ECE 2025 Fall 2003 HW # 11

(11.1) $y(t) = x(t+1) + 2x(t) + x(t-2)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

(a) $h(t) = \delta(t+1) + 2\delta(t) + \delta(t-2)$

(b) $$H(j\omega) = \int_{-\infty}^{\infty} \left[\delta(t+1) + 2\delta(t) + \delta(t-2)\right] e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} \delta(t+1)\, e^{-j\omega t}\,dt + \int_{-\infty}^{\infty} 2\delta(t)\, e^{-j\omega t}\,dt + \int_{-\infty}^{\infty} \delta(t-2)\, e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} \delta(t+1)\, e^{-j\omega(-1)}\,dt + \int_{-\infty}^{\infty} 2\delta(t)\, e^{-j\omega(0)}\,dt + \int_{-\infty}^{\infty} \delta(t-2)\, e^{-j\omega(2)}\,dt$$

$$= e^{j\omega} + 2\,\cancel{e} + e^{-j2\omega}$$

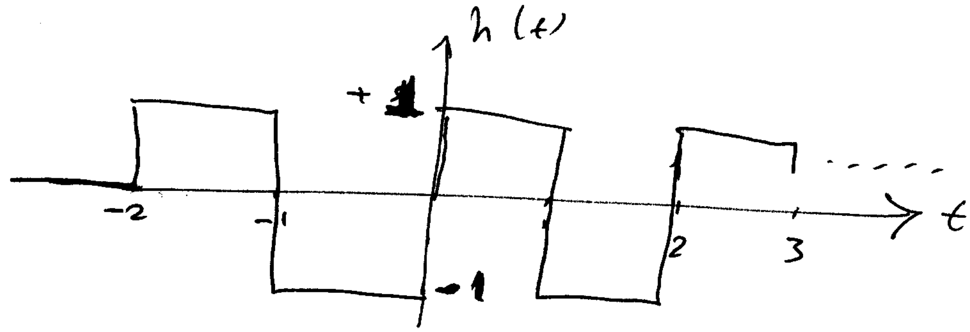
(c) $$y(t) = e^{j\omega(t+1)} + 2e^{j\omega t} + e^{j\omega(t-2)}$$

$$= e^{j\omega t}\left[e^{j\omega} + 2 + e^{-j2\omega}\right]$$

$$= H(j\omega)\, e^{j\omega t}.$$

11.3 $h(t) = \text{sign}[\sin(\pi t)]\, u(t+2)$

(a)



(b) NO. $\int_{-\infty}^{\infty} |h(t)| \not< \infty$

(c) $H_2(j\omega) = e^{-j\omega c} \Rightarrow h_2(t) = \delta(t-c)$

h(t) given above

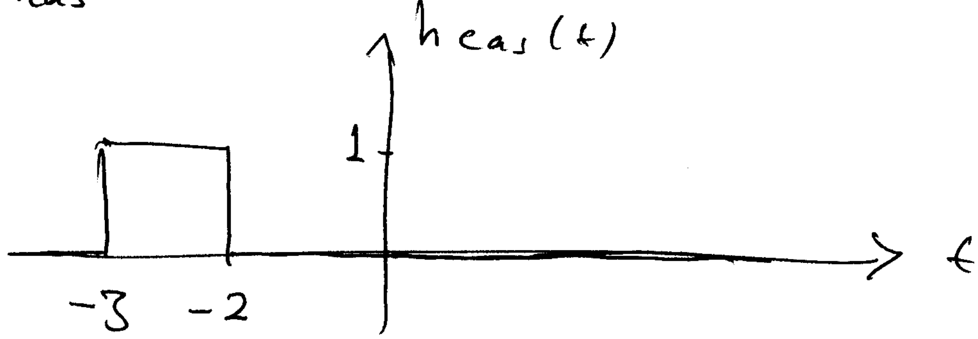
$\Rightarrow h_{eq}(t) = h_1(t) * h_2(t)$

h(t) of the cascade system

Remember that $x(t) * \delta(t - t_d) = x(t - t_d)$

$\Rightarrow c \geq 2$ to make the complete cascaded system causal.

