ECE-2025 Fall-2003

Lecture 25
Review: Digital Filtering of Analog Signals
1-Dec-03

Info: Web-CT, Lab, HW

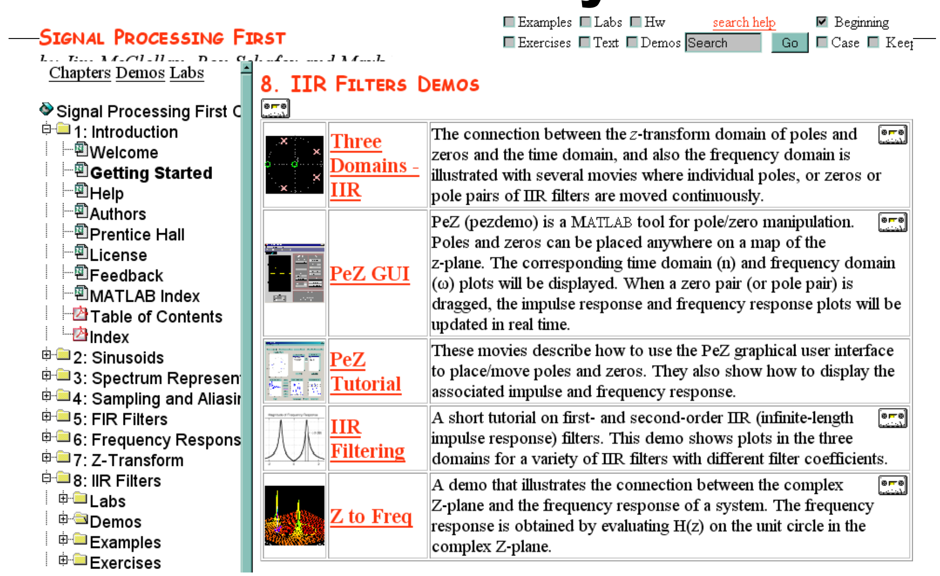
- HW #12 due (in Lecture) on **last** day, 5-Dec
- **Labs: deadline to count for grade: 5-Dec**
  - **Missing Lab → "F"**
- Grade Totals posted
  - Max is 70 points now
  - Recitation will be 5 points
  - Lowest HW will be dropped
  - All PreLabs are finalized now
- Labs being held this week and during 1-4 Dec
  - Fill out 3 Course Evaluation Surveys

FINAL EXAM

- **Final Exam is either:**
  - **Thurs @ 8am, 11-Dec: (for 10am Lecture section)**
  - **Friday @ 8am, 12-Dec: (for 11am Lecture)**
    - Review Session on Wednesday, 10-Dec (7:30pm)
- FORMULA PAGE: **ONE** page **HAND-WRITTEN**
    - **Tables 11.2 and 11.3 will be supplied with the exam**
- COVERAGE / EMPHASIS?
  - **Fourier Transform**
    - Sampling, Filtering & Spectrum
  - Digital Filters: IIR & FIR & H(z)
  - Sampling & Aliasing
  - Hard problems from Quizzes #2 and #3.
  - **Homework** & Old Quizzes & **PreLab Questions**

Demos about IIR Systems

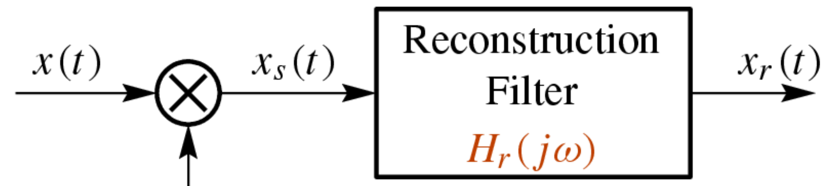
Lecture



LECTURE OBJECTIVES

- Sampling Theorem Revisited
  - GENERAL: in the FREQUENCY DOMAIN
  - Fourier transform of sampled signal
  - Reconstruction from samples
- **Effective Frequency Response**
- Important FT properties
  - Convolution ←→ multiplication
  - Frequency shifting

Sampling: Freq. Domain



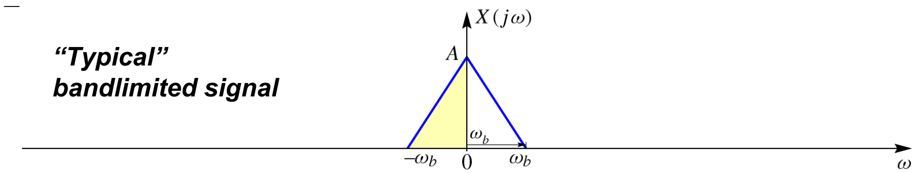
$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

EXPECT FREQUENCY SHIFTING !!!

