

Lecture 25
Review: Digital Filtering
of Analog Signals
1-Dec-03

Info: Web-CT, Lab, HW

- HW #12 due (in Lecture) on **last** day, 5-Dec
- **Labs: deadline to count for grade: 5-Dec**
 - **Missing Lab → “F”**
- Grade Totals posted
 - Max is 70 points now
 - Recitation will be 5 points
 - Lowest HW will be dropped
 - All PreLabs are finalized now
- Labs being held this week and during 1-4 Dec
 - Fill out 3 Course Evaluation Surveys

FINAL EXAM

- **Final Exam is either:**
 - **Thurs @ 8am, 11-Dec:** (for 10am Lecture section)
 - **Friday @ 8am, 12-Dec:** (for 11am Lecture)
 - Review Session on Wednesday, 10-Dec (7:30pm)
- **FORMULA PAGE: ONE page HAND-WRITTEN**
 - Tables 11.2 and 11.3 will be supplied with the exam
- **COVERAGE / EMPHASIS?**
 - **Fourier Transform**
 - Sampling, Filtering & Spectrum
 - Digital Filters: IIR & FIR & H(z)
 - Sampling & Aliasing
 - Hard problems from Quizzes #2 and #3.
 - **Homework** & Old Quizzes & **PreLab Questions**

Demos about IIR Systems

Lecture

SIGNAL PROCESSING FIRST

Examples
 Labs
 Hw
 Exercises
 Text
 Demos
 Beginning
 Case
 Keep

1: Introduction 2: Sinusoids 3: Spectrum Represent 4: Sampling and Alias 5: FIR Filters 6: Frequency Respons 7: Z-Transform 8: IIR Filters Labs Demos Exercises

Chapters Demos Labs

- Signal Processing First C
 - 1: Introduction
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 - Getting Started
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 - 3: Spectrum Represent
 - 4: Sampling and Alias
 - 5: FIR Filters
 - 6: Frequency Respons
 - 7: Z-Transform
 - 8: IIR Filters
 - Labs
 - Demos
 - Examples
 - Exercises

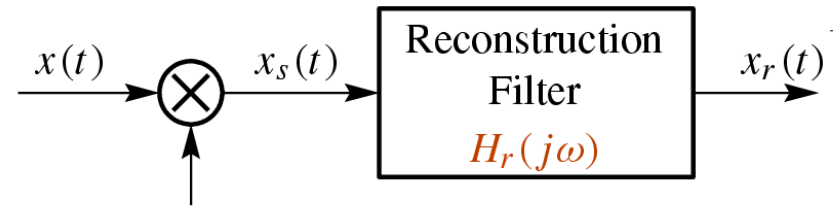
8. IIR FILTERS DEMOS

	Three Domains - IIR	The connection between the z-transform domain of poles and zeros and the time domain, and also the frequency domain is illustrated with several movies where individual poles, or zeros or pole pairs of IIR filters are moved continuously.
	PeZ GUI	PeZ (pezdemo) is a MATLAB tool for pole/zero manipulation. Poles and zeros can be placed anywhere on a map of the z-plane. The corresponding time domain (n) and frequency domain (ω) plots will be displayed. When a zero pair (or pole pair) is dragged, the impulse response and frequency response plots will be updated in real time.
	PeZ Tutorial	These movies describe how to use the PeZ graphical user interface to place/move poles and zeros. They also show how to display the associated impulse and frequency response.
	IIR Filtering	A short tutorial on first- and second-order IIR (infinite-length impulse response) filters. This demo shows plots in the three domains for a variety of IIR filters with different filter coefficients.
	Z to Freq	A demo that illustrates the connection between the complex Z-plane and the frequency response of a system. The frequency response is obtained by evaluating H(z) on the unit circle in the complex Z-plane.

