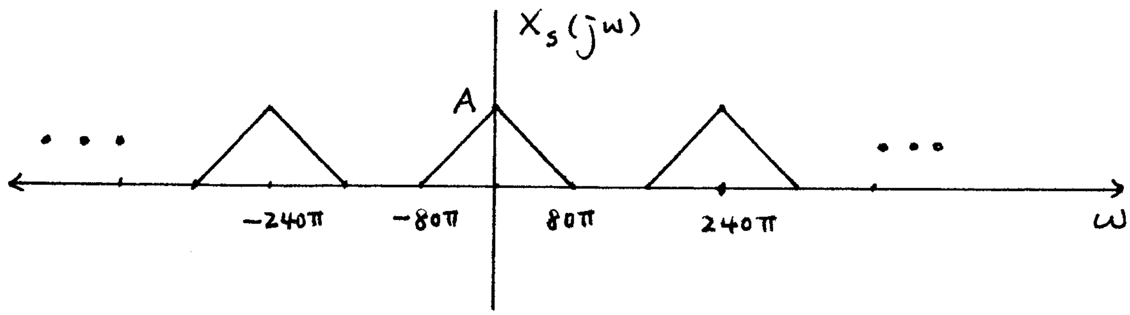
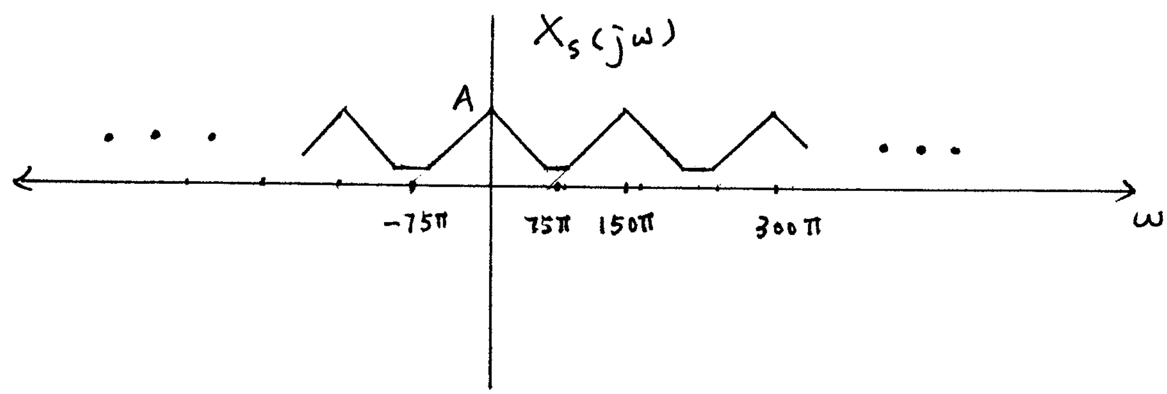
Problem 12.1

(a) The Nyquist rate is $80\pi \times 2 = 160\pi$

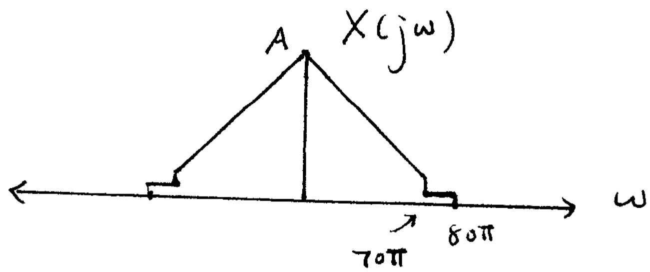
Let $\omega_s = 1.5 \times 160\pi = 240\pi$. Then



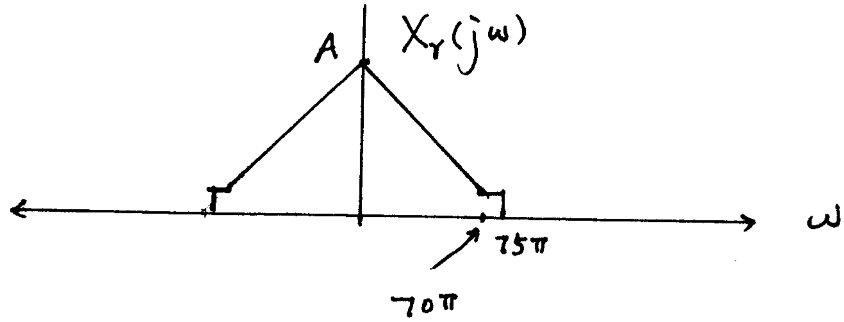
(b) Let $\omega_s = 150\pi$



aliasing occurs because a different $X(j\omega)$ may result in the same $X_s(j\omega)$ as above. for example, $X(j\omega)$ given as follows (shown with larger scale)



