HW#12 ECE2025 Fall 2003

Note: You could also get the answer to part (b) by applying the "multiplication in time corresponds to convolution in frequency" property and convolving the FT of sin(4*pi*t) with the FT of sin(50*pi*t) - try it and make sure you get the same answer! That may be faster on an exam. (Hint. Hint.)

Problem 12.1.

P-11.6(b) Find the FT of $x(t) = \sin(4\pi t)\sin(50\pi t)$

First, use trigonometric identity $\sin\alpha \sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$

We find, $x(t) = \frac{1}{2}\cos(46\pi t) - \frac{1}{2}\cos(54\pi t)$

$$X(j\omega) = \frac{\pi}{2}\delta(\omega - 46\pi) + \frac{\pi}{2}\delta(\omega+46\pi) - \frac{\pi}{2}\delta(\omega-54\pi) - \frac{\pi}{2}\delta(\omega+54\pi)$$

P-11.6(c) $x(t) = \frac{\sin 4\pi t}{\pi t}\sin(50\pi t)$

FT of $\frac{\sin 4\pi t}{\pi t}$ is $u(\omega+4\pi) - u(\omega-4\pi)$

FT of $\sin(50\pi t)$ is $-j\pi\delta(\omega-50\pi) + j\pi\delta(\omega+50\pi)$

Recall that

$$y(t)p(t) \overset{F}{\longleftrightarrow} \frac{1}{2\pi} Y(j\omega) * P(j\omega)$$

Thus
$$X(j\omega) = \frac{1}{2\pi}(-j\pi)\cdot\left[u(\omega+4\pi-50\pi) - u(\omega-4\pi-50\pi)\right] + \frac{1}{2\pi}(j\pi)\left[u(\omega+4\pi+50\pi) - u(\omega-4\pi+50\pi)\right]$$

$$= \frac{-j}{2}\left[u(\omega-46\pi) - u(\omega-54\pi)\right] + \frac{j}{2}\left[u(\omega+54\pi) - u(\omega+46\pi)\right]$$

~~EA~~

Problem 12.2

P-11.8 (a) $X(j\omega) = \frac{e^{-3j\omega}}{2+j\omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} e^{-2(t-3)} \cdot u(t-3) = x(t)$

(b) $X(j\omega) = \frac{j\omega}{2+j\omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} x(t) = \frac{d}{dt}\left[e^{-2t}u(t)\right]$

$= -2e^{-2t}u(t) + e^{-2t}\delta(t) = e^{-2t}\left(-2u(t) + \delta(t)\right)$

(c) $X(j\omega) = \frac{j\omega}{2+j\omega}e^{-3j\omega}$: delay the answer in (b) by 3

or take the derivative of the answer in (a).

$x(t) = e^{-2(t-3)}\left(-2u(t-3) + \delta(t-3)\right)$

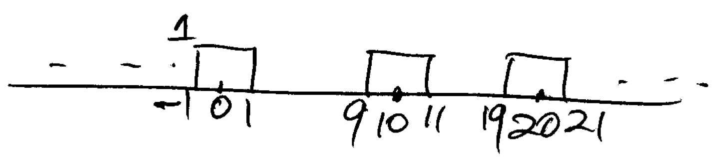
(d) $X(j\omega) = \left(\frac{2\sin\omega}{\omega}\right) \sum_{k=-\infty}^{\infty} \left(\frac{\pi}{5}\right) \delta\left(\omega - \frac{2\pi k}{10}\right)$

$\frac{2\sin\omega}{\omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} u(t+1) - u(t-1)$

$\sum_{k=-\infty}^{\infty} \left(\frac{\pi}{5}\right) \delta\left(\omega - \frac{2\pi k}{10}\right) \overset{\mathcal{F}^{-1}}{\longleftrightarrow} \sum_{n=-\infty}^{\infty} \delta(t - 10n)$

Convolution in time:

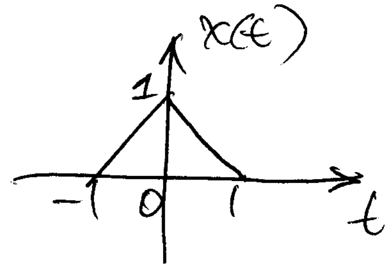
$x(t) = \sum_{n=-\infty}^{\infty} \left[u(t+1-10n) - u(t-1-10n)\right]$