LECTURE OBJECTIVES

- **Sampling Theorem** Revisited
 - GENERAL: in the **FREQUENCY DOMAIN**
 - Fourier transform of sampled signal
 - Reconstruction from samples
- **Effective Frequency Response**
- Important FT properties
 - Convolution \leftrightarrow multiplication
 - Frequency shifting

Sampling: Freq. Domain



$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

**EXPECT
FREQUENCY
SHIFTING !!!**

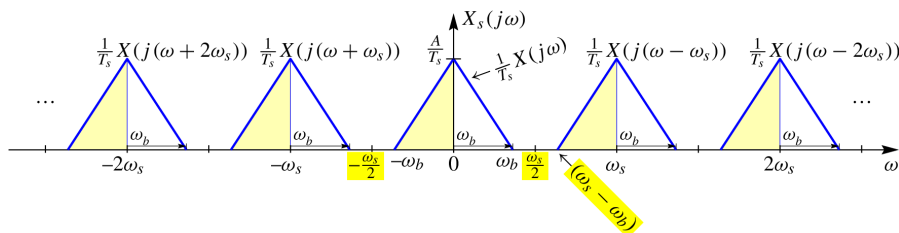
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

Frequency-Domain Representation of Sampling

“Typical” bandlimited signal



$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

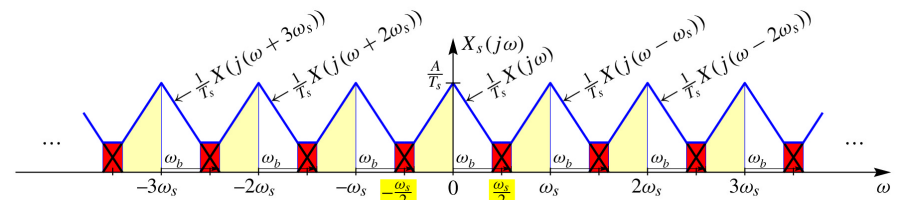


Aliasing Distortion

“Typical” bandlimited signal



- If $\omega_s < 2\omega_b$, the copies of $X(j\omega)$ overlap, and we have **aliasing distortion**.



Reconstruction of $x(t)$

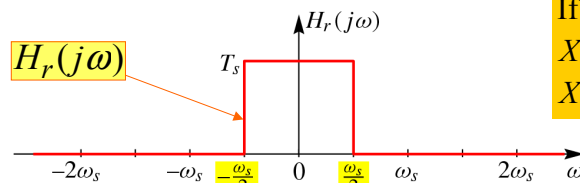
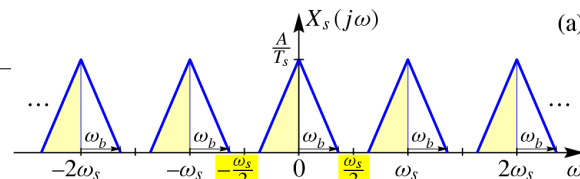


$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

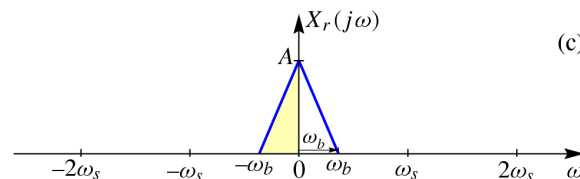
$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega)X_s(j\omega)$$

Reconstruction: Frequency-Domain

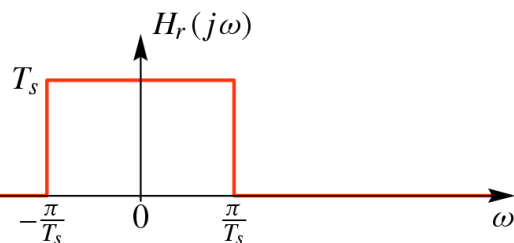


If $\omega_s > 2\omega_b$, the copies of $X(j\omega)$ do not overlap, so $X_r(j\omega) = H_r(j\omega)X_s(j\omega)$

