HW#8, Fall, 2003

problem 8.2 Solutions

c) $y_1[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$

$H_2(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$

$H_1(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$

$H(z) = H_1(z)\,H_2(z) = \frac{1}{9}(1+z^{-1}+z^{-2})(1+z^{-1}+z^{-2})$

$= \frac{1}{9}(1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4})$

d) $y[n] = \frac{1}{9}(x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4])$

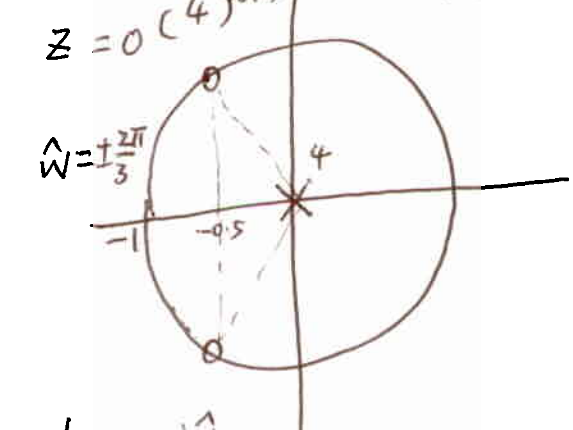
No, this is not a 6-point averager. It's a weighted 5-point Averager.

e) $H(z) = \frac{1}{9}(1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}) = \frac{1}{9} \cdot \frac{(z^2+z+1)(z^2+z+1)}{z^4}$

Zeros of $H(z)$, z_1, z_2, z_3, z_4, $z_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j = e^{\pm j\frac{2}{3}\pi}$

$z_{3,4} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j = e^{\pm j\frac{2}{3}\pi}$

poles of $H(z)$, $z = 0$ (4 poles)

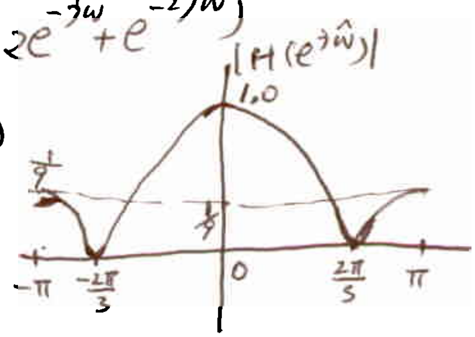


f) $H(e^{j\hat{w}}) = H(z)\big|_{z=e^{j\hat{w}}}$

$= \frac{1}{9}(1 + 2e^{-j\hat{w}} + 3e^{-2j\hat{w}} + 2e^{-3j\hat{w}} + e^{-4j\hat{w}})$

$= \frac{1}{9}e^{-2j\hat{w}}(e^{2j\hat{w}} + 2e^{j\hat{w}} + 3 + 2e^{-j\hat{w}} + e^{-2j\hat{w}})$

$H(e^{j\hat{w}}) = \frac{1}{9}e^{-2j\hat{w}}(2\cos(2\hat{w}) + 4\cos\hat{w} + 3)$



HW# 8 , Fall 2003

problem 8.4

$$H(z) = (1+z^{-2})(1-4z^{-2}) = 1-3z^{-2}-4z^{-4})$$

$$H(e^{j\hat{w}}) = 1-3e^{-2j\hat{w}}-4e^{-4j\hat{w}}$$

$$h[n] = \delta[n]-3\delta[n-2]-4\delta[n-4]$$

$$x[n] = 20-20\delta[n]+20\cos(0.5\pi n+\tfrac{\pi}{4})$$

$$= x_1[n] + x_2[n]$$

$$= -20\delta[n] + (20+20\cos(0.5\pi n+\tfrac{\pi}{4}))$$

$$y[n] = y_1[n] + y_2[n]$$

$$y_1[n] = h[n] \cdot x_1[n]$$

$$= (-20\delta[n])(\delta[n]-3\delta[n-2]-4\delta[n-4])$$

$$= -20\delta[n]+60\delta[n-2]+80\delta[n-4]$$

$$y_2[n] = H(e^{j0}) \cdot 20 + H(e^{j\times 0.5\pi})(20\cos(0.5\pi n)+\tfrac{\pi}{4})$$