(c)



(a) Use long division as in Example 8.11

$$
\begin{array}{r}
-1.3 \\
0.77z^{-1}+1 \;\big)\overline{\,-z^{-1}+1\,} \\
\underline{-z^{-1}-1.3} \\
2.3
\end{array}
$$

QUOTIENT

REMAINDER

NOTE: $\frac{1}{0.77} = 1.2987 \approx 1.3$

$$H_a(z) = -1.3 + \frac{2.3}{1+0.77z^{-1}}$$

Use z-Transform pair: $\frac{b}{1-az^{-1}} \longleftrightarrow b\,a^n u[n]$

$$h_a[n] = -1.3\,\delta[n] + 2.3\,(-0.77)^n u[n]$$

(b) Use long division:

$$H_b(z) = \frac{1+0.8z^{-1}}{1-0.9z^{-1}} = -\frac{8}{9} + \frac{17/9}{1-0.9z^{-1}}$$

$$\Rightarrow h_b[n] = -\frac{8}{9}\,\delta[n] + \frac{17}{9}(0.9)^n u[n]$$

(c) Use the shifting property: $z^{-n_0}H(z) \longleftrightarrow h[n-n_0]$

$$H_c(z) = z^{-2}\left(\frac{1}{1-0.9z^{-2}}\right) = z^{-2}G_c(z)$$

$g_c[n] = (0.9)^n u[n].$

$$h_c[n] = g_c[n-2] = (0.9)^{n-2}u[n-2]$$

This signal starts at $n=2$

(d) This is an FIR filter.

Invert term by term:

$$H_d(z) = 1 - z^{-1} + 2z^{-3} - 3z^{-4}$$

$\delta[n]$ $\quad -\delta[n-1]$ $\quad 2\delta[n-3]$ $\quad -3\delta[n-4]$

$$h_d[n] = \delta[n] - \delta[n-1] + 2\delta[n-3] - 3\delta[n-4]$$

Problem 12.3:

Characterize each system ($S_1 \rightarrow S_7$)

S_1: $H_1(z) = \dfrac{\frac{1}{2} + \frac{1}{2}z^{-1}}{1 - 0.9z^{-1}}$ $\Rightarrow$ pole at $z = 0.9$, zero at $z = -1$

$H_1(e^{j\hat{\omega}})$ is a LPF with a null at $\hat{\omega} = \pi$.

S_2: $H_2(z) = \dfrac{9 + 10z^{-1}}{1 + 0.9z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$, zero at $z = -10/9$

$H_2(e^{j\hat{\omega}})$ is an all-pass filter

S_3: $H_3(z) = \dfrac{\frac{1}{2}(1 - z^{-1})}{1 + 0.9z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$, zero at $z = 1$

$H_3(e^{j\hat{\omega}})$ is a HPF with a null at $\hat{\omega} = 0$.

S_4: $H_4(z) = \frac{1}{4}(1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})$

$= \frac{1}{4}(1 + z^{-1})^4$ $\Rightarrow$ 4 zeros at $z = -1$

$H_4(e^{j\hat{\omega}})$ is a LPF with null at $\hat{\omega} = \pi$.

DC value: $H_4(e^{j0}) = 4$.

S_5: $H_5(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} = \dfrac{1 + z^{-5}}{1 + z^{-1}}$

has 4 zeros around the unit circle.
No zero at $z = -1$; others at $e^{j(2\pi k/5 - \pi/5)}$

$H_5(e^{j\hat{\omega}})$ is a HPF with nulls at $\hat{\omega} = \pm\frac{\pi}{5}, \pm\frac{3\pi}{5}$

S_6: $H_6(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \dfrac{1 - z^{-4}}{1 - z^{-1}}$

has 3 zeros around the unit circle at $z = \pm j, -1$

$H_6(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{2}, \pi$

S_7: $H_7(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} = \dfrac{1 - z^{-6}}{1 - z^{-1}}$

has 5 zeros around the unit circle at $z = e^{j\pi k/3}$

$H_7(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{3}, \pm 2\frac{\pi}{3}, \pi$

PZ#1: S_7 PZ#3: S_2 PZ#5: S_5

PZ#2: S_1 PZ#4: S_6 PZ#6: S_3

Problem 12.4:

Characterize each system ($S_1 \to S_7$)

S_1: $H_1(z) = \dfrac{\frac{1}{2} + \frac{1}{2} z^{-1}}{1 - 0.9 z^{-1}}$ $\Rightarrow$ pole at $z = 0.9$
zero at $z = -1$

$H_1(e^{j\hat{\omega}})$ is a LPF with a null at $\hat{\omega} = \pi$.

