GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2004
Lab #10: GUIs for Continuous-Time Signals & Systems

Date: 29-Mar – 1-Apr 2004

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time. You **MUST** complete the online Pre-Post-Lab exercise on Web-CT at the beginning of your scheduled lab session. You can use MATLAB and also consult your lab report or any notes you might have, but you cannot discuss the exercises with any other students. You will have approximately 20 minutes at the beginning of your lab session to complete the online Pre-Post-Lab exercise. The Pre-Post-Lab exercise for this lab includes some questions about concepts from the previous Lab report as well as questions on the Pre-Lab section of this lab.

The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. After completing the warm-up section, turn in the verification sheet to your TA.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports but the submitted work should be original and it should be your own work.

The lab report for this week will be an **Informal Lab Report.** It is only necessary to turn in Section 4 as this week's lab report. The report will **due the next time your lab meets: 5–8 April**.

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1 Introduction

This lab concentrates on the use of three MATLAB GUIs for convolution:

1. **`cconvdemo:`** GUI for continuous-time convolution.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \qquad (1)$$

2. **`CLTIdemo:`** GUI for continuous-time filtering of sinusoids with a linear, time-invariant (LTI) system

$$y(t) = A|H(j\omega_1)|\cos(\omega_1 t + \phi + \angle H(j\omega_1))$$

   when the input signal is a sinusoid, $x(t) = A\cos(\omega_1 t + \phi)$.

3. **`FseriesDemo:`** GUI for continuous-time Fourier Series.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

   where $x(t)$ is a periodic signal and ω_0 is the fundamental frequency, $\omega_0 = 2\pi/T$.

Each one of these demos illustrates an important point about the behavior of a continuous-time signals and systems. They also provide a convenient way to visualize the output of a continuous-time LTI systems.

All of these demos are available in the *SP-First* Toolbox, or they can be downloaded from the following web page:

`http://users.ece.gatech.edu/mcclella/matlabGUIs/index.html`

2 Pre-Lab: Run the GUIs

Several GUIs have been introduced during lectures over the past few weeks. The first objective of this lab is to demonstrate usage of several GUIs.

2.1 Continuous-Time Convolution Demo

In this demo, you can select an input signal $x(t)$, as well as the impulse response of an **ANALOG** filter $h(t)$. Then the demo shows the "flipping and shifting" used when a convolution integral is performed. Figure 1 shows the interface for the `cconvdemo` GUI.