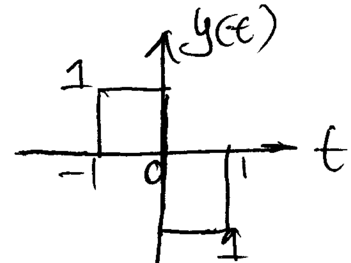
Problem 12.3

P-11.15

$$x(t) = \begin{cases} 1-|t|, & |t|<1 \\ 0, & \text{elsewhere} \end{cases}$$



Let $y(t) = \frac{dx(t)}{dt}$

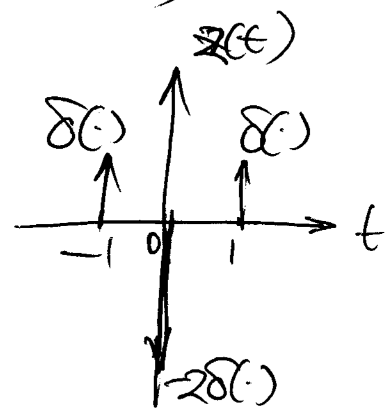


$= \left(u(t+1) - u(t)\right)$

$-\left(u(t) - u(t-1)\right) = u(t+1) - 2u(t) + u(t-1)$

Let $z(t) = \frac{dy(t)}{dt}$

$= \delta(t+1) - 2\delta(t) + \delta(t-1)$



(a) $Y(j\omega) = (j\omega)\, X(j\omega)$

$Y(j\omega) = \left(\pi\delta(\omega) + \frac{1}{j\omega}\right)e^{j\omega} - 2\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$

$+ \left(\pi\delta(\omega) + \frac{1}{j\omega}\right)e^{-j\omega}$

$= \pi\delta(\omega) + \frac{e^{j\omega}}{j\omega} - 2\pi\delta(\omega) - \frac{2}{j\omega} + \pi\delta(\omega) + \frac{e^{-j\omega}}{j\omega}$

$= \frac{2\cos\omega - 2}{j\omega}$

Therefore, $X(j\omega) = \frac{1}{j\omega} Y(j\omega) = \frac{2\cos\omega - 2}{(j\omega)^2} = \frac{2 - 2\cos\omega}{\omega^2}$

(b) $Z(j\omega) = e^{j\omega} - 2 + e^{-j\omega} = 2\cos\omega - 2$

$$Z(j\omega) = (j\omega)^2 X(j\omega) \Rightarrow X(j\omega) = -\frac{1}{\omega^2} Z(j\omega)$$

$$= \frac{2 - 2\cos\omega}{\omega^2}$$

Same answer as in (a)!

Problem 12.4

(a) $H(j\omega) = (j\omega) \cdot H_{lp}(j\omega)$, where

$$H_{lp}(j\omega) = \begin{cases} 1, & |\omega| < \omega_0 \\ 0, & \text{otherwise} \end{cases} \xleftrightarrow{\mathcal{F}^{-1}} h_{lp}(t) = \frac{\sin(\omega_0 t)}{\pi t}$$

$$h(t) = \frac{d}{dt} h_{lp}(t) = \frac{1}{\pi t^2}\left[\omega_0 t \cos(\omega_0 t) - \sin(\omega_0 t)\right]$$

When $t=0$, use L'Hopital's rule to find

$$h(0) = \lim_{t \to 0} \frac{1}{2\pi t}\left[\omega_0 \cos(\omega_0 t) - \omega_0^2 t \sin(\omega_0 t) - \omega_0 \cos(\omega_0 t)\right]$$

