) Ae^{j\varphi} e^{j\omega t}$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

Frequency Response

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When does $H(j\omega)$ Exist?

- When is $|H(j\omega)| < \infty$

$$|H(j\omega)| = \left| \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \right| \leq \int_{-\infty}^{\infty} |h(\tau)| |e^{-j\omega\tau}| d\tau$$

$$|H(j\omega)| \leq \int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

- Thus the frequency response exists if the LTI system is a **stable** system.

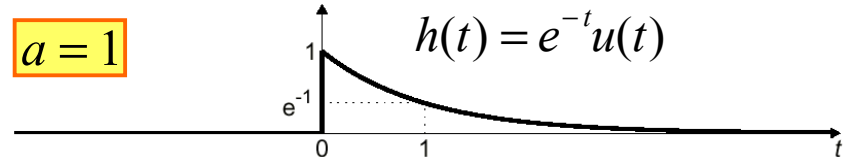
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$$h(t) = e^{-at} u(t) \Leftrightarrow H(j\omega) = \frac{1}{a + j\omega}$$

- Suppose that $h(t)$ is:



$$H(j\omega) = \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) e^{-j\omega\tau} d\tau = \int_0^{\infty} e^{-(a+j\omega)\tau} d\tau$$

$a > 0$

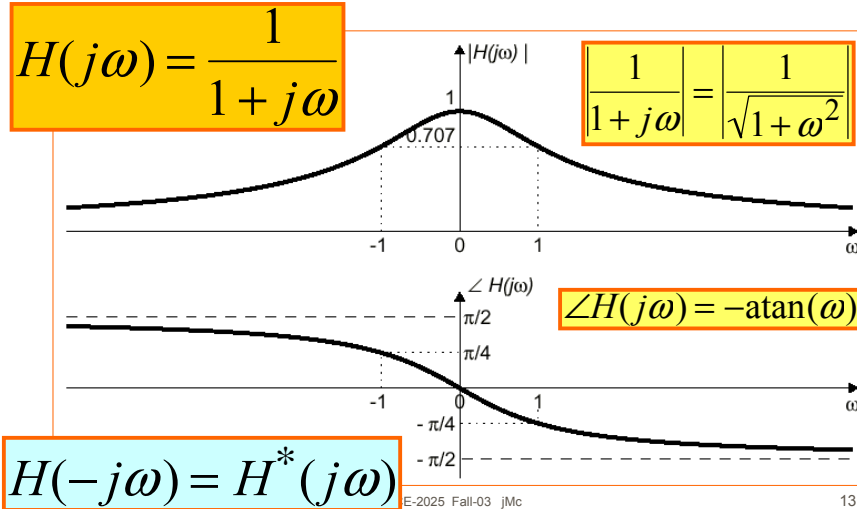
$$H(j\omega) = \frac{e^{-(a+j\omega)\tau}}{-(a+j\omega)} \Big|_0^{\infty} = \frac{e^{-a\tau} e^{-j\omega\tau}}{-(a+j\omega)} \Big|_0^{\infty} = \frac{1}{a + j\omega}$$

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Magnitude and Phase Plots



Freq Response of Integrator?

- Impulse Response
 - $h(t) = u(t)$
- NOT a Stable System
 - Frequency response $H(j\omega)$ does NOT exist

$$h(t) = e^{-at} u(t) \Leftrightarrow H(j\omega) = \frac{1}{a+j\omega} \rightarrow \frac{1}{j\omega}?$$

Need another term

“Leaky” Integrator (a is small)
Cannot build a perfect Integral

$$a \rightarrow 0$$

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Ideal Delay:

$$y(t) = x(t - t_d)$$

$$H(j\omega) = \int_{-\infty}^{\infty} \delta(\tau - t_d) e^{-j\omega\tau} d\tau = e^{-j\omega t_d}$$

$$H(j\omega) = e^{-j\omega t_d}$$

$$x(t) = e^{j\omega t} \mapsto$$

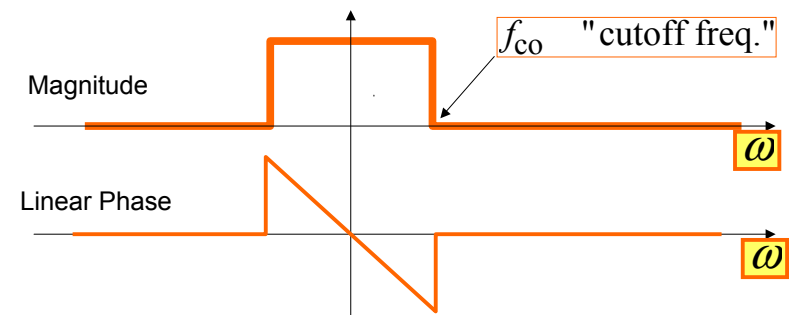
$$H(j\omega)$$

$$y(t) = e^{j\omega(t-t_d)} = e^{-j\omega t_d} e^{j\omega t}$$

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Ideal Lowpass Filter w/ Delay

$$H_{LP}(j\omega) = \begin{cases} e^{-j\omega t_d} & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

