HW #6

Problem 6.1.

(a) $y[n] = 3x[n] - 4x[n-2] + x[n-7]$ (1a)

To find the impulse response, let $x[n] = \delta[n]$, and $h[n] = y[n]$

i.e., $h[n] = 3\delta[n] - 4\delta[n-2] + \delta[n-7]$

(b) In $y[n] = \sum_{k=0}^{M} b_k x[n-k]$

$b_0 = 3$, $b_1 = 0$, $b_2 = -4$, $b_3 = 0$, $b_4 = 0$, $b_5 = 0$, $b_6 = 0$, $b_7 = 1$

(c) Order $M = 7$, length $L = 8$

(d) Substitute $x[n] = 3$ into (1a), we obtain

$y[n] = 3*3 - 4*3 + 3 = 0.$

(e) $y[n] = 9x[n+3] - 3x[n] + 2x[n-4]$ cannot be put into the form $y[n] = \sum_{k=0}^{M} b_k x[n-k]$ (causal system), since the term $9x[n+3]$ is non-causal.

Problem 6.2

$x[n] = 4\delta[n+2] + \delta[n+1] + \delta[n-1] + 4\delta[n-2]$

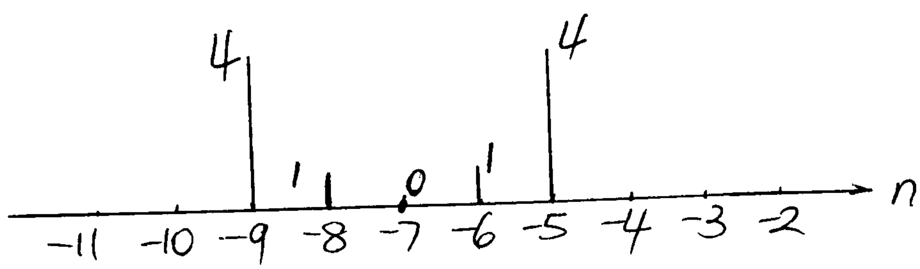
$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2]$

Convolution table:

n	-2	-1	0	1	2	3	4
$x[n]$	4	1	0	1	4	0	0
$h[n]$	0	0	1	-1	2	0	0
$h[0]x[n-0]$	4	1	0	1	4	0	0
$h[1]x[n-1]$	0	-4	-1	0	-1	-4	0
$h[2]x[n-2]$	0	0	8	2	0	2	8
$y[n]$	4	-3	7	3	3	-2	8

(b) $x[n] = n^2 \cdot (u[n+2] - u[n-3])$

(c) $x[n] * \delta[n+7] = x[n+7] = 4\delta[n+9] + \delta[n+8] + \delta[n+6] + 4\delta[n+5]$



Problem 6.3

(a)

n	-2	-1	0	1	2	3	4
$h[n]$	0	0	1	-1	2	0	0
$x[n]$	4	1	0	1	4	0	0
$x[-2]h[n+2]$	4	-4	8	0	0	0	0
$x[-1]h[n+1]$	0	1	-1	2	0	0	0
$x[0]h[n]$	0	0	0	0	0	0	0
$x[1]h[n-1]$	0	0	0	1	-1	2	0
$x[2]h[n-2]$	0	0	0	0	4	-4	8
$y[n]$	4	-3	7	3	3	-2	8