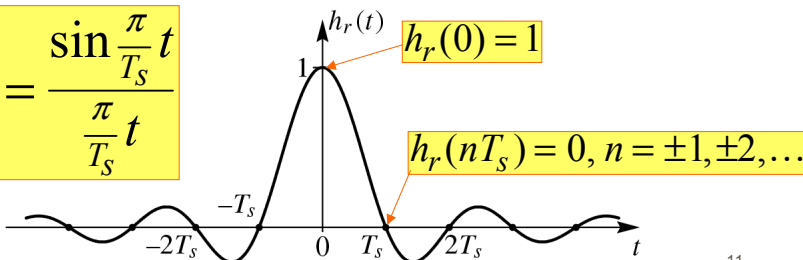


Ideal Reconstruction Filter

$$H_r(j\omega) = \begin{cases} T_s & |\omega| < \frac{\pi}{T_s} \\ 0 & |\omega| > \frac{\pi}{T_s} \end{cases}$$



$$h_r(t) = \frac{\sin \frac{\pi}{T_s} t}{\frac{\pi}{T_s} t}$$



Signal Reconstruction

$$x_r(t) = h_r(t) * x_s(t) = h_r(t) * \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)h_r(t - nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \frac{\pi}{T_s} (t - nT_s)}{\frac{\pi}{T_s} (t - nT_s)}$$

Ideal bandlimited interpolation formula

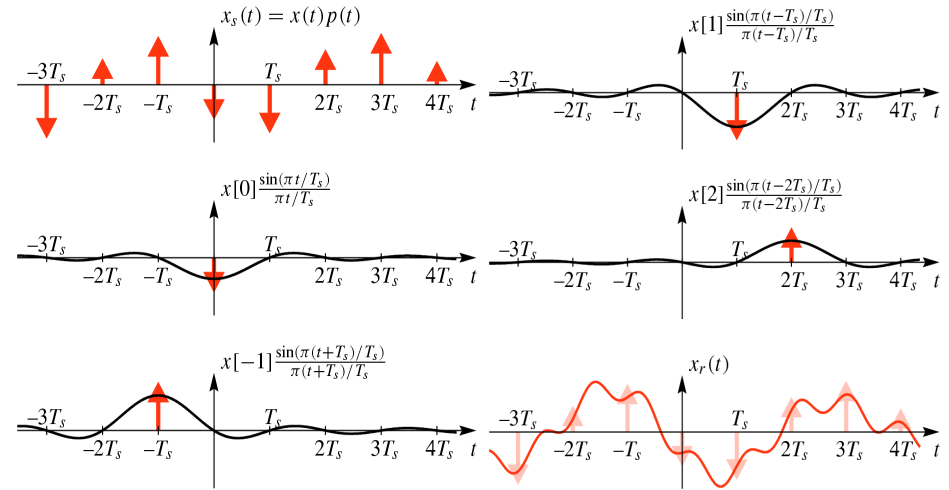
Shannon Sampling Theorem

- **“SINC” Interpolation** is the ideal
 - PERFECT RECONSTRUCTION
 - of BANDLIMITED SIGNALS

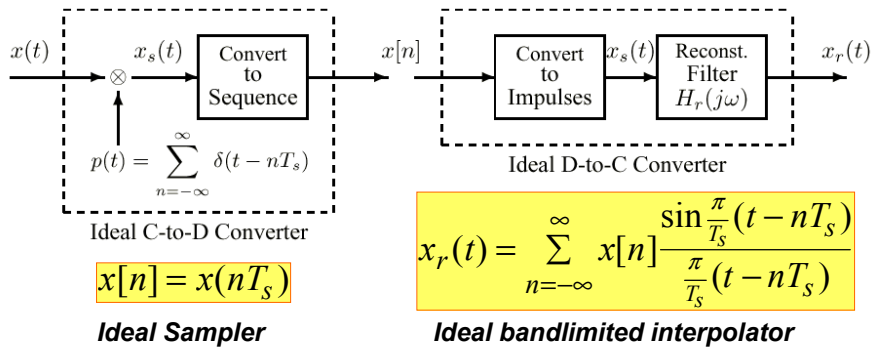
A signal $x(t)$ with bandlimited Fourier transform such that $X(j\omega) = 0$ for $|\omega| \geq \omega_b$ can be reconstructed exactly from samples taken with sampling rate $\omega_s = 2\pi/T_s \geq 2\omega_b$ using the following bandlimited interpolation formula:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \left[\frac{\pi}{T_s} (t - nT_s) \right]}{\frac{\pi}{T_s} (t - nT_s)}.$$