$$= \lim_{t \to 0} \frac{-\omega_0^2}{2\pi} \sin(\omega_0 t) = 0$$

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Problem 12.4 (a) codes:

```
w0=2*pi;
period=2*pi/w0;
t=-5*period:period/100:5*period;
h=1./(pi*t.^2).*(w0*t.*cos(w0*t)-sin(w0*t));
index=find(t==0);
h(index)=0;
plot(t,h); grid
xlabel('t'); ylabel('h(t)')
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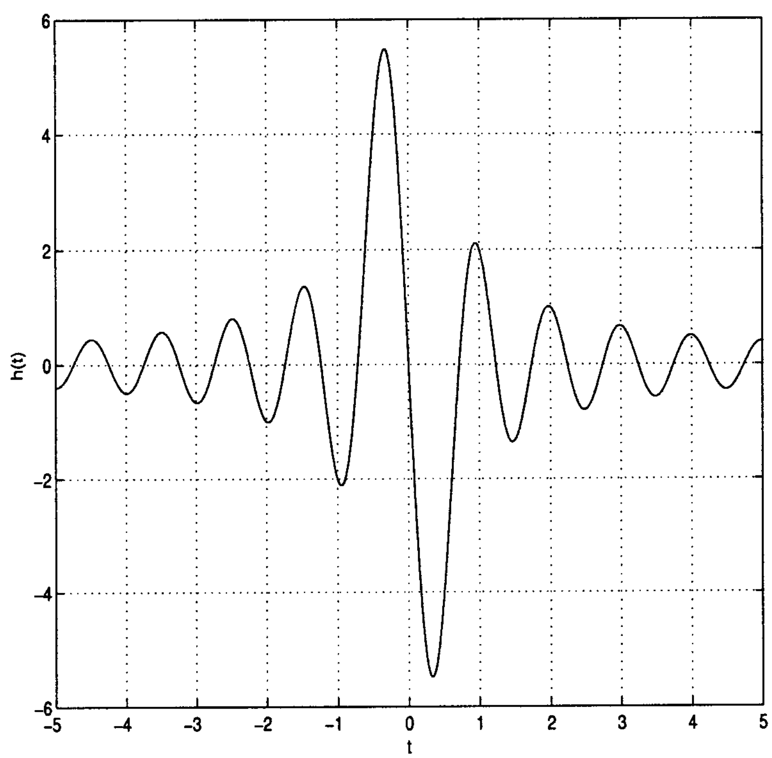


(b) $x(t) = A\exp(-t/T_2)\sin(\omega_0 t)u(t)$

Let $y(t) = A\cdot e^{-\frac{t}{T_2}}u(t) \xleftrightarrow{\mathcal{F}} Y(j\omega) = \frac{A}{\frac{1}{T_2}+j\omega}$

$x(t) = y(t)\sin(\omega_0 t) = \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)y(t)$

$X(j\omega) = \frac{1}{2j}\left[Y(j(\omega-\omega_0)) - Y(j(\omega+\omega_0))\right]$

$= \frac{A}{2j}\left[\frac{1}{\frac{1}{T_2}+j(\omega-\omega_0)} - \frac{1}{\frac{1}{T_2}+j(\omega+\omega_0)}\right]$

(c) $x(t) = e^{-8t}u(t) - e^{-8t}u(t-2)$

$= e^{-8t}u(t) - e^{-8(t-2)}u(t-2)\cdot e^{-16}$

$X(j\omega) = \frac{1}{8+j\omega} - \frac{e^{-2j\omega}\cdot e^{-16}}{8+j\omega} = \frac{1-e^{-16}e^{-2j\omega}}{8+j\omega}$

