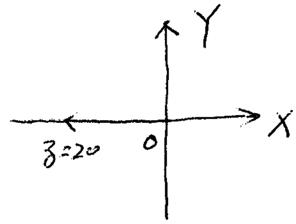
ECE2025 Problem Set#1 Solution

Problem 1.1

(a) $r=\sqrt{x^2+y^2}=\sqrt{(-20)^2+0^2}=20$,

$\theta=\arctan\left(\frac{y}{x}\right)=\arctan\left(\frac{0}{-20}\right)+\pi=\pi$,

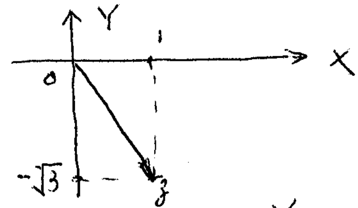
$z=20\angle\pi=20e^{j\pi}$.



(b) $r=\sqrt{x^2+y^2}=\sqrt{(1)^2+(-\sqrt{3})^2}=2$,

$\theta=\arctan\left(\frac{y}{x}\right)=\arctan\left(\frac{-\sqrt{3}}{1}\right)=-\pi/3$,

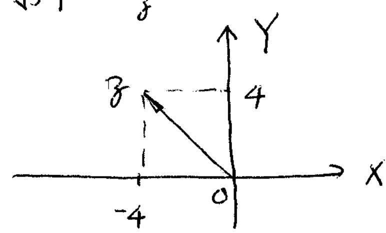
$z=2\angle-\pi/3=2e^{-j\pi/3}$.



(c) $r=\sqrt{x^2+y^2}=\sqrt{(-4)^2+4^2}=4\sqrt{2}$,

$\theta=\arctan\left(\frac{y}{x}\right)+\pi=\arctan\left(\frac{4}{-4}\right)+\pi=3\pi/4$,

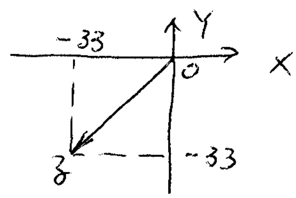
$z=4\sqrt{2}\angle3\pi/4=4\sqrt{2}e^{j3\pi/4}$.



(d) $r=\sqrt{x^2+y^2}=\sqrt{(-33)^2+(-33)^2}=33\sqrt{2}$,

$\theta=\arctan\left(\frac{y}{x}\right)+\pi=\arctan\left(\frac{-33}{-33}\right)+\pi=5\pi/4$,

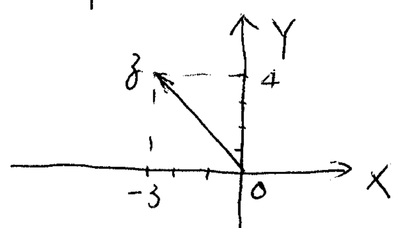
$z=33\sqrt{2}\angle5\pi/4=33\sqrt{2}e^{j5\pi/4}$.



(e) $r=\sqrt{x^2+y^2}=\sqrt{(-3)^2+4^2}=5$,

$\theta=\arctan\left(\frac{y}{x}\right)+\pi=\arctan\left(\frac{4}{-3}\right)+\pi\approx2.21$

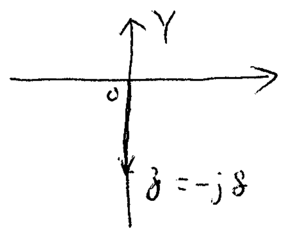
$z=5\angle2.21=5e^{j2.21}$.



(f) $r=\sqrt{x^2+y^2}=\sqrt{0^2+(-8)^2}=8$,

$\theta=\arctan\left(\frac{y}{x}\right)=\arctan\left(\frac{-8}{0}\right)=-\pi/2$,

$z=8\angle-\pi/2=8e^{-j\pi/2}$.



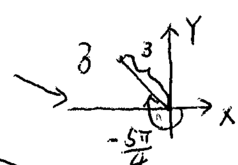
Problem 1.2

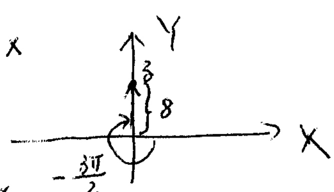
(a) $z=3\cos(-5\pi/4)+j3\sin(-5\pi/4)=-\frac{3}{2}\sqrt{2}+j\frac{3}{2}\sqrt{2}$.

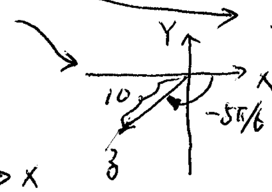
(b) $z=8\cos(-3\pi/2)+j8\sin(-3\pi/2)=j8$.