Reconstruction in Time-Domain



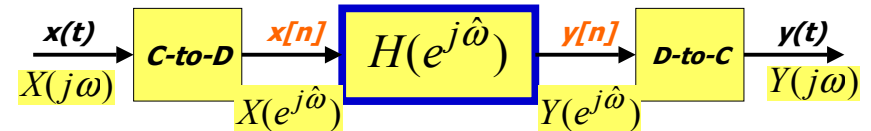
Ideal C-to-D and D-to-C



$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega) X_s(j\omega)$$

DT Filtering of CT Signals

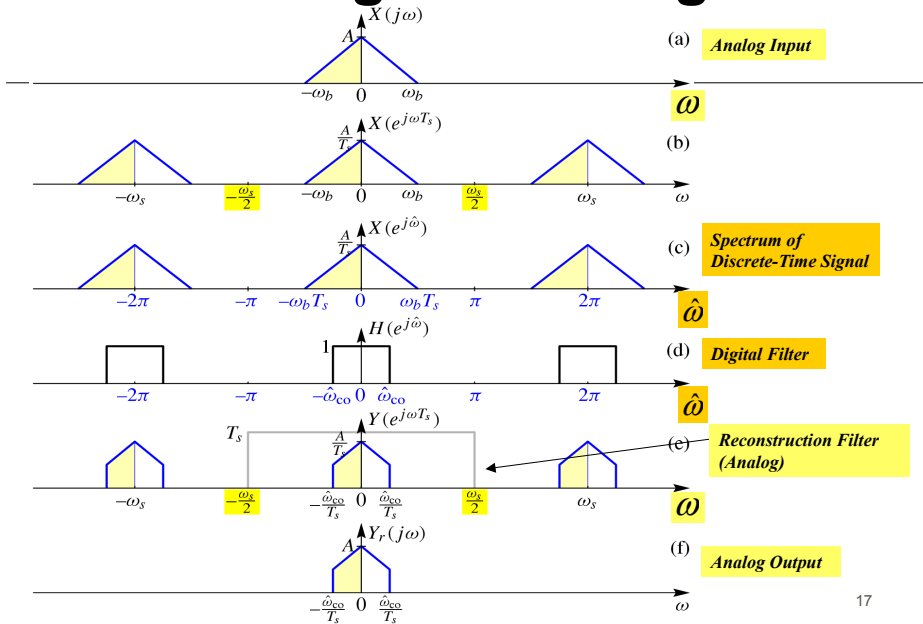


If no aliasing occurs in sampling $x(t)$, then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega) X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2} \omega_s \\ \text{UNDEFINED} \\ \text{NOT LTI} & |\omega| > \frac{1}{2} \omega_s \end{cases}$$