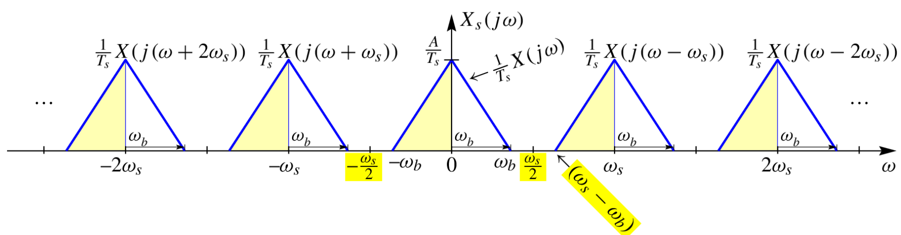
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

Frequency-Domain Representation of Sampling

"Typical" bandlimited signal

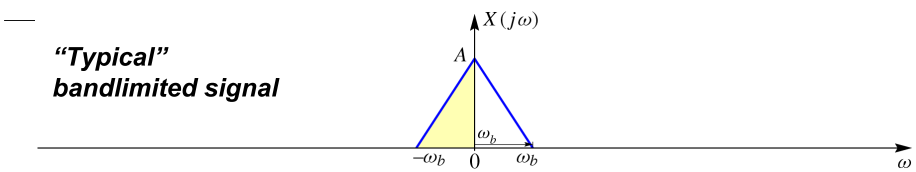


$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

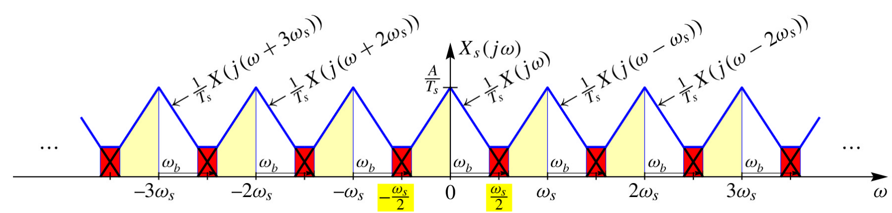


Aliasing Distortion

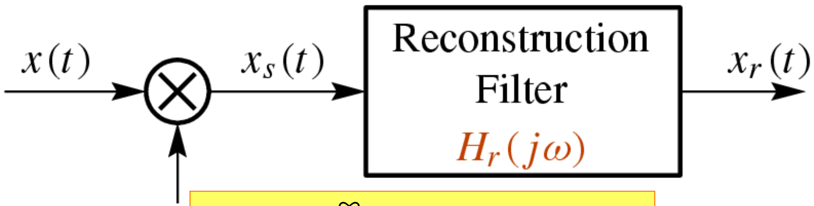
"Typical" bandlimited signal



- If $\omega_s < 2\omega_b$, the copies of $X(j\omega)$ overlap, and we have aliasing distortion.



Reconstruction of *x*(*t*)