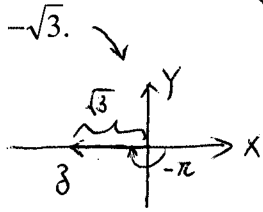
(c) $z=10\cos(-5\pi/6)+j10\sin(-5\pi/6)=-5\sqrt{3}-j5$.

(d) $z=\sqrt{3}\cos(31\pi)+j\sqrt{3}\sin(31\pi)=-\sqrt{3}$.









Problem 1.3 $z_1 = -5 + j5 = 5\sqrt{2}e^{j3\pi/4}$, $z_2 = 5\sqrt{2}e^{-j(7\pi/2)} = 5\sqrt{2}e^{j(\pi/2)} = j5\sqrt{2}$

(a) $z_1^* = (-5 + j5)^* = -5 - j5 = 5\sqrt{2}\angle -3\pi/4.$

(b) $jz_2 = j\left(5\sqrt{2}e^{-j(7\pi/2)}\right) = e^{j\pi/2}5\sqrt{2}e^{-j(7\pi/2)} = 5\sqrt{2}e^{-j(6\pi/2)} = 5\sqrt{2}\angle\pi = -5\sqrt{2}.$

(c) $z_2/z_1 = 5\sqrt{2}e^{j\pi/2}/\left(5\sqrt{2}e^{j3\pi/4}\right) = 1\angle -\pi/4 = \dfrac{\sqrt{2}}{2} - j\dfrac{\sqrt{2}}{2}.$

(d) $z_2^2 = \left(5\sqrt{2}e^{j(\pi/2)}\right)^2 = 50\angle\pi = -50.$

(e) $1/z_1 = 1/\left(5\sqrt{2}e^{j3\pi/4}\right) = \dfrac{\sqrt{2}}{10}\angle -3\pi/4 = -\dfrac{1}{10} - j\dfrac{1}{10}.$

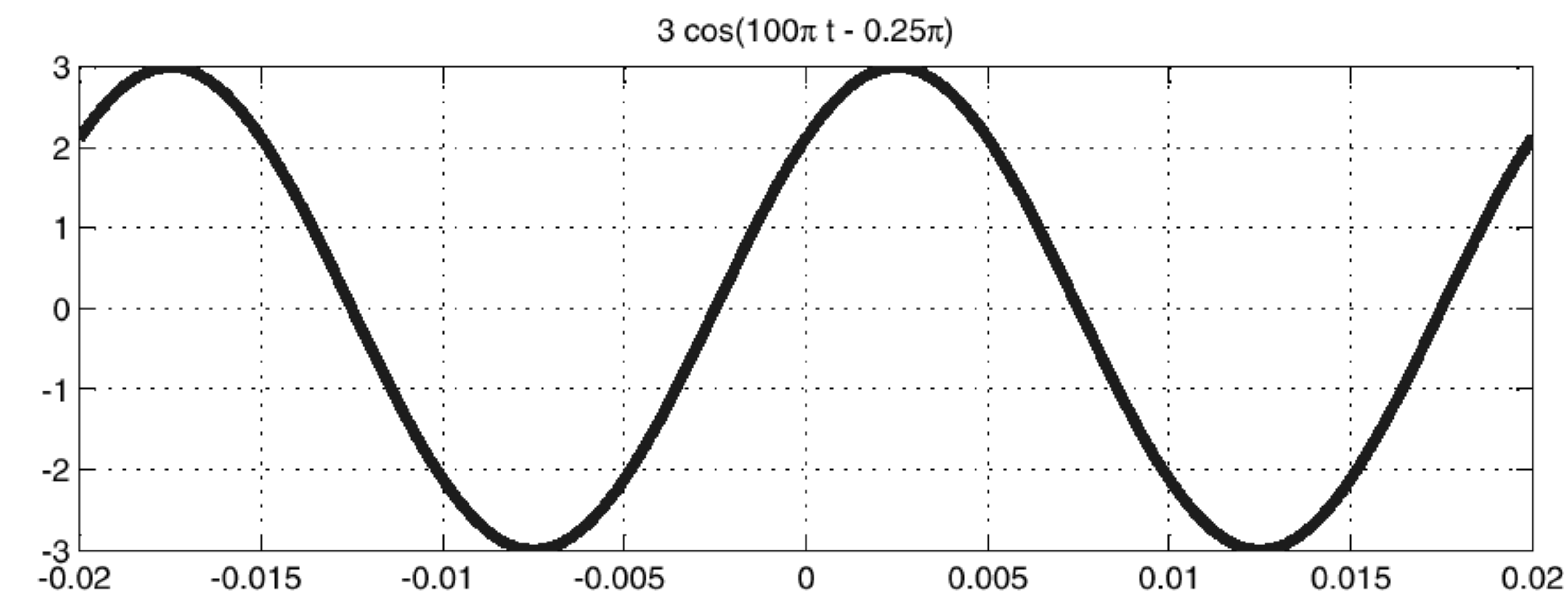
(f) $z_1 z_2 = \left(5\sqrt{2}e^{j3\pi/4}\right)\bullet 5\sqrt{2}e^{j\pi/2} = 50\angle 5\pi/4 = -25\sqrt{2} - j25\sqrt{2}.$