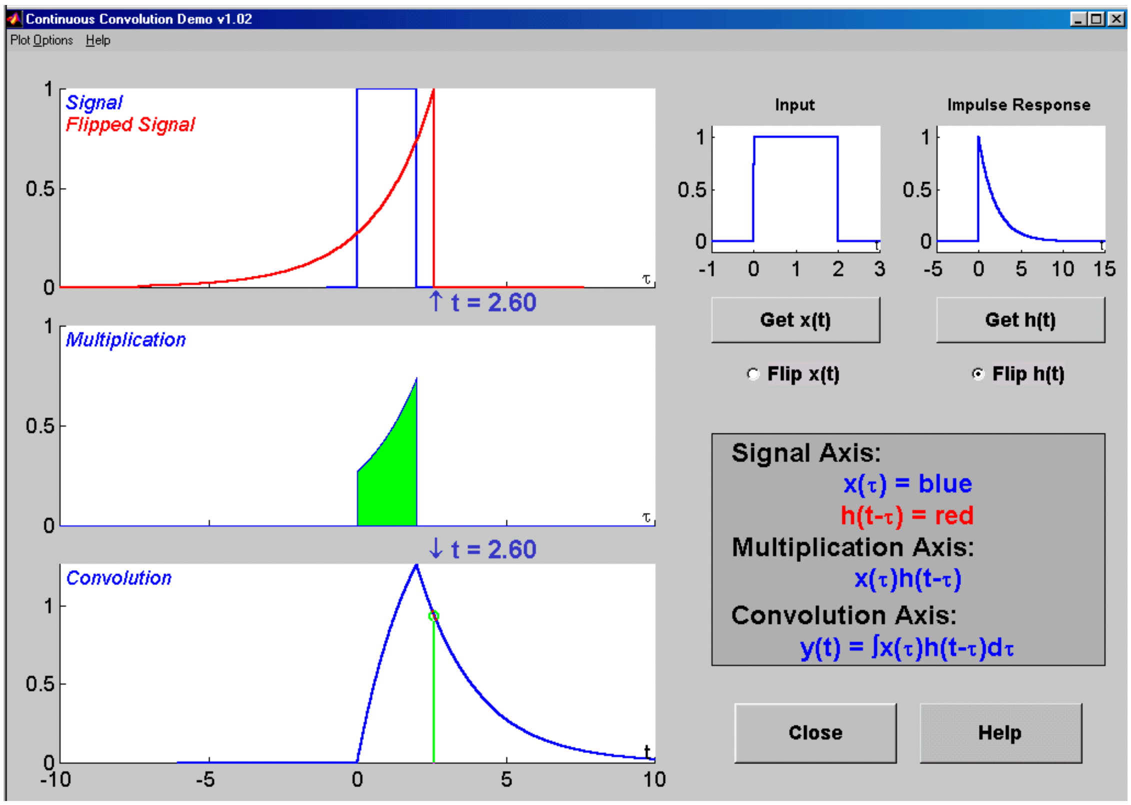

Figure 1: Interface for the continuous-time convolution GUI **`cconvdemo`**.

In the Pre-Lab, you should perform the following steps with the `cconvdemo` GUI.

(a) Set the input to a 4-second pulse $x(t) = u(t) - u(t-4)$.

(b) Set the filter's impulse response to a shifted impulse, i.e., $h(t) = \delta(t-3)$. Use the GUI to produce the output signal.

(c) Use the GUI to produce the output signal. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution.

(d) Set the input to a different shifted impulse, i.e., $x(t) = \delta(t-2)$. Use the GUI to produce the output signal. Notice that when flipping and sliding that there is only one time where the signals overlap.

(e) Compare the outputs from parts (c) and (d). Use properties of the impulse signal to explain the different outputs.

2.2 Sinusoidal Response (CLTIdemo)

In this demo, you can select an input signal that is a sinusoid, and see the change created by the frequency response. This demo reinforces the concept that "sinusoid in gives sinusoid out." Figure 2 shows the interface for the `CLTIdemo`. We know that if the input to an LTI continuous-time system is a sinusoid of