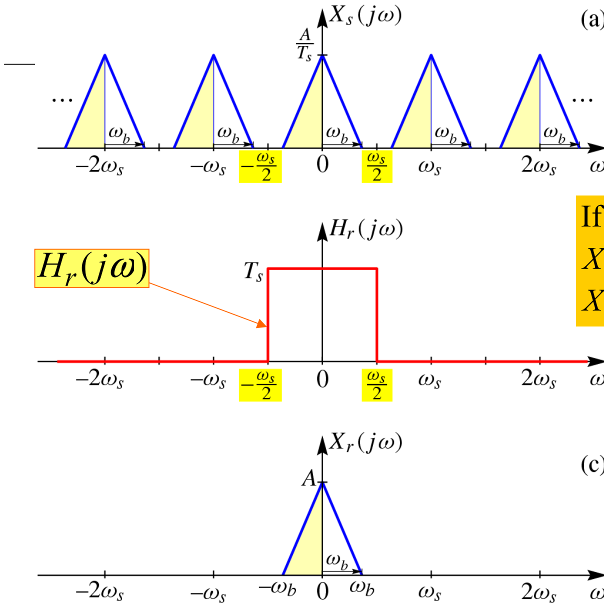


$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s)$$

$$X_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega)X_s(j\omega)$$

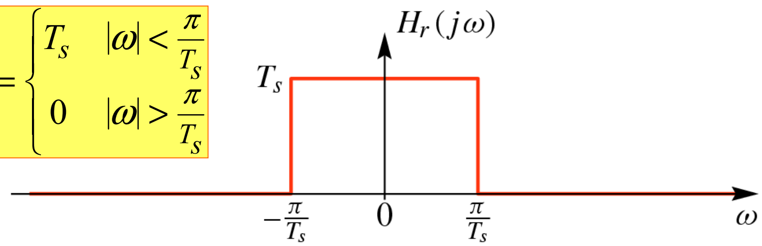
Reconstruction: Frequency-Domain



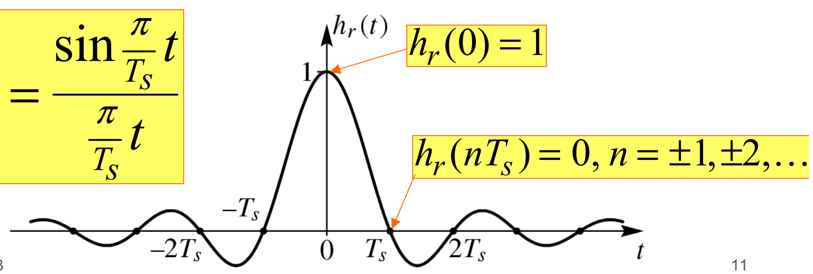
If $\omega_s > 2\omega_b$, the copies of $X(j\omega)$ do not overlap, so $X_r(j\omega) = H_r(j\omega)X_s(j\omega)$

Ideal Reconstruction Filter

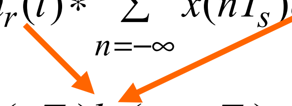
$$H_r(j\omega) = \begin{cases} T_s & |\omega| < \frac{\pi}{T_s} \\ 0 & |\omega| > \frac{\pi}{T_s} \end{cases}$$



$$h_r(t) = \frac{\sin\frac{\pi}{T_s}t}{\frac{\pi}{T_s}t}$$



Signal Reconstruction



$$x_r(t) = h_r(t) * x_s(t) = h_r(t) * \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)h_r(t-nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\frac{\sin\frac{\pi}{T_s}(t-nT_s)}{\frac{\pi}{T_s}(t-nT_s)}$$

Ideal bandlimited interpolation formula

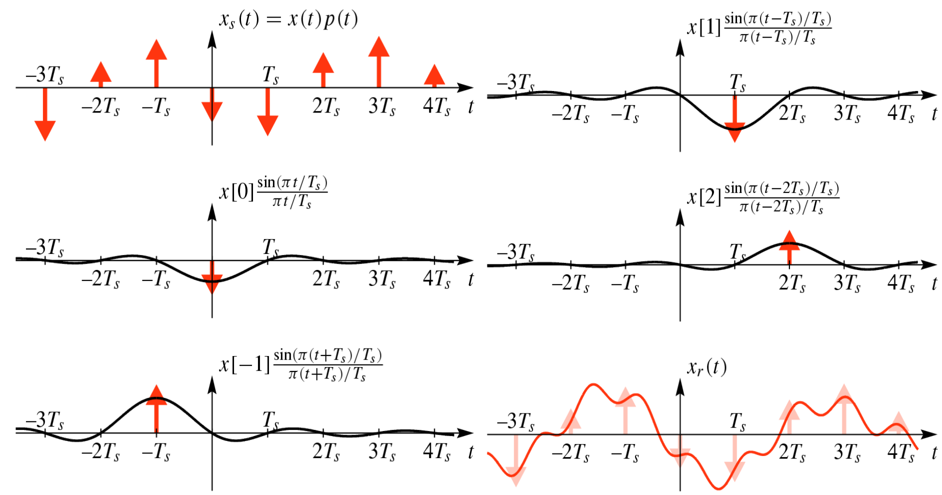
Shannon Sampling Theorem

- **"SINC" Interpolation** is the ideal
  - PERFECT RECONSTRUCTION
  - of BANDLIMITED SIGNALS