DT Filtering of a CT Signal



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EFFECTIVE Freq. Response

- Assume NO Aliasing, then
 - ANALOG FREQ \leftrightarrow DIGITAL FREQ

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

DIGITAL FILTER

$H(e^{j\omega T_s})$ vs. ω

ANALOG FREQUENCY

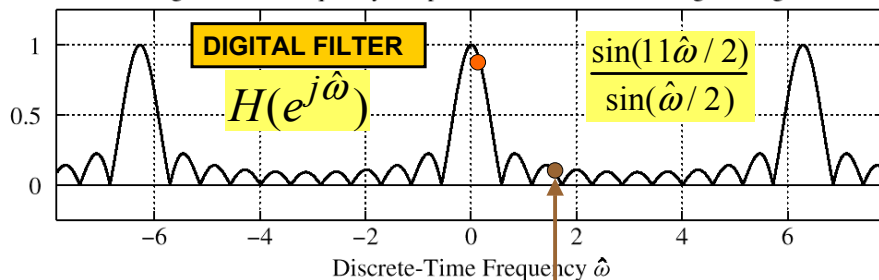
- So, we can plot:
- Scaled Freq. Axis

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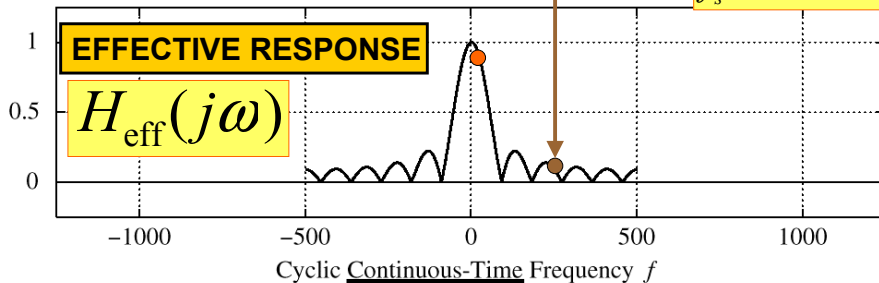
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Magnitude of Frequency Response for 11-Point Running Averager



Equivalent Continuous-Time Frequency Response for $f_s = 1000$ Hz



H_eff for 11-pt Averager

- Frequency Response for Discrete-time

$$H(e^{j\hat{\omega}}) = \frac{\sin(11\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s} = \frac{\omega}{1000}$$

- Analog Frequency Response

$$H_{\text{eff}}(j\omega) = \frac{\sin(11\omega/2000)}{\sin(\omega/2000)}$$

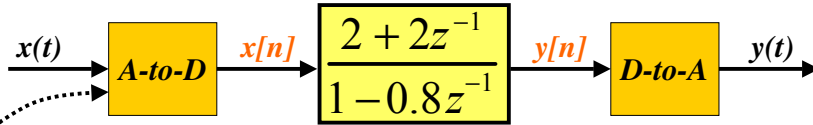
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POP QUIZ

- Given:



- Find the output, $y(t)$

- When

$$x(t) = \cos(2\pi(1000)t)$$

$$f_s = 5000 \text{ Hz}$$

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Effective Frequency Response

- Given:

$$H(z) = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}}$$

**NO Aliasing,
Because
 $2(1000) < 5000$**

- The discrete-time frequency response is

$$H(e^{j\hat{\omega}}) = \frac{2 + 2e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$$

- Then the Effective Frequency Response is

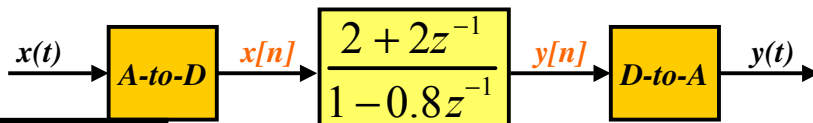
$$H(j\omega) = \frac{2 + 2e^{-j\omega/5000}}{1 - 0.8e^{-j\omega/5000}}$$

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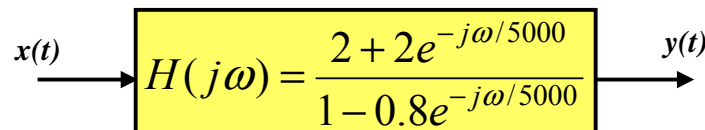
Equivalent Systems

- Given:



$$f_s = 5000 \text{ Hz}$$

- “Effective Analog System” for $\omega < (2\pi f_s)/2$



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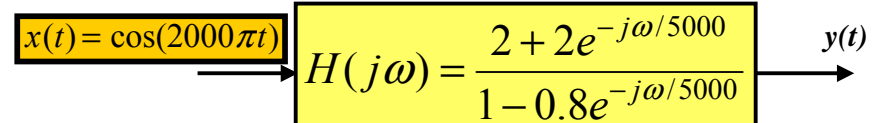
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POP QUIZ ANSWER

$$f_s = 5000 \text{ Hz}$$

- Given:



$$H(j\omega) \Big|_{\omega=2\pi(1000)} = \frac{2 + 2e^{-j2000\pi/5000}}{1 - 0.8e^{-j2000\pi/5000}} = 3.02e^{-j0.452\pi}$$

- The output is

$$y(t) = 3.02 \cos(2000\pi t - 0.452\pi)$$

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