(d) $h_{eas}(t) = h(t) * [\delta(t) + \delta(t+1)]$



PROBLEM 11.4 Solution:

The impulse response is $h(t) = \dfrac{\sin 400\pi t}{\pi t}$

(a) Frequency Response will be (from Fourier Transform table)

$$H(j\omega) = u(\omega + 400\pi) - u(\omega + 400\pi)$$

which is a rectangle of height one, over the region $|\omega| < 400\pi$.

(b) For the half-wave rectified sinusoid, $x(t) = \begin{cases} \sin(\omega_0 t) & \text{for } 0 \le t < T_0/2 \\ 0 & \text{for } T_0/2 \le t < T_0 \end{cases}$

The fundamental frequency is $\omega_0 = 2\pi/T_0$, and the Fourier Series coefficients are given by:

$$a_k = \begin{cases} \dfrac{-1}{\pi(k^2-1)} & \text{for } k \text{ even} \\ -j\dfrac{k}{4} & \text{for } k = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

When $\omega_0 = 150\pi$, the Fourier Transform is

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - 150\pi k)$$

In order to find $y(t)$, find its Fourier transform, $Y(j\omega) = H(j\omega)X(j\omega)$ and realize that multiplication by $H(j\omega)$ will eliminate all the impulses in $X(j\omega)$ except for those between $\omega = -400\pi$ and $\omega = +400\pi$. Since $|\omega_0 k| = |150\pi k| < 400\pi \Longrightarrow |k| < 2\frac{2}{3}$, we see that only the indices $k = -2, -1, 0, 1, 2$ will be included in the output:

$$\begin{aligned} Y(j\omega) = H(j\omega)X(j\omega) &= \sum_{k=-2}^{2} 2\pi a_k \delta(\omega - 150\pi k) \\ &= 2\pi a_{-2}\delta(\omega + 300\pi) + 2\pi a_{-1}\delta(\omega + 150\pi) + 2\pi a_0 \delta(\omega) + 2\pi a_1 \delta(\omega - 150\pi) + 2\pi a_2 \delta(\omega - 300\pi) \\ &= 2\pi(-1/3\pi)\delta(\omega + 300\pi) + 2\pi(j/4)\delta(\omega + 150\pi) + 2\pi(1/\pi)\delta(\omega) \\ &\qquad + 2\pi(-j/4)\delta(\omega - 150\pi) + 2\pi(-1/3\pi)\delta(\omega - 300\pi) \end{aligned}$$

Taking the inverse transform we obtain

$$\begin{aligned} y(t) &= (-1/3\pi)e^{-j300\pi t} + (j/4)e^{-j150\pi t} + (1/\pi) + (-j/4)e^{j150\pi t} + (-1/3\pi)e^{j300\pi t} \\ &= \frac{1}{\pi} + \frac{1}{2}\cos(150\pi t - \pi/2) - \frac{2}{3\pi}\cos(300\pi t) \end{aligned}$$

11.5 $h(t) = u(t+100) - u(t-100)$

a) $H(j\omega) = \dfrac{\sin(100\,\omega)}{\omega/2}$ from FT table

b) $x(t) = 3 + \cos(0.01\pi t) + \frac{1}{4}\cos(0.02\pi t) + \frac{1}{9}\cos(0.03\pi t) + \frac{1}{16}\cos(0.04\pi t)$

$@\ \omega = 0 \Rightarrow H(j0) = \lim_{\omega \to 0} \dfrac{\sin(100\,\omega)}{\omega/2} = 200$

$@\ \omega = \pi \dfrac{k}{100} \Rightarrow H\left(j\dfrac{\pi k}{100}\right) = \dfrac{\sin(\pi k)}{\left(\frac{\pi k}{100}\right)/2} = 0 \quad k = 1,2,3,4$

Thus, $y(t) = 3(200) = 600$

c) $x(t) = x_1(t) + x_2(t)$

$x_1(t) = \dfrac{1600}{\sqrt{2}}\cos\left(\dfrac{25}{10000}\pi t\right)$, $x_2(t) = 800\,\delta(t-50)$