$$= (-6)\times 20 + 0\,(20\cos(0.5\pi n)+\tfrac{\pi}{4})$$

$$= -120$$

$$y[n] = y_1[n] + y_2[n]$$

$$= -20\delta[n]+60\delta[n-2]+80\delta[n-4]-120$$

$$= -120-20\delta[n]+60\delta[n-2]+80\delta[n-4]$$

HW#8, Fall, 2003

problem 8.5

a) $H(z) = z^{-2}\left(1 - \frac{1}{\sqrt{3}} z^{-1}\right)$

$\underline{H(e^{j\hat{\omega}}) = e^{-2j\hat{\omega}}\left(1 - \frac{1}{\sqrt{3}} e^{-j\hat{\omega}}\right)}$

b) $x(t) = \sqrt{3}\cos(500\pi t)$

$f_s = \frac{1}{T_s} = \frac{1}{0.001} = 1000$

$\hat{\omega} = \frac{\omega}{f_s} = \frac{500\pi}{1000} = 0.5\pi,$

$\underline{x[n] = \sqrt{3}\cos(0.5\pi n)}$

c) $H(e^{j\hat{\omega}}) = e^{-2j\hat{\omega}}\left(1 - \frac{1}{\sqrt{3}} e^{-j\hat{\omega}}\right)$

$H(e^{j\times 0.5\pi}) = e^{-j\pi}\left(1 - \frac{1}{\sqrt{3}} e^{-j\frac{\pi}{2}}\right) = -1 + \frac{1}{\sqrt{3}} e^{-j\frac{\pi}{2}}$

$= -1 - \frac{1}{\sqrt{3}} j = \frac{2}{\sqrt{3}} e^{-5/6\,\pi}$

$y[n] = \frac{2}{\sqrt{3}} \sqrt{3} \cos(0.5\pi n - 5/6\,\pi)$

$\underline{y[n] = 2\cos\left(0.5\pi n - \frac{5\pi}{6}\right)}$

d) $y(t) = 2\cos(0.5\pi \times 1000 - 5\pi/6)$

$\underline{y(t) = 2\cos(500\pi t - 5\pi/6)}$

e) $x(t) = \cos(2000\pi t)$ $\quad$ $x[n] = \cos\left(\frac{2000}{1000}\pi n\right)$

$\because f_s = \frac{1}{T_s} = 1000$ $\quad$ $= \cos(2\pi \cdot n)$

$x[n] = 1$

$H(e^{j2\pi}) = H(e^{j\cdot 0}) = \left(1 - \frac{1}{\sqrt{3}}\right) = 0.422$

$y[n] = 0.422\cos(2\pi n) = 0.422$

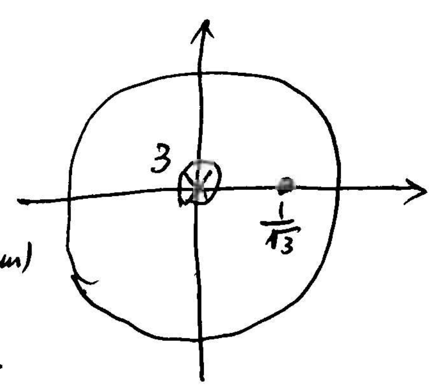
$\underline{y(t) = 0.422}$

f) $H(z) = z^{-2}\left(1 - \frac{1}{\sqrt{3}} z^{-1}\right)$

$= \frac{(z - \frac{1}{\sqrt{3}})}{z^3}$

poles, $z = 0$ (3 of them)

$\underline{\text{zeros, } z = \frac{1}{\sqrt{3}}}$



HW#8, Fall, 2003

problem 8.6

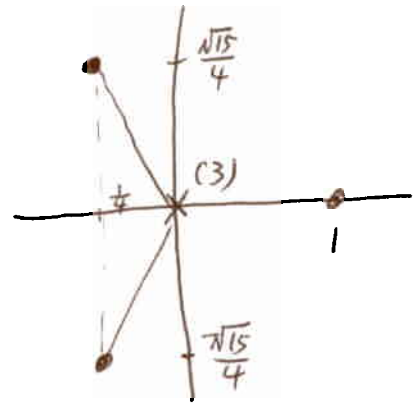
(a) $H(z) = 1 - (\frac{1}{2})z^{-1} + (\frac{1}{2})z^{-2} - z^{-3}$

$= (1 - z^{-1})(1 + \frac{1}{2}z^{-1} + z^{-2})$

zeros at $z_1 = 1$

$z_{2,3} = -\frac{1}{4} \pm \frac{\sqrt{15}}{4}j$

poles at $z = 0$, (3 of them)