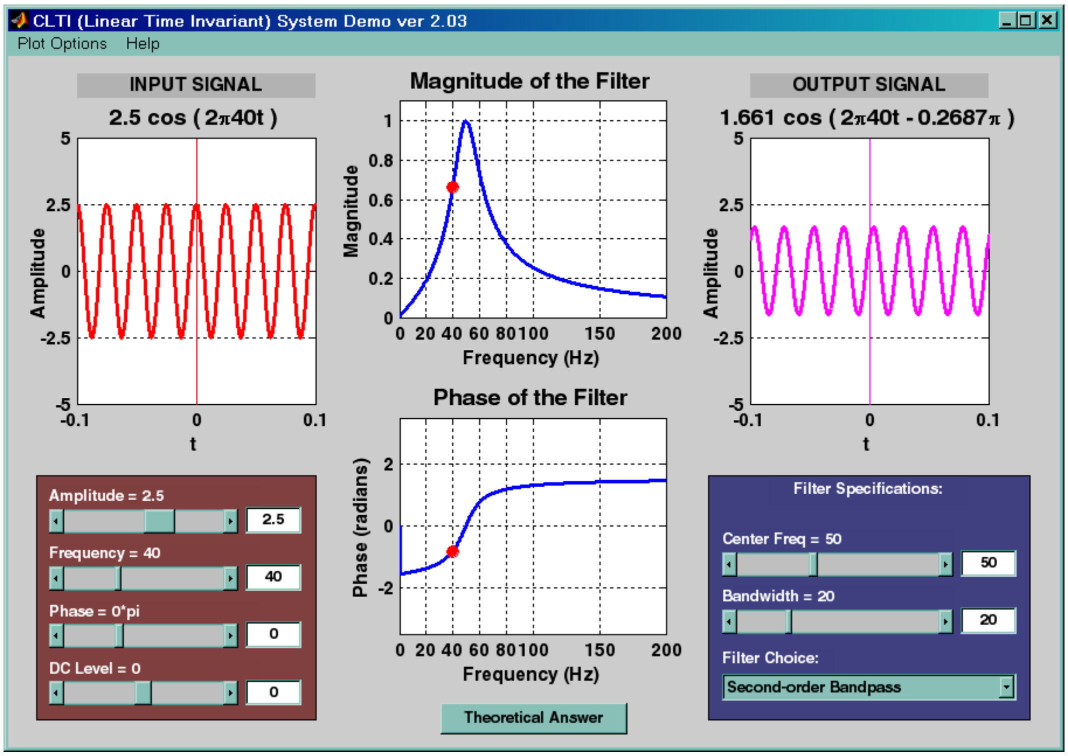

Figure 2: `CLTIdemo` interface for continuous-time frequency response.

the form

$$x(t) = A + B\cos(\omega_1 t + \phi) \qquad -\infty < t < \infty \tag{2}$$

then the corresponding output is also a sinusoid:

$$y(t) = AH(j0) + B|H(j\omega_1)|\cos\left(\omega_1 t + \phi + \angle H(j\omega_1)\right) \qquad -\infty < t < \infty, \tag{3}$$

where $H(j\omega)$ is the *frequency response* of the continuous-time LTI system. The **`CLTIdemo`** GUI illustrates this for a variety of simple analog filters.

(a) Use the `CLTIdemo` GUI to find the output of a first-order lowpass filter by selecting "First-Order Lowpass" from the menu and setting the cutoff frequency to 30 Hz. Recall that the frequency response of this lowpass filter is

$$H(j\omega) = \frac{1}{j\omega + a} \tag{4}$$

where a is the cutoff frequency in rads/sec.

(b) Set the input to

$$x(t) = 1.0 + \cos(20\pi t).$$

Look at the output and compare its amplitude and phase to the input amplitude and phase. Click the box labeled "Theoretical Answer" to see a formula for the output $y(t)$.
Note: The GUI input frequencies are in hertz, which is $f = \omega/(2\pi)$; ω would have units of rad/s.

(c) Keeping the DC level and the amplitude of the cosine the same, use the slider to increase the input frequency and observe the change in the output. Keep increasing the slider until the frequency is $\omega = 80\pi$ rad/s (or $f = 40$ Hz). Compare the output in this case to the output at the original frequency of $\omega = 20\pi$. If you were to describe the output as having a "ripple", does the ripple increase or decrease as ω increases?

(d) Repeat the previous part with the filter set to "Ideal Lowpass" with a cutoff frequency of 30 Hz. Start with the input signal from part (a).

(e) Set the frequency of the input back to $\omega = 20\pi$ and change the filter to "First-Order Highpass" with a cutoff frequency of 30 Hz. Observe the output as the frequency is increased. What is the DC component of the output? Does the amplitude of the output sinusoid get bigger or smaller as the frequency is increased?

(f) Convince yourself that the following frequency response is a first-order HPF:

$$H(j\omega) = \frac{j\omega}{j\omega + b}$$

where the parameter b is the cutoff frequency of the HPF in rad/s.

2.3 Spectrum from Fourier Series

Use the `FSeriesDemo` GUI (Fig. 3) to show the spectrum for a 50% duty cycle square wave. Notice that the GUI will also show the resynthesized signal for a finite number of coefficients.