$x_1(t) \rightarrow \boxed{h(t)} \rightarrow y_1(t)$

$x_2(t) \rightarrow \boxed{h(t)} \rightarrow y_2(t)$

$y_1(t) = ?$

$@\ \omega = \dfrac{25}{10000}\pi \Rightarrow H\left(j\dfrac{25}{10000}\pi\right) = \dfrac{\sin\left(\frac{\pi}{4}\right)}{0.00125\pi} = \dfrac{\sqrt{2}}{0.0025\pi}$

$\Rightarrow y_1(t) = \dfrac{1600}{\sqrt{2}} \dfrac{\sqrt{2}}{0.0025\pi}\cos(0.0025\pi t) = \dfrac{64 \times 10^4}{\pi}\cos(0.0025\pi t)$

11.5 Cont'd

$y_2(t) = ?$

$y_2(t) = [u(t+100) - u(t-100)] * 800\,\delta(t-50)$

$= 800[u(t+50) - u(t-150)]$

$\Rightarrow y(t) = y_1(t) + y_2(t)$ (LTI)

$= \frac{64 \times 10^4}{\pi} \cos(0.0025\pi t) + 800[u(t+50) - u(t-150)]$

PROBLEM 11.6 Solution:

The input signal is $x(t) = 200\cos(40t)$, and the corresponding output signal is $y(t) = 4\cos(40t+\phi)$, but the phase is unknown. The impulse response of the system is $h(t) = e^{-at}u(t)$, but a is unknown.

(a) The value of a can be found from the magnitude of the frequency response. For the given impulse response, the form of the frequency response is

$$H(j\omega) = \frac{1}{a + j\omega}$$

When the input signal is a sinusoid of frequency $\omega = 40$, the amplitude and phase of the output signal are determined by the value of the frequency response at $\omega = 40$. In particular, the amplitude of the output is the amplitude of the input multiplied by the magnitude of $H(j40)$. Thus

$$4 = 200|H(j40)| = 200\left|\frac{1}{a+j40}\right|$$

We can solve for a by squaring both sides

$$4^2 = (200)^2\left|\frac{1}{a+j40}\right|^2 = \frac{(200)^2}{a^2+(40)^2}$$

Which gives

$$a^2 + (40)^2 = (50)^2 \qquad \Longrightarrow \qquad a = 30$$

(b) To find the phase, we use the value of a from part (a) and calculate the phase of $H(j\omega)$ at $\omega = 40$.

$$\angle H(j\omega) = \angle\left\{\frac{1}{30+j\omega}\right\} = -\angle\{30+j40\} = -0.927 \text{ rads} = -0.295\pi \text{ rads} = -53.13^\circ$$

Since the input signal has zero phase, the output signal is $y(t) = 4\cos(40t - 0.295\pi)$

(c) This part is unrelated to parts (a) and (b). When $x(t) = 2^t$, we suspect that the Fourier transform does not exist because 2^t blows up, i.e., $\lim\limits_{t\to\infty} 2^t \to \infty$. It will be sufficient to show that the Fourier transform does not exist at one value of ω. Let's pick $\omega = 0$. The Fourier integral becomes:

$$\begin{aligned}
X(j0) &= \int_{-\infty}^{\infty} x(t)e^{-j0t}dt = \int_{-\infty}^{\infty} 2^t dt \\
&= \lim_{q\to\infty}\int_{-q}^{q} e^{(\ln 2)t}dt = \lim_{q\to\infty}\left.\frac{e^{(\ln 2)t}}{\ln 2}\right|_{-q}^{q} = \lim_{q\to\infty}\frac{e^{(\ln 2)q} - e^{(\ln 2)(-q)}}{\ln 2} \\
&= \frac{\lim\limits_{q\to\infty} e^{(\ln 2)q} - \lim\limits_{q\to\infty} e^{-(\ln 2)q}}{\ln 2} = \frac{\lim\limits_{q\to\infty} e^{(\ln 2)q} - 0}{\ln 2} \to \infty
\end{aligned}$$

Thus, the Fourier integral does not exist at $\omega = 0$, so $x(t) = 2^t$ does not have a Fourier transform.