Problem 12.5

(a)(i) $x(t) = \frac{a}{\pi(a^2+t^2)} \xleftrightarrow{\mathcal{F}} X(j\omega) = e^{-a|\omega|}$

$h(t) = \frac{b}{\pi(b^2+t^2)} \xleftrightarrow{\mathcal{F}} H(j\omega) = e^{-b|\omega|}$

$y(t) = x(t) * h(t) \xleftrightarrow{\mathcal{F}} Y(j\omega) = X(j\omega)H(j\omega)$

$= e^{-(a+b)|\omega|}$

Hence, $y(t) = \frac{a+b}{\pi((a+b)^2+t^2)}$

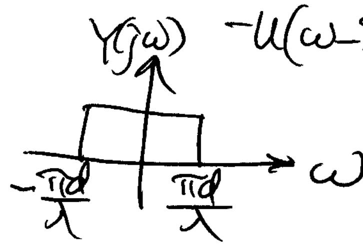
(ii) $y(t)$ has a form that is similar to that of $x(t)$.

(b)

$$x(t)=\left[\frac{\sin\left(\frac{\pi d}{\lambda}t\right)}{\frac{\pi d}{\lambda}t}\right]^2$$

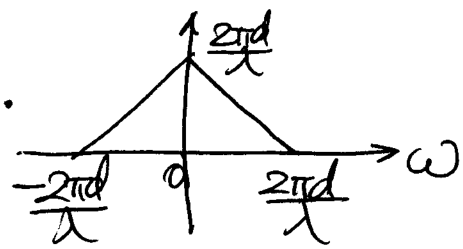
Let $x(t)=[y(t)]^2$, then

$$y(t)=\frac{\sin\left(\frac{\pi d}{\lambda}t\right)}{\pi t}\cdot\frac{\lambda}{d} \xleftrightarrow{F} Y(j\omega)=\frac{\lambda}{d}\cdot\left[u\left(\omega+\frac{\pi d}{\lambda}\right)-u\left(\omega-\frac{\pi d}{\lambda}\right)\right]$$



$$X(j\omega)=\frac{1}{2\pi}Y(j\omega)*Y(j\omega)$$

$$=\frac{1}{2\pi}\left(\frac{\lambda}{d}\right)^2\cdot$$