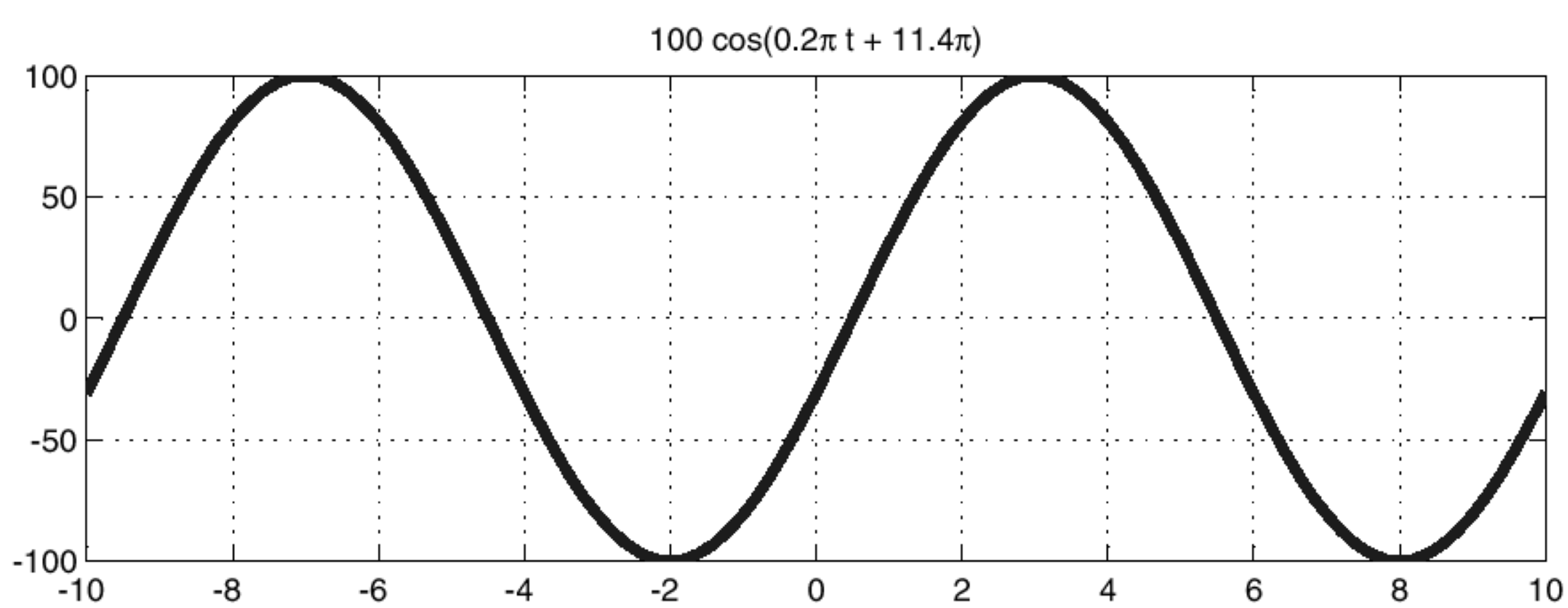
(g) $z_1 + z_2^* = (-5 + j5) + \left(-j5\sqrt{2}\right) = -5 - j5(\sqrt{2} - 1) \approx -5 - j2.07 = 202.5°.$

(h) $|z_2|^2 = \left(5\sqrt{2}e^{j\pi/2}\right)\bullet\left(5\sqrt{2}e^{j\pi/2}\right) = 50 = 50\angle 0.$