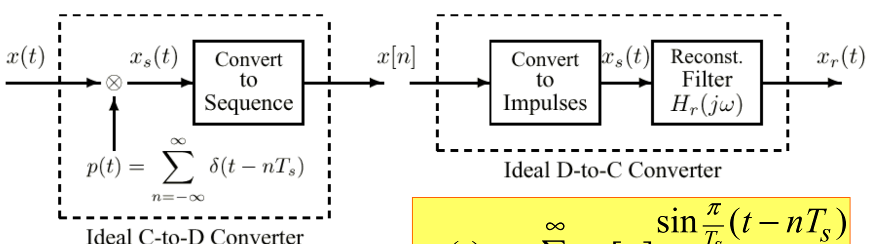
A signal $x(t)$ with bandlimited Fourier transform such that $X(j\omega) = 0$ for $|\omega| \geq \omega_b$ can be reconstructed exactly from samples taken with sampling rate $\omega_s = 2\pi/T_s \geq 2\omega_b$ using the following bandlimited interpolation formula:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin\left[\frac{\pi}{T_s}(t - nT_s)\right]}{\frac{\pi}{T_s}(t - nT_s)}.$$

Reconstruction in Time-Domain



Ideal C-to-D and D-to-C



$x[n] = x(nT_s)$

Ideal Sampler

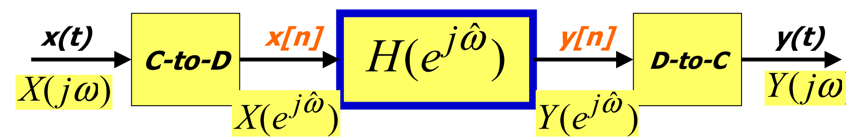
$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin\frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

Ideal bandlimited interpolator

$$X_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega)X_s(j\omega)$$

DT Filtering of CT Signals

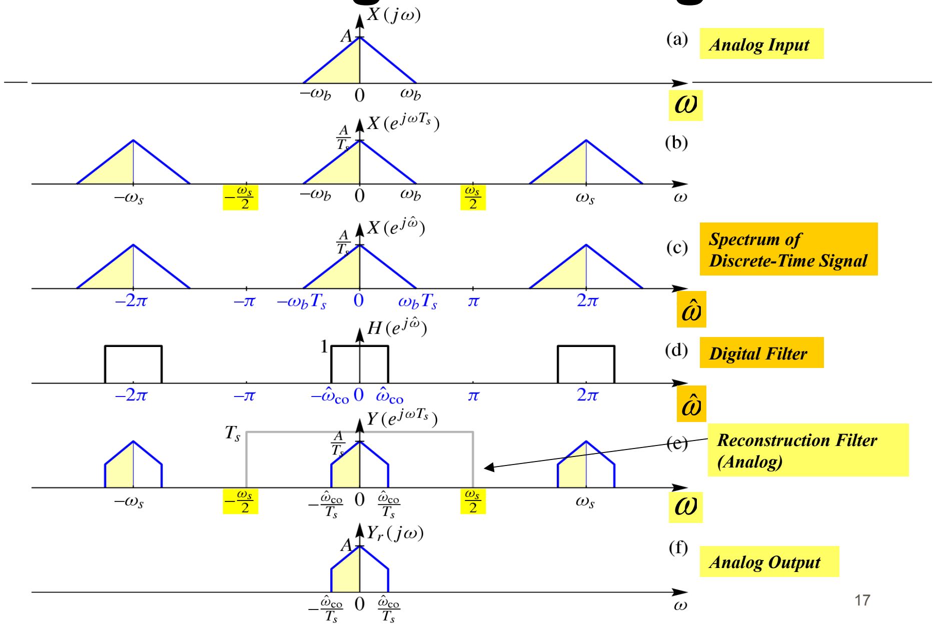


If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ \textit{UNDEFINED NOT LTI} & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

DT Filtering of a CT Signal



EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <--> DIGITAL FREQ

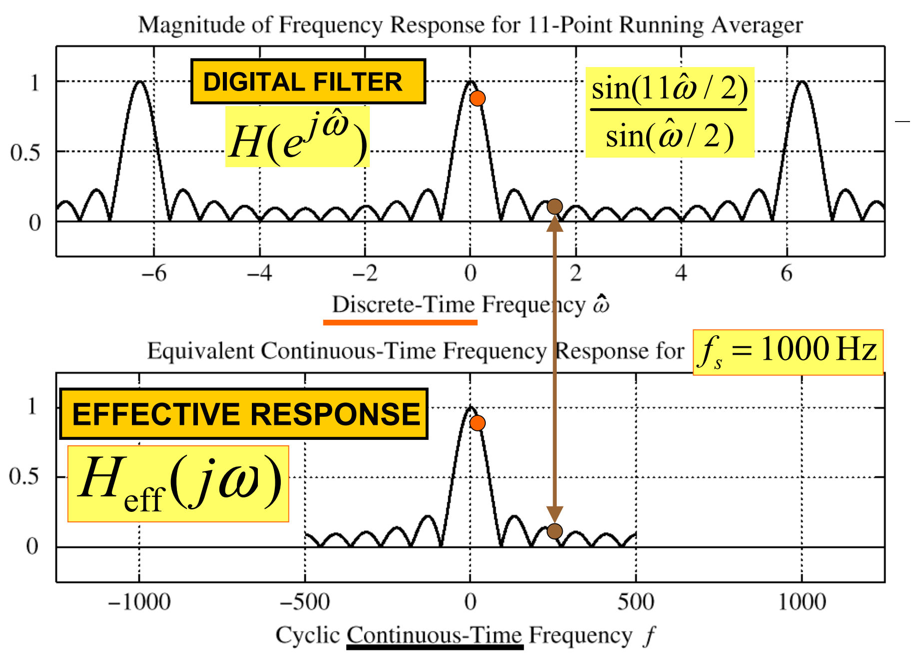
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