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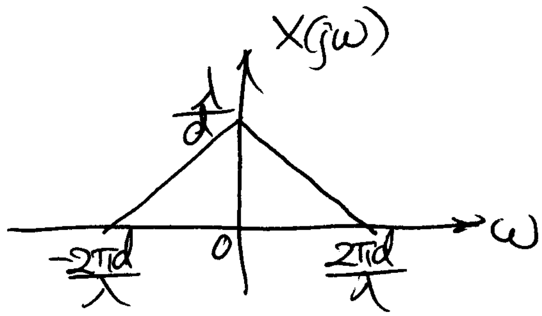


$$X(j\omega)=\begin{cases}\frac{\lambda}{d}-\frac{1}{2\pi}\cdot\frac{\lambda^2}{d^2}|\omega|, & |\omega|<\frac{2\pi d}{\lambda}\\ 0, & \text{otherwise}\end{cases}$$

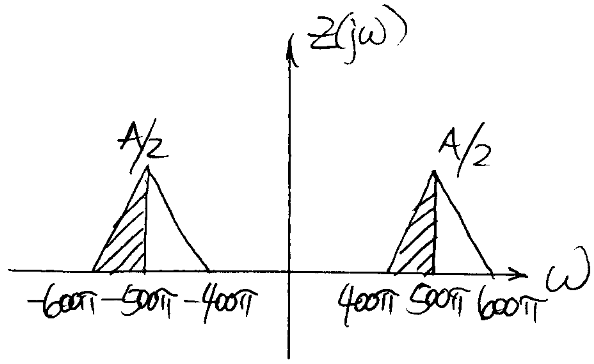
Problem 12.6

Let $z(t) = x(t)\cos(\omega_c t)$

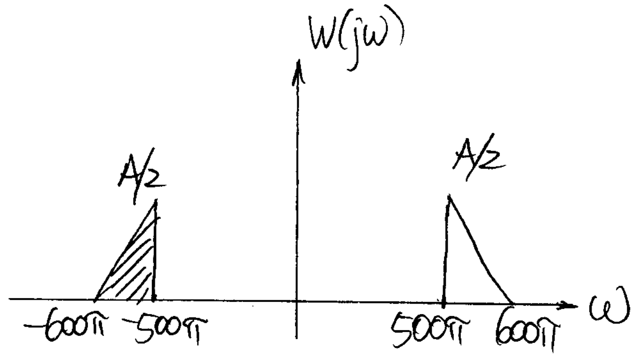
$w(t) = z(t) * h_{1}(t)$

$v(t) = w(t)\cos(\omega_c t)$

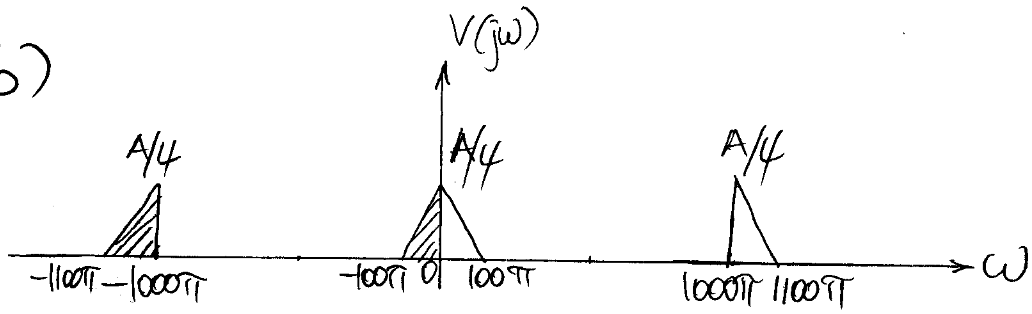
$y(t) = v(t) * h_d(t)$



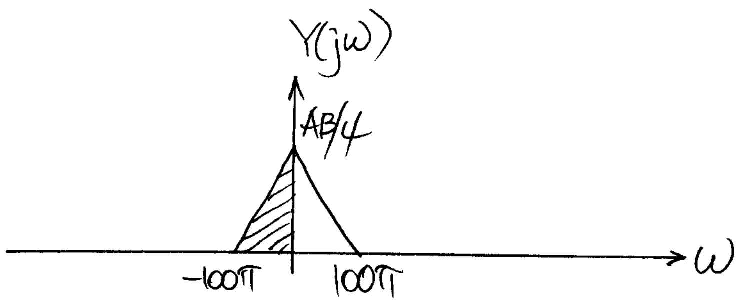
(a)



(b)



(c)



Want $y(t) = x(t)$

$\Longleftrightarrow Y(j\omega) = X(j\omega)$

Hence, $AB/4 = A$

$\Rightarrow B = 4$

(d) Recall that in DSBAM transmission, the transmitted signal occupies twice the bandwidth of the information signal.

In part (a), we see that in SSB transmission, the transmitted signal $w(t)$ occupies the same bandwidth as the information signal $x(t)$. Therefore, SSB is more spectrally effecient.

Problem 12.7

Exercise 12.8 We have:

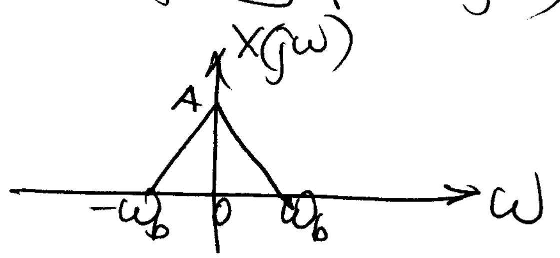