(b) In MATLAB,

```
conv([4 1 0 1 4], [1 -1 2])
conv([1 -1 2], [4 1 0 1 4])
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both yield

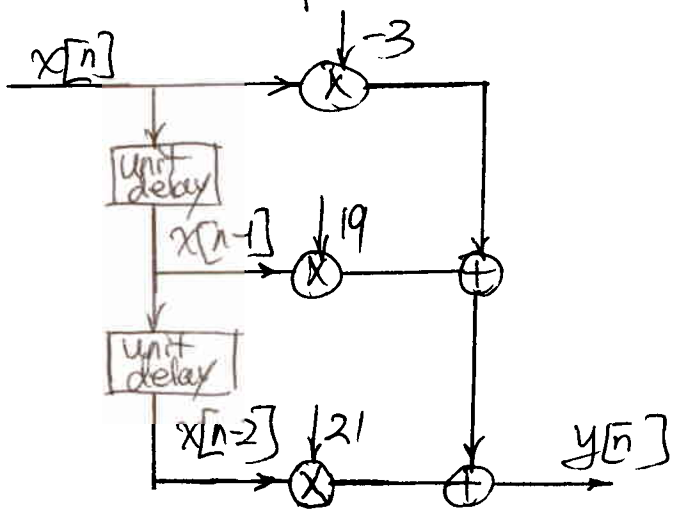
[4 -3 7 3 3 -2 8]

Problem 6.4

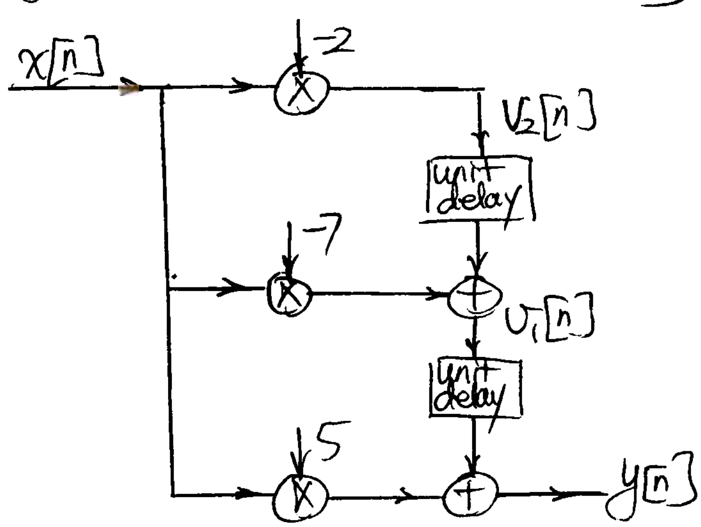
(a) $h[n] = -3\delta[n] + 19\delta[n-1] + 21\delta[n-2]$

$\Rightarrow y[n] = -3x[n] + 19x[n-1] + 21x[n-2]$

Direct-form implementation:



(b) $y[n] = 5x[n] - 7x[n-1] - 2x[n-2]$



(c) $h[n] = \sin(\pi n/10)\, u[n-5]$

The system is causal, since $h[n]=0,\ \forall n<0$.

(d) $h[n] = (0.4)^n u[n+3]$

The system is non-causal, since $h[n] \neq 0$ for $n=-3,-2,-1$.

Problem 6.5

$$x_1[n] = u[n], \qquad y_1[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$$

Let $x_3[n] = u[n-4]$ and $y_3[n]$ be the corresponding output.

By time-invariance, $y_3[n] = y_1[n-4]$

$$= \delta[n-4] + 2\delta[n-5] - \delta[n-6]$$

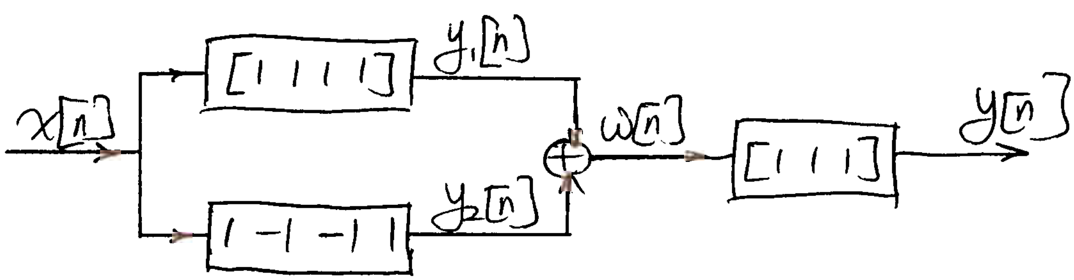
Now, $x_2[n] = 3u[n] - 2u[n-4]$

By linearity, $y_2[n] = 3y_1[n] - 2y_3[n]$

$$= 3\delta[n] + 6\delta[n-1] - 3\delta[n-2]$$
$$-2\delta[n-4] - 4\delta[n-5] + 2\delta[n-6]$$

Problem 6.6

(a)



$$y_1[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$
$$y_2[n] = x[n] - x[n-1] - x[n-2] + x[n-3]$$
$$w[n] = y_1[n] + y_2[n]$$
$$y[n] = w[n] + w[n-1] + w[n-2]$$

(b) Overall impulse response is

$$h[n] = \big((\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]) + (\delta[n] - \delta[n-1] - \delta[n-2] + \delta[n-3])\big) * (\delta[n] + \delta[n-1] + \delta[n-2])$$
$$= 2\sum_{k=0}^{5} \delta[n-k]$$
$$y[n] = x[n] * h[n] = 2\sum_{k=0}^{5} x[n-k]$$
$$= 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4] + 2x[n-5]$$

Problem 6.7

(a) $h[n] = \delta[n-1]$, $x[n] = u[n]$

$\Rightarrow y[n] = x[n] * h[n] = u[n-1]$

(b) $h[n] = \delta[n] - \delta[n-1]$, $x[n] = u[n]$

$\Rightarrow y[n] = x[n] * h[n] = u[n] - u[n-1] = \delta[n]$

(c) $h[n] = \delta[n-1] - 0.5\,\delta[n-2]$, $x[n] = 0.5^n u[n]$

$$\Rightarrow y[n] = x[n] * h[n] = 0.5^{n-1} u[n-1] - 0.5 \cdot 0.5^{n-2} u[n-2]$$
$$= 0.5^{n-1}(u[n-1] - u[n-2]) = 0.5^{n-1} \cdot \delta[n-1] = \delta[n-1]$$

Since $0.5^{n-1}\big|_{n=1} = 0.5^0 = 1$.