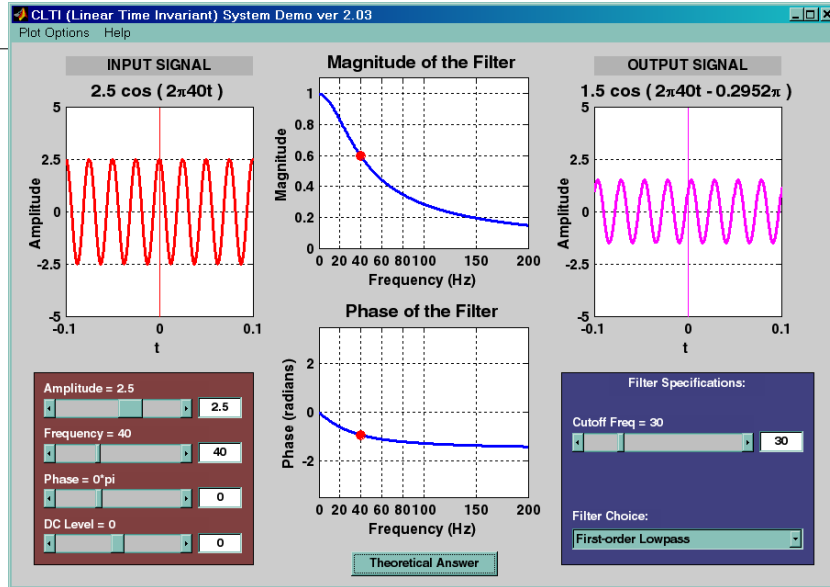


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Sinusoid in Gives Sinusoid out



Example: Ideal Low Pass

$$H_{LP}(j\omega) = \begin{cases} e^{-j3\omega} & |\omega| < 2 \\ 0 & |\omega| > 2 \end{cases}$$

$$x(t) = 10e^{j\pi/3} e^{j1.5t} \mapsto y(t) = H(j1.5)10e^{j\pi/3} e^{j1.5t}$$

$$y(t) = (e^{-j4.5})10e^{j\pi/3} e^{j1.5t} = 10e^{j\pi/3} e^{j1.5(t-3)}$$

Cosine Input

$$x(t) = A\cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

$$y(t) = H(j\omega_0) \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + H(-j\omega_0) \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

Since $H(-j\omega_0) = H^*(j\omega_0)$

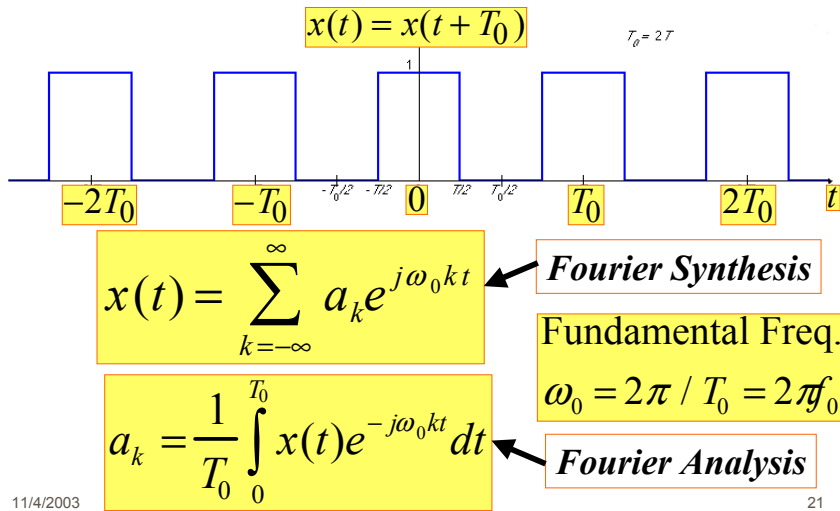
$$y(t) = A |H(j\omega_0)| \cos(\omega_0 t + \phi + \angle H(j\omega_0))$$