S_2: $H_2(z) = \dfrac{9 + 10 z^{-1}}{1 + 0.9 z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$
zero at $z = -10/9$

$H_2(e^{j\hat{\omega}})$ is an all-pass filter

S_3: $H_3(z) = \dfrac{\frac{1}{2}(1 - z^{-1})}{1 + 0.9 z^{-1}}$ $\Rightarrow$ pole at $z = -0.9$
zero at $z = 1$

$H_3(e^{j\hat{\omega}})$ is a HPF with a null at $\hat{\omega} = 0$.

S_4: $H_4(z) = \frac{1}{4}(1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})$
$= \frac{1}{4}(1 + z^{-1})^4$ $\Rightarrow$ 4 zeros at $z = -1$

$H_4(e^{j\hat{\omega}})$ is a LPF with null at $\hat{\omega} = \pi$.
DC value: $H_4(e^{j0}) = 4$.

S_5: $H_5(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} = \dfrac{1 + z^{-5}}{1 + z^{-1}}$

has 4 zeros around the unit circle.
No zero at $z = -1$; others at $e^{j(2\pi k/5 - \pi/5)}$

$H_5(e^{j\hat{\omega}})$ is a HPF with nulls at $\hat{\omega} = \pm\frac{\pi}{5}, \pm\frac{3\pi}{5}$

S_6: $H_6(z) = 1 + z^{-1} + z^{-2} + z^{-3} = \dfrac{1 - z^{-4}}{1 - z^{-1}}$

has 3 zeros around the unit circle at $z = \pm j, -1$

$H_6(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{2}, \pi$

S_7: $H_7(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} = \dfrac{1 - z^{-6}}{1 - z^{-1}}$

has 5 zeros around the unit circle at $z = e^{j\pi k/3}$

$H_7(e^{j\hat{\omega}})$ is a LPF with nulls at $\hat{\omega} = \pm\frac{\pi}{3}, \pm 2\frac{\pi}{3}, \pi$

(A) S_1	(C) S_6	(E) S_5
(B) S_3	(D) S_2	(F) S_4

Problem 12.5

Let $a = 0.25$

(a)

n	$n<0$	0	1	2	3	4	5	6
$\delta[n]$	0	1	0	0	0	0	0	0
$h[n-2]$	0	0	0	1	0	a	0	a^2
$h[n]$	0	1	0	a	0	a^2	0	a^3

$$\Rightarrow \quad h[n] = \begin{cases} a^{n/2} u[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

(b) $y[n] = 0.25\, y[n-2] + x[n]$

$$\Rightarrow \quad Y(z) = 0.25\, z^{-2}\, Y(z) + X(z)$$

$$\Rightarrow \quad (1 - 0.25\, z^{-2})\, Y(z) = X(z)$$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.25\, z^{-2}}$$

$$= \frac{z^2}{z^2 - 0.25}$$

poles @ $\pm\sqrt{0.25}$

zeros @ $0, 0$

(c) First solve it using Z-transform

$$Y(z) = H(z)\,X(z) = \frac{1}{1-0.25z^{-2}} \cdot \frac{1}{1+z^{-1}}$$

$$= \frac{Az^{-1}+B}{1-0.25z^{-2}} + \frac{C}{1+z^{-1}}$$

where $A = \frac{1}{3}$, $B = -\frac{1}{3}$, $C = \frac{4}{3}$

Recall $\frac{1}{1-az^{-1}} \leftrightarrow a^n u[n]$

$$\frac{1}{1-az^{-2}} \leftrightarrow \begin{cases} \sqrt{a}^{\,n}\, u[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Define

$$w[n] = \begin{cases} \sqrt{0.25}^{\,n}\, u[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Thus,

$$y[n] = \frac{1}{3} w[n-1] - \frac{1}{3} w[n] + \frac{4}{3}(-1)^n u[n]$$

Second, find $y[n]$ by directly evaluating the difference equation.

n	$n<0$	0	1	2	3	4	5	6	7
$x[n]$	0	1	-1	1	-1	1	-1	1	-1
$y[n-2]$	0	0	0	1	-1	$1+a$	$-(1+a)$		
$y[n]$	0	1	-1	$1+a$	$-(1+a)$	$1+a+a^2$	$-(1+a+a^2)$		

$$\Rightarrow \quad y[n] = \begin{cases} \dfrac{1-a^{n/2+1}}{1-a}\, u[n], & n \text{ even} \quad (a=0.25) \\[2ex] -\,y[n-1] & n \text{ odd} \end{cases}$$

(d)