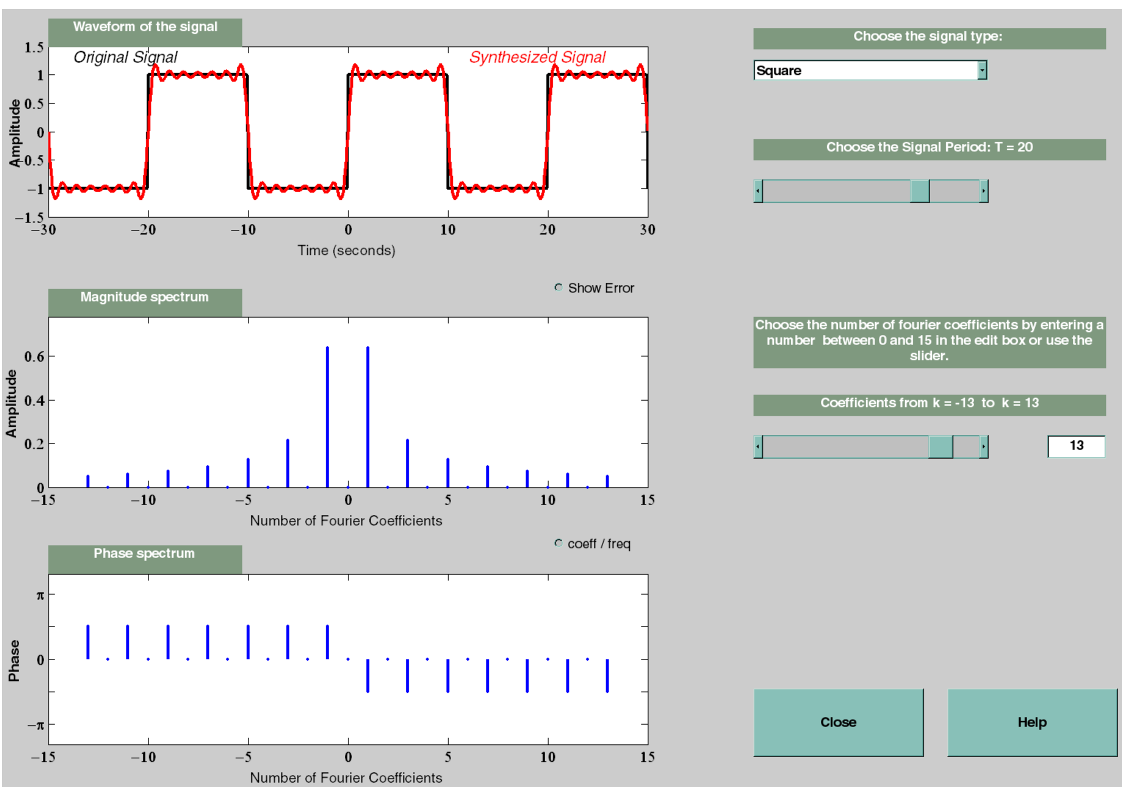

Figure 3: `FSeriesDemo` interface for Fourier Series synthesis.

3 Warm-up: Compute Outputs with the GUIs

The objective of the warm-up in this lab is to use the GUIs to study how systems process different kinds of signals. Write down your observations on the *Verification Sheet.*

3.1 Convolving Continuous-Time Signals

In the `cconvdemo` GUI, you can select an input signal $x(t)$, as well as the impulse response of an ***analog*** filter $h(t)$. Then the demo shows the "flipping and shifting" used when a convolution integral is performed. Figure 1 shows the interface for the `cconvdemo` GUI.

3.1.1 Convolving Rectangles

In the Warm-up, you should perform the following steps with the `cconvdemo` GUI.

(a) Set the input to a 4-second pulse $x(t) = u(t) - u(t-4)$.