Review Fourier Series

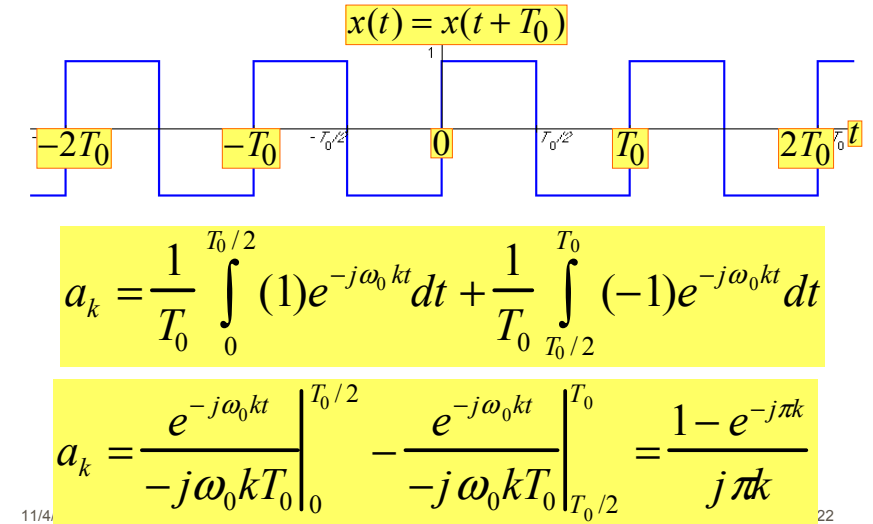
- ANALYSIS
 - Get representation from the signal
 - Works for PERIODIC Signals
- Fourier Series
 - INTEGRAL over one period