- So, we can plot:
- Scaled Freq. Axis

$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

DIGITAL FILTER

ANALOG FREQUENCY



H_{eff} for 11-pt Averager

- Frequency Response for Discrete-time

$$H(e^{j\hat{\omega}}) = \frac{\sin(11\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

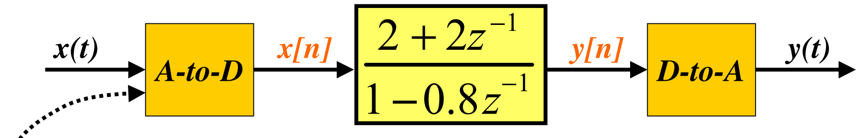
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s} = \frac{\omega}{1000}$$

- Analog Frequency Response

$$H_{\text{eff}}(j\omega) = \frac{\sin(11\omega/2000)}{\sin(\omega/2000)}$$

POP QUIZ

- Given:



- Find the output, y(t)
  - When $x(t) = \cos(2\pi(1000)t)$

$f_s = 5000\,\text{Hz}$

Effective Frequency Response

- Given: $H(z) = \frac{2+2z^{-1}}{1-0.8z^{-1}}$

NO Aliasing, Because 2(1000) < 5000

- The discrete-time frequency response is

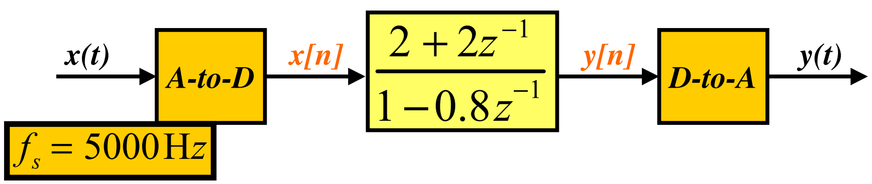
$$H(e^{j\hat{\omega}}) = \frac{2+2e^{-j\hat{\omega}}}{1-0.8e^{-j\hat{\omega}}}$$

- Then the Effective Frequency Response is

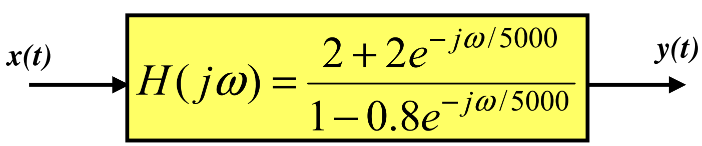
$$H(j\omega) = \frac{2+2e^{-j\omega/5000}}{1-0.8e^{-j\omega/5000}}$$

Equivalent Systems

- Given:



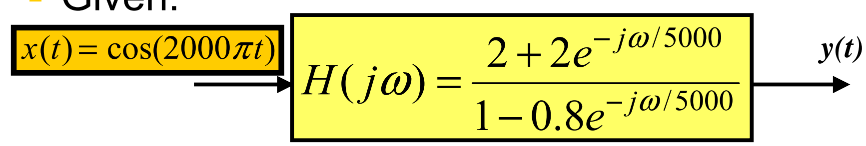
- "Effective Analog System" for $\omega<(2\pi f_s)/2$



POP QUIZ ANSWER

$f_s = 5000\,\text{Hz}$

- Given:



$$H(j\omega)\Big|_{\omega=2\pi(1000)} = \frac{2+2e^{-j2000\pi/5000}}{1-0.8e^{-j2000\pi/5000}}$$

$$= 3.02e^{-j0.452\pi}$$

- The output is

$$y(t) = 3.02\cos(2000\pi t - 0.452\pi)$$