(b) Set the filter's impulse response to a 2-second pulse with amplitude $\frac{1}{2}$, i.e., $h(t) = \frac{1}{2}\{u(t) - u(t-2)\}$.

(c) Use the GUI to produce the output signal. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution.

(d) Set the filter's impulse response to a 4-second pulse with amplitude $\frac{1}{4}$, i.e., $h(t) = \frac{1}{4}\{u(t) - u(t-4)\}$. Use the GUI to produce the output signal.

(e) Set the filter's impulse response to a shifted impulse, i.e., $h(t) = \delta(t-3)$. Use the GUI to produce the output signal.

(f) Compare the outputs from parts (c), (d) and (e). Notice the different shapes (triangle, rectangle or trapezoid), the different maximum values, and the different lengths of the outputs. Be prepared to explain these differences.
If the duration of $x(t)$ is T_x and the duration of $h(t)$ is T_h, what will the duration of $y(t)$ be?

3.1.2 Convolving Exponentials

In the warm-up, you should perform the following steps with the `cconvdemo` GUI.

(a) Set the input to a rectangular pulse: $x(t) = \{u(t) - u(t-3)\}$.

(b) Set the filter's impulse response to an exponential: $h(t) = e^{-0.25t}\{u(t) - u(t-7)\}$.

(c) Use the GUI to produce a plot of the output signal. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution. Note: if you move the hand tool past the end of the plot, the plot will automatically scroll in that direction.

(d) The top panel is a plot of $x(\tau)$, overlayed with the "flipped" impulse response $h(t-\tau)$ used to produce the "flip and slide" effect of convolution. The middle panel shows the integrand which is the product of $x(\tau)$ and $h(t-\tau)$. The top two plots are functions of τ, while the bottom plot of $y(t)$ is a function of t. Observe that the output $y(t)$ is composed of five distinct regions: no overlap (on the left side), partial overlap (on the left side), complete overlap, partial overlap (on the right side), and no overlap (on the right side). If you substitute $x(t)$ and $h(t)$ from from parts (a) and (b) into Eq. (1), you can show that the output is given by the piecewise equation

$$y(t) = \begin{cases} 0 & t < T_0 & \textit{Region } 1 \\ \int_{L_1}^{L_2} e^{-0.25(t-\tau)} d\tau & T_1 \le t < T_2 & \textit{Region } 2 \\ \int_{L_3}^{L_4} e^{-0.25(t-\tau)} d\tau & T_3 \le t < T_4 & \textit{Region } 3 \\ \int_{L_5}^{L_6} e^{-0.25(t-\tau)} d\tau & T_5 \le t < T_6 & \textit{Region } 4 \\ 0 & T_7 \le t & \textit{Region } 5 \end{cases} \tag{5}$$

Use the GUI to observe that $y(t)$ does indeed have five distinct regions, and use it to confirm that

$$T_0 = 0,\ T_1 = 0,\ T_2 = 3,\ T_3 = 3,\ T_4 = 7,\ T_5 = 7,\ T_6 = 10,\ \text{and } T_7 = 10$$

are the correct values for the boundaries of the regions. Then determine the limits of integration for each integral above. Make sure that you are flipping and sliding $h(t)$. *Also, notice that the limits of integration might depend on the variable t.*

Instructor Verification (separate page)

3.2 Frequency Response of an Analog Filter

In the lab project, you will use a continuous-time LTI system for filtering. In this section of the warm-up, we will investigate the following frequency response:

$$H(j\omega) = \frac{b}{a + j\omega} \tag{6}$$

where a controls the bandwidth of the filter.

(a) Make a plot of the magnitude and phase of $H(j\omega)$ versus ω in rad/s. Pick the parameters of the frequency response to be $a = 40\pi$, and $b = 40\pi$. In order to get values for the plot, you should evaluate the $H(j\omega)$ formula directly for a dense grid of frequencies. Use a range of frequencies that extends from -500 rad/s to $+500$ rad/s.[1] From the plot of $|H(j\omega)|$ versus ω, determine what kind of filter $H(j\omega)$ is.