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\omega_0 k t} dt$$

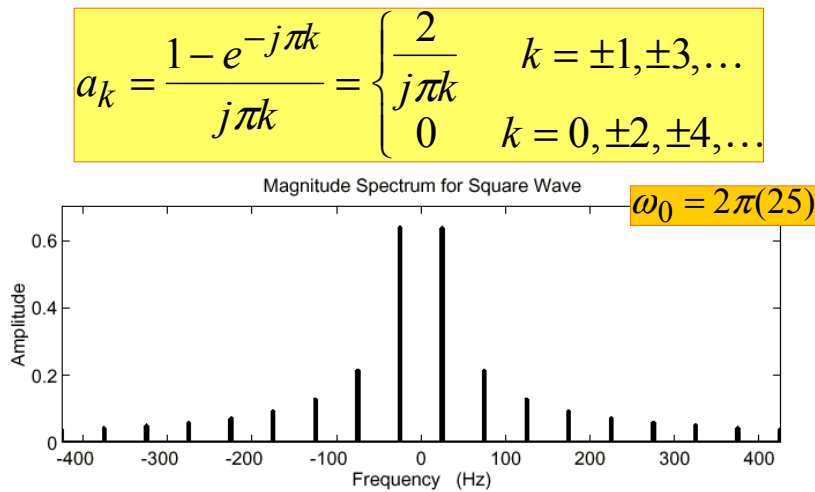
General Periodic Signals



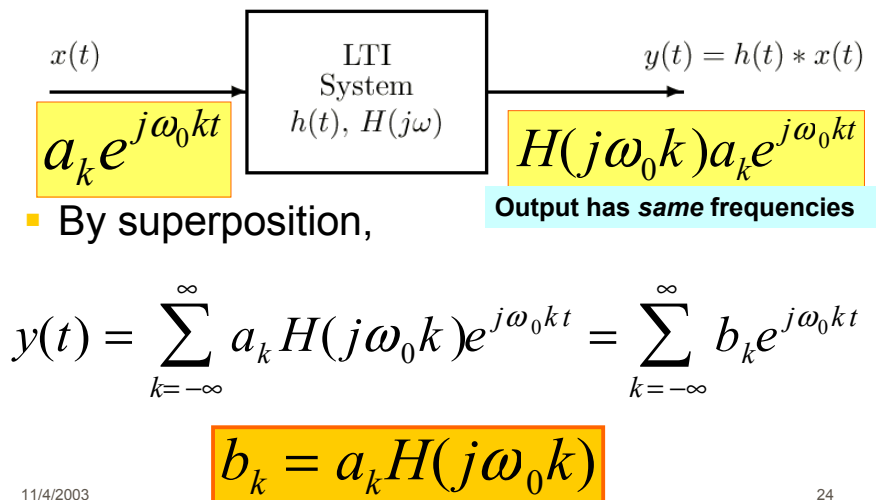
Square Wave Signal



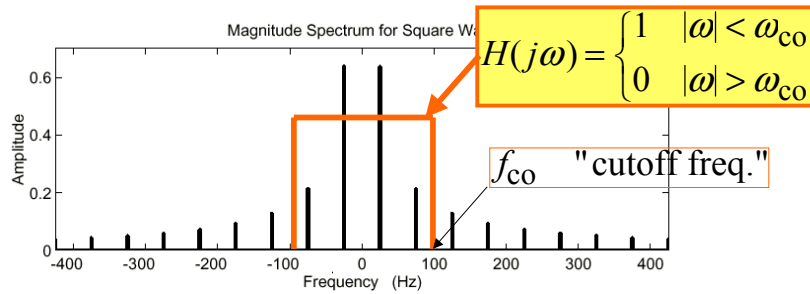
Spectrum from Fourier Series



LTI Systems with Periodic Inputs

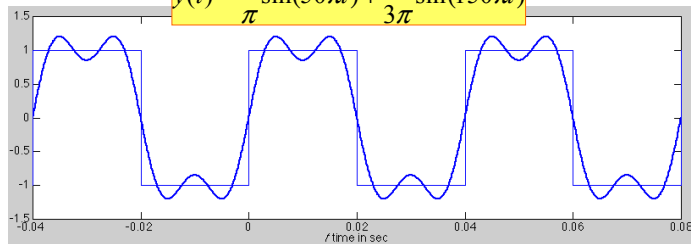


Ideal Lowpass Filter (100 Hz)

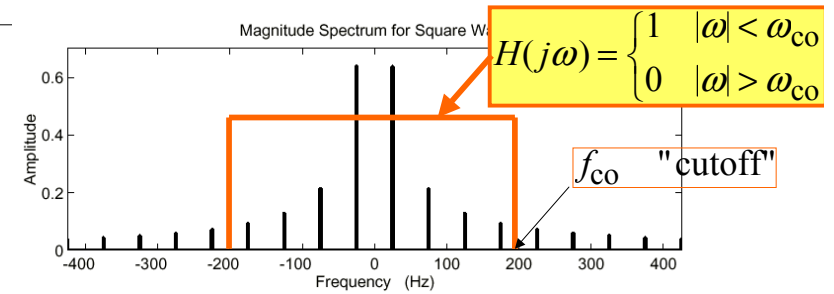


$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

$$y(t) = \frac{4}{\pi} \sin(50\pi t) + \frac{4}{3\pi} \sin(150\pi t)$$

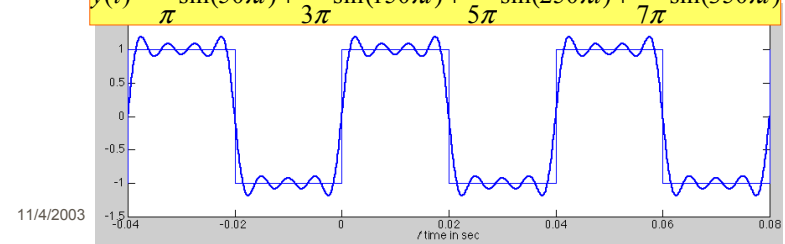


Ideal Lowpass Filter (200 Hz)

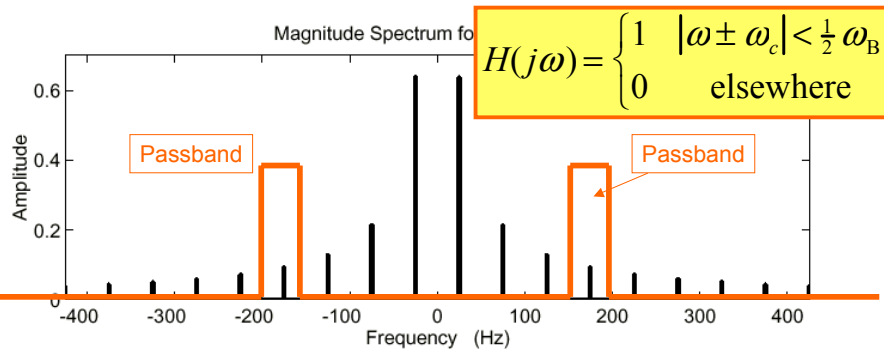


$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

$$y(t) = \frac{4}{\pi} \sin(50\pi t) + \frac{4}{3\pi} \sin(150\pi t) + \frac{4}{5\pi} \sin(250\pi t) + \frac{4}{7\pi} \sin(350\pi t)$$



Ideal Bandpass Filter



$$H(j\omega) = \begin{cases} 1 & |\omega \pm \omega_c| < \frac{1}{2} \omega_B \\ 0 & \text{elsewhere} \end{cases}$$

What is the output signal ?

$$y(t) = \frac{2}{j7\pi} e^{j2\pi(175)t} - \frac{2}{j7\pi} e^{-j2\pi(175)t} = \frac{4}{7\pi} \cos(2\pi(175)t - \frac{1}{2}\pi)$$

Example

$$H(j\omega) = e^{-j\omega t_d}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \mapsto y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j\omega_0 k t}$$

$$b_k = a_k H(j\omega_0 k) = a_k e^{-j\omega_0 k t_d}$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k e^{-j\omega_0 k t_d} e^{j\omega_0 k t} = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k (t - t_d)}$$

$$\therefore y(t) = x(t - t_d)$$