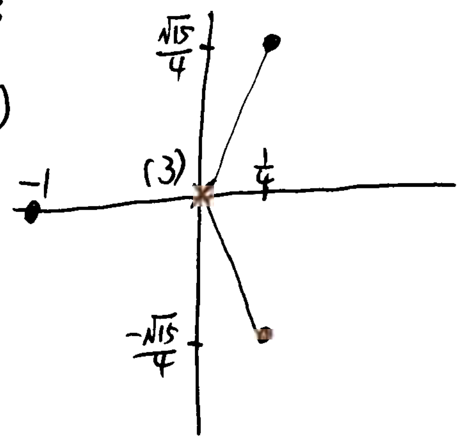
b) $H(z) = 1 + (\frac{1}{2})z^{-1} + (\frac{1}{2})z^{-2} + z^{-3}$

$= (1 + z^{-1})(1 - \frac{1}{2}z^{-1} + z^{-2})$

Zeros at $z_1 = -1$

$z_{2,3} = \frac{1}{4} \pm \frac{\sqrt{15}}{4}j$

poles at $z = 0$, (3 of them)

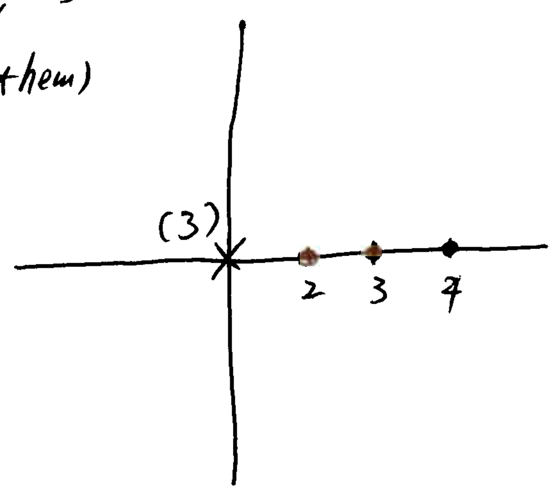


c) $H(z) = 1 - 9z^{-1} + 26z^{-2} - 24z^{-3}$

$= \frac{1}{z^3}(z-2)(z-3)(z-4)$

zeros at $z_1 = 2$, $z_2 = 3$, $z_3 = 4$

poles at $z = 0$, (3 of them)



HW#8, Fall, 2003

problem 8.7

a) <u>Fred</u>, zeros at $\{1, e^{\pm j\frac{2\pi}{7}}, e^{\pm j\frac{4\pi}{7}}, e^{\pm j 6\pi/7}\}$

$H(z) = 1 - z^{-7}$ by using MATLAB

$H(z) = \sum_{k=0}^{7} b_k z^{-k}$

$b_k = \{1, 0, 0, 0, 0, 0, 0, -1\}$

a) <u>Wilma</u>, zeros at $\{e^{\pm j\frac{\pi}{4}}, e^{\pm j\frac{\pi}{2}}, e^{\pm j\frac{3}{4}\pi}, e^{-j\pi}\}$

using matlab

$H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7}$

$H(z) = \sum_{k=0}^{7} b_k z^{-k}$

$b_k = \{1, 1, 1, 1, 1, 1, 1\}$

a) <u>Barney</u>, zeros at $\{e^{\pm 3/4\pi \cdot j}\}$

using matlab

$H(z) = 1 + \sqrt{2} z^{-1} + z^{-2}$

$H(z) = \sum_{k=0}^{2} b_k z^{-k}$

$b_k = \{1, \sqrt{2}, 1\}$

a) <u>Betty</u>, roots at $\{\frac{1}{2} e^{\pm 3/4\pi \cdot j}\}$

using matlab

$H(z) = 1 + \frac{1}{\sqrt{2}} z^{-1} + \frac{1}{4} z^{-2}$

$b_k = \{1, \frac{1}{\sqrt{2}}, \frac{1}{4}\}$

b). Mad Murdock is Barney because they both have 2 zeros

Faceman Peck is Wilma because they both have 6 zeros and $H(e^{j \cdot 0}) \neq 0$.

Hannibal Smith is Fred because they both have 7 zeros and $H(e^{j \cdot 0}) = 0$

B.A. Baracus is Betty because they both have 2 roots and no nulls.