(b) Determine the peak value of the magnitude (frequency) response and the location of the peak. Use the algebraic form of the frequency response formula $H(j\omega)$ to explain that the peak value is correct.

3.3 Sinusoidal Response of LPF

The **`CLTIdemo`** GUI can implement the LPF defined by Eq. (6) if you choose the filter named "First-Order Lowpass."

(a) Use the `CLTIdemo` GUI to create a first-order lowpass filter by selecting "First-Order Lowpass" from the menu and setting the cutoff frequency to 20 Hz.[2] This should be the same frequency response as in Section 3.2.

[1] You can plot the frequency response versus frequency in hertz or radians/sec. Either way is acceptable, but make sure that you label the horizontal axis.

[2] The `CLTIdemo` GUI will convert the frequencies from hertz to rad/s.

(b) Set the input signal to

$$x(t) = 1.0 + \cos(40\pi t)$$

Look at the output and compare its amplitudes and phases to the input amplitudes and phases. Click the box labeled "Theoretical Answer" and record the result.

(c) Now change the input signal to $x(t) = \cos(80\pi t)$, and record the numerical values of the output signal's amplitude and phase. Repeat for $x(t) = \cos(120\pi t)$, again recording the amplitude and phase of the output signal.

(d) Now consider the case where the input signal $x(t)$ is a square wave with a period of 1/20 secs. Although the **`FseriesDemo`** GUI in Section 2.3 cannot handle this value for the period, it can still be used to make a plot the spectrum of this type of signal. The spectrum values plotted are the magnitudes of $\{a_k\}$ for this square wave which are[3]

$$a_k = \frac{1 - e^{-j\pi k}}{j\pi k} - \delta[k]$$

Use the $\{a_k\}$ coefficients and the frequency response of the filter in order to write the first few terms of the output signal $y(t)$ as a sum of cosines.

$$y(t) = B_0 + B_1 \cos(\omega_0 t + \psi_1) + B_2 \cos(2\omega_0 t + \psi_2) + B_3 \cos(3\omega_0 t + \psi_3) + \dots$$

Use the values of the frequency response from the **`CLTIdemo`** GUI and the $\{a_k\}$ coefficients above to determine the numerical values for ω_0, B_0, B_1, B_2, B_3, and ψ_1, ψ_2, and ψ_3.

Instructor Verification (separate page)

The calculation above amounts to an analysis of how you can "filter" the periodic input signal (the square wave from Section 2.3) through a continuous-time LTI system whose frequency response is given in Eq. (6) of Section 3.2. Since this is an analog system, we cannot do the actual filtering in MATLAB; instead, we can only calculate what the output signal would be by finding the Fourier Series of the output.

4 Lab Exercises

In each of the following exercises, you should make a screen shot of the final picture produced by the GUI to validate that you were able to do the implementation. In all cases, you will have to do some mathematical calculations to verify that the MATLAB GUI result is correct.

4.1 Convolving Exponentials

In the first exercise, you should perform the following steps with the **`cconvdemo`** GUI.

(a) Set the input to an exponential: $x(t) = 6e^{-0.5t}\{u(t) - u(t-4)\}$.

(b) Set the filter's impulse response to an exponential: $h(t) = e^{0.5t}\{u(t+2) - u(t-2)\}$.

(c) Use the GUI to produce a plot of the output signal. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution. Note: if you move the hand tool past the end of the plot, the plot will automatically scroll in that direction.

[3] Section 3-6.1 of the *SP-First* textbook has the derivation of the Fourier Series coefficients for the square wave.