$$y(t) = x(t)\cos(\omega_c t)$$

$$w(t) = y(t)\cos(\omega_c t + \phi) = \frac{e^{j\phi}}{2} y(t) e^{j\omega_c t} + \frac{e^{-j\phi}}{2} y(t) e^{-j\omega_c t}$$

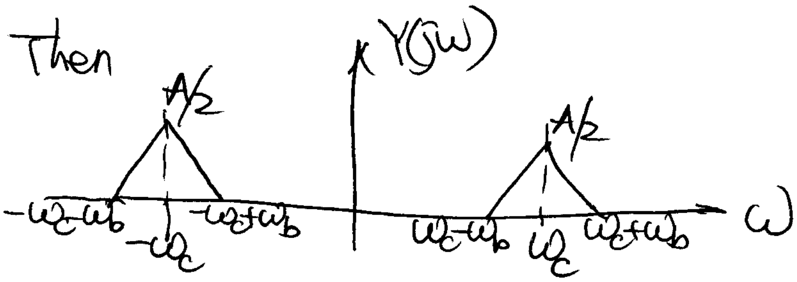
$$\Rightarrow W(j\omega) = \frac{e^{j\phi}}{2} Y(j(\omega - \omega_c)) + \frac{e^{-j\phi}}{2} Y(j(\omega + \omega_c))$$

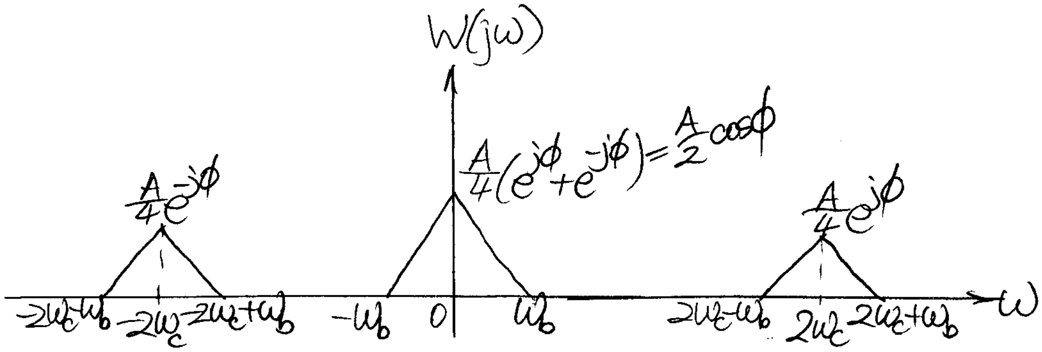
$$v(t) = w(t) * h(t)$$

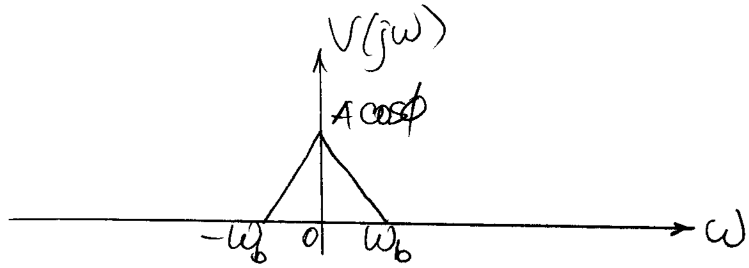
Assume the following for $X(j\omega)$:



Then







(assume that the LPF has gain G=2).

Since $V(j\omega) = \cos(\phi)\, X(j\omega)$

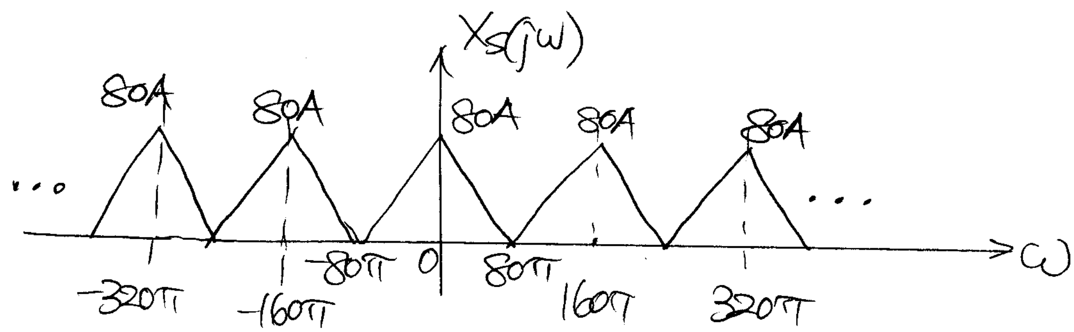
we have $v(t) = \cos(\phi)\, x(t)$.

Problem 12.9

P-12.12

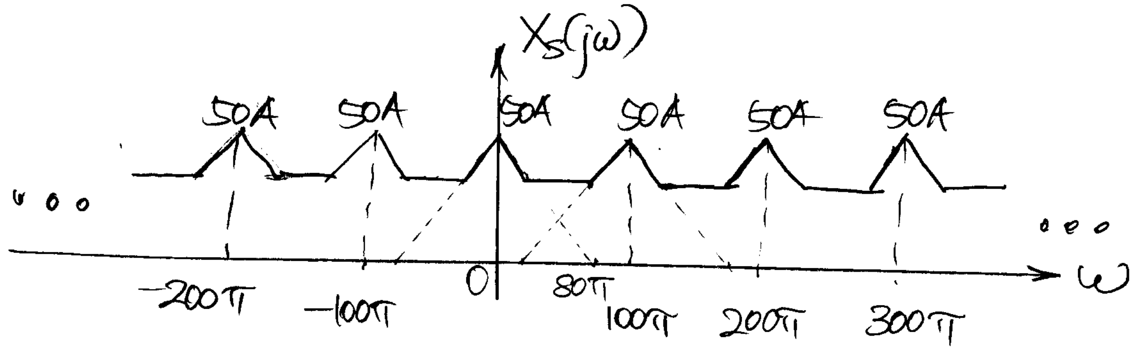
(a) Recall $X_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_c(j(\omega - k\omega_s))$

Nyquist rate: $\omega_s = 2 \times 80\pi = 160\pi = \frac{2\pi}{T_s} \Rightarrow \frac{1}{T_s} = 80$



(b) $\omega_s = 100\pi < 2 * 80\pi$: Aliasing will occur

$\omega_s = \frac{2\pi}{T_s} \Rightarrow \frac{1}{T_s} = 50$



(c)