(i) $z_2 + z_2^* = j5\sqrt{2} + \left(j5\sqrt{2}\right)^* = 0.$

Problem 1.4



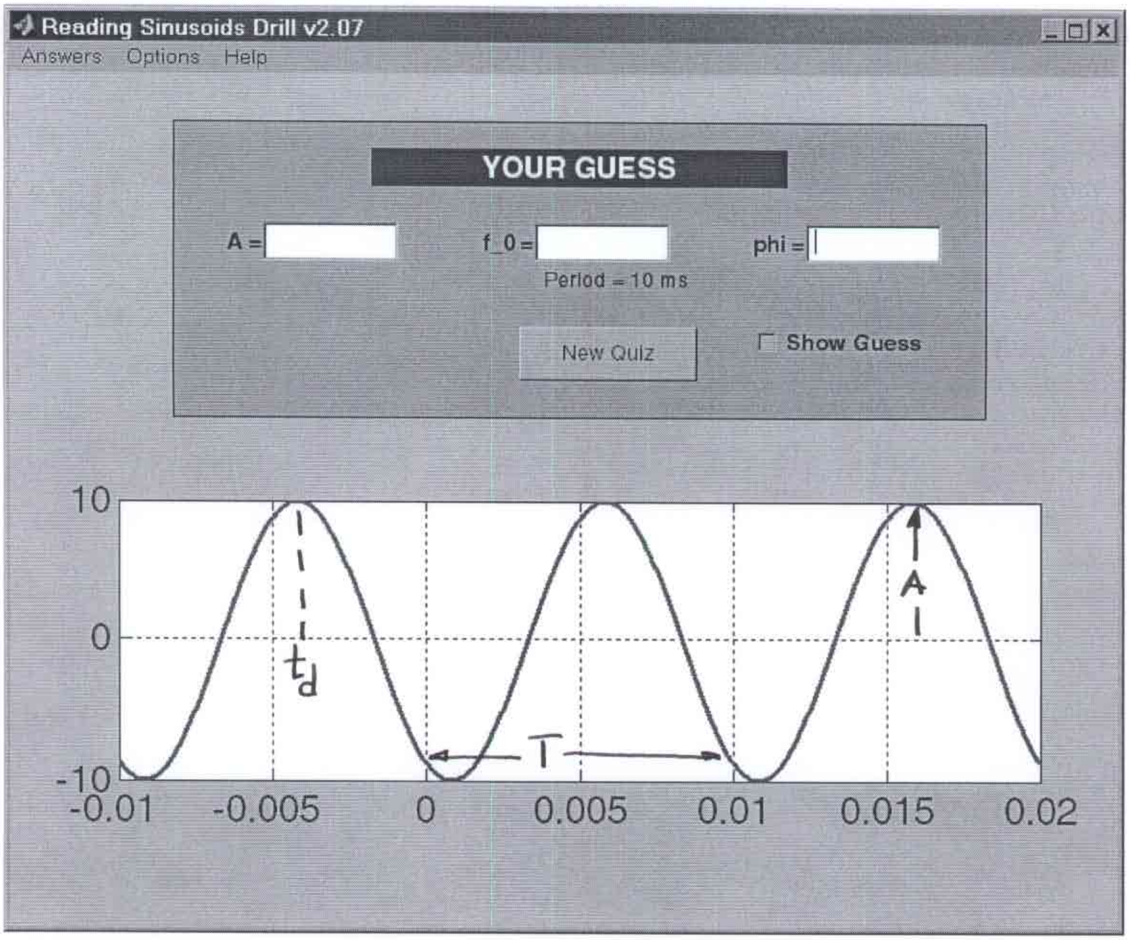


PROBLEM 1.5*:

The waveform in the following figure (generated from the MATLAB GUI `sindrill`) can be expressed as

$$x(t) = A\cos[\omega_0(t - t_d)] = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, f_0, t_d, and ϕ. Choose the value of ϕ such that $-\pi < \phi \le \pi$.



$$T = 0.01 \text{ secs.} \quad \Longrightarrow \quad f_0 = \frac{1}{T} = \frac{1}{0.01} = 100 \text{ Hz} \quad \Longrightarrow \quad \omega_0 = 2\pi f_0 = 2\pi(100) = 200\pi \text{ rad/s}$$

$$t_d = -0.004 \text{ secs.} \quad \Longrightarrow \quad \phi = -\omega_0 t_d = -200\pi(-0.004) = +0.8\pi \text{ radians}$$

$$A = 10$$

$$\Longrightarrow \quad x(t) = 10\cos(200\pi t + 0.8\pi) = 10\cos(200\pi(t + 0.004))$$

Note: the true answer for ϕ is actually $\phi = 5\pi/6 = 0.833333\pi$, but it would be very hard to measure a time shift of 0.004166666 secs on the plot above.