(d) The top panel is a plot of $x(\tau)$, overlayed with the "flipped" impulse response $h(t-\tau)$ used to produce the "flip and slide" effect of convolution. The middle panel shows the integrand, $J(\tau,t)$, which is the product of $x(\tau)$ and $h(t-\tau)$. The top two plots are functions of τ, while the bottom plot of $y(t)$ is a function of t. Observe that the output $y(t)$ is composed of five distinct regions: no overlap (on the left side), partial overlap (on the left side), complete overlap, partial overlap (on the right side), and no overlap (on the right side). If you substitute $x(t)$ and $h(t)$ from from parts (a) and (b) into Eq. (1), you can show that the output is given by the piecewise equation

$$y(t) = \begin{cases} 0 & t < T_0 & \mathit{Region}\ 1 \\ \int_{L_1}^{L_2} J(\tau,t)d\tau & T_1 \le t < T_2 & \mathit{Region}\ 2 \\ \int_{L_3}^{L_4} J(\tau,t)d\tau & T_3 \le t < T_4 & \mathit{Region}\ 3 \\ \int_{L_5}^{L_6} J(\tau,t)d\tau & T_5 \le t < T_6 & \mathit{Region}\ 4 \\ 0 & T_7 \le t & \mathit{Region}\ 5 \end{cases} \tag{7}$$

Use the GUI to observe that $y(t)$ does indeed have five distinct regions, and use it to help you figure out the values for the boundaries of the regions and the limits of integration for each part. Determine whether you are flipping $x(t)$ or $h(t)$. *Hint: The limits of integration might depend on the variable t.*

You should discover that one of the regions is actually a point, but explain how to find the FIVE regions for this convolution integral. Use the plots of $x(\tau)$ and $h(t-\tau)$ together with the corresponding plot of $y(t)$ to complete the following table with the correct values for the time region boundaries and integral limits in Eq. (7).

	$T_0 =$			Region 1
$T_1 =$	$T_2 =$	$L_1 =$	$L_2 =$	Region 2
$T_3 =$	$T_4 =$	$L_3 =$	$L_4 =$	Region 3
$T_5 =$	$T_6 =$	$L_5 =$	$L_6 =$	Region 4
	$T_7 =$			Region 5

(e) Write down the expression for the integrand which was denoted by $J(\tau,t)$ above.

(f) Finally, determine the mathematical formula for the convolution in each of the five regions. Use the GUI to help in setting up the integrals, but carry out the mathematics of the integrals by hand.

4.2 Continuous-Time Convolution: Transient and Steady State

(a) Use the **cconvdemo** to set up the problem of finding the output of an analog filter whose impulse response is

$$h(t) = 0.4\pi e^{-0.4\pi t}u(t)$$

when the input is

$$x(t) = \sin(2\pi t)u(t)$$

When entering the signals, you must make them finite-duration, so make the sinusoid 100 secs. long and the exponential 10 secs.

(b) Use the GUI to observe the output signal and the *transient* region at the start of the convolution. Determine the length of the transient region (in secs).
Note: Normally the transient regions is a region of partial overlap, while the region of complete overlap is called the *steady state region*. However, in this case, an approximation is needed because the steady-state behavior appears to start after a short amount of time, so find the start-up duration where the output has not settled into into its steady-state sinusoidal form.

(c) Perform the mathematics of the convolution integral to get the *exact analytic form* of the output signal and verify that the GUI is correct. Also verify that the starting and ending times of the transient region of the output signal are (approximately) correct.
Hint: it is rather difficult to convolve a one-sided sinusoid with a one-sided exponential, but there is a simplification available if complex exponentials are used. Since $\sin(2\pi t)$ is the imaginary part of $e^{j2\pi t}$, and ***since the impulse response is real-valued*** the desired convolution can be obtained as

$$x(t) * h(t) = \Im m \left\{ e^{j2\pi t}u(t) * 0.4\pi e^{-0.4\pi t}u(t) \right\}$$

Thus we would convolve two one-sided exponentials for which there is a standard form.

(d) Express your final answer in the form

$$y(t) = M\cos(2\pi t + \psi)u(t) + Ce^{-0.4\pi t}u(t)$$

by giving the numerical values for M, ψ, and C.

4.3 Frequency Response: Finding the Steady State Without Convolution

Unfortunately, the **`CLTIdemo`** GUI cannot directly implement the system in Section 4.2. However, we can make a system that is equivalent. Let the frequency response of the system be

$$H(j\omega) = \frac{40\pi}{40\pi + j\omega}$$

and the input signal be the sinusoid

$$x(t) = \sin(200\pi t)$$

(a) Set up this problem in the **`CLTIdemo`** GUI, and then determine the formula for the output signal.

(b) Do an evaluation of the frequency response (by hand) to verify that the numbers in the GUI are correct. Evaluate the frequency response at the "frequency of interest."

(c) Compare the output from the **`CLTIdemo`** GUI to the values of M and ψ found in Section 4.2(d). Explain why there is a match.

4.4 Summarize with a Concept Map

For the Summary section of your lab report, draw a concept map that contains at least five concepts that were used during this lab. Since experts use many links between concepts, try to produce a map that has a high "link to node" ratio. Use the Concept Navigation Tool (***CNT***) to produce the map.

Possible concept names might be: Transient, Steady-State, Bandpass Filter, Lowpass Filter, Highpass Filter, Unit Step, Unit Impulse, Passband, Stopband, Sinusoid, Phasor Addition, z-Transform, Fourier Transform, Convolution, FIR Filter, IIR Filter, One-Sided Exponential, Fourier Series, Fundamental Frequency, Frequency Response, Causality, Impulse Response, Aliasing, Nulling Filter, Convolution, Linearity, Time-Invariance, Hamming Window, Sinc Function, Running Average, First Difference, and Stem Plot. This is meant to be suggestive of concept names, but you are not restricted to this list.

Please print out your concept map for your lab report, but also try to ***save it to the web*** by using that option in ***CNT.*** Include an identifier that refers to Lab #10.

Lab #10
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INSTRUCTOR VERIFICATION PAGE

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ______________________________ Date of Lab: __________

Part 3.1.2: Demonstrate that you can run the continuous-time convolution demo. Explain how to find the FIVE regions for this convolution integral. Use the plots of $x(\tau)$ and $h(t-\tau)$ together with the corresponding plot of $y(t)$ to complete the following table with the correct values for the integral limits in Eq. (5).

	$T_0 = 0$			Region 1
$T_1 = 0$	$T_2 = 3$	$L_1 =$	$L_2 =$	Region 2
$T_3 = 3$	$T_4 = 7$	$L_3 =$	$L_4 =$	Region 3
$T_5 = 7$	$T_6 = 10$	$L_5 =$	$L_6 =$	Region 4
	$T_7 = 10$			Region 5

Note that the area under the curve in the middle plot is shaded green. When you set the time indicator to $t = 5$, how is the shaded area related to $y(5)$? How does the shaded area tell you the limits of integration?

Explain the above answers to your TA.

Verified:____________________ Date/Time:__________

Part 3.3: Find the $\{b_k\}$ Fourier Series coefficients of the output signal. Plot the spectrum versus frequency for the output signal. List the values of ω_0, a_k , $H(jk\omega_0)$, b_k , B_k and ψ_k in the table below. *Note:* $B_k \neq b_k$.

	$\omega_0 =$	$H(jk\omega_0)$			
$k = 0$	$a_0 =$		$b_0 =$	$B_0 =$	
$k = 1$	$a_1 =$		$b_1 =$	$B_1 =$	$\psi_1 =$
$k = 2$	$a_2 =$		$b_2 =$	$B_2 =$	$\psi_2 =$
$k = 3$	$a_3 =$		$b_3 =$	$B_3 =$	$\psi_3 =$

Verified:____________________ Date/Time:__________