

EE-2025

Spring-2004

LECTURE #3
Phasor Addition Theorem
12-Jan-04

Help Sessions/Recitation

- **Bring Calculator to Recitation**
 - **Complex arithmetic**
- VanLeer 361 Sunday, 2 PM
- VanLeer 361 Mon, Tues, Wed @ 6 PM
 - These are set up in VL-361 which has computers, so students can ask questions about the labs and about MATLAB. Please take advantage of these tutorial sessions.
 - If no one attends, the TA will depart after 20 mins.

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Web-CT Info

- Check the Bulletin Board for msgs
- SP First URL:
www.ece.gatech.edu/~spfirst
 - Username = gt, password = student
- Lectures are being posted
 - PDF format (4 per page)
- Get PDF file of Lab #1 from WebCT
- Get PDF file of HW #2 from WebCT
 - **HW #1 due this week (in Recitation)**

Homework Formatting

- Include a Cover page with
 - Name
 - Lab section, ie, L05, L20, etc.
 - Recitation Prof's name
 - **Download example from Web-CT**
- Write on **ONE** side only
 - Use Engineer's paper or plain paper
- STAPLE

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Lab Info

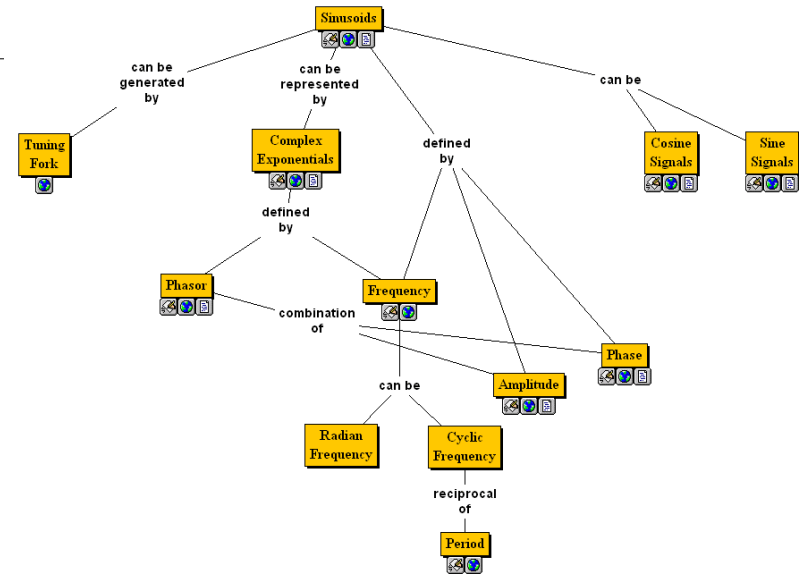
- **Be prepared for on-Line Pre-Lab Questions**
 - Start within first 15 minutes of Lab
- Lab #1 Verifications: Turn in at end of Lab
 - Counts as part of the Lab Report score
- Lab Report due one week later
- Lab **FAQs** are posted, but relate to old labs
- Learn **your** Lab TA's format requirements

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Concept Map for Chapter 2



JUMP

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 2, Section 2-6
- Other Reading:
 - Appendix A: Complex Numbers
 - Appendix B: MATLAB
 - Next Lecture: start Chapter 3

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LECTURE OBJECTIVES

- Phasors = Complex Amplitude
 - Complex Numbers **represent** Sinusoids

$$z(t) = Xe^{j\omega t} = (Ae^{j\phi})e^{j\omega t}$$

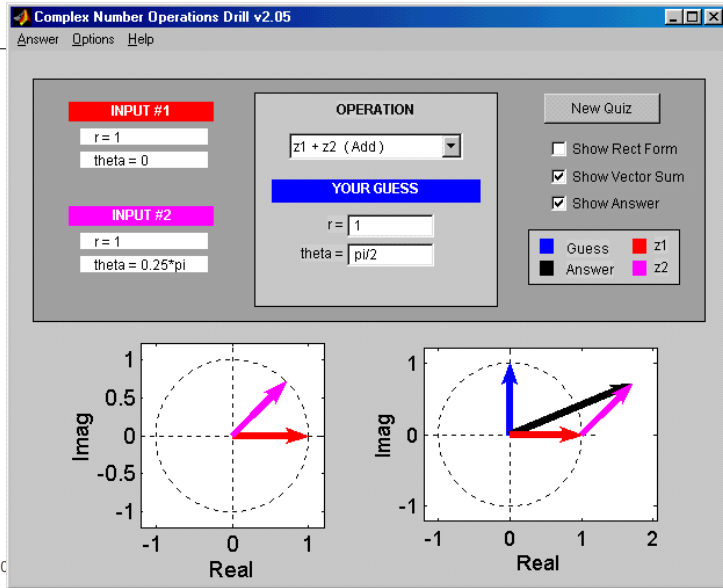
- Develop the ABSTRACTION:
 - Adding Sinusoids = Complex Addition
 - **PHASOR ADDITION THEOREM**

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Z DRILL (Complex Arith)



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AVOID Trigonometry

- Algebra, even complex, is **EASIER !!!**
- Can you recall $\cos(\theta_1 + \theta_2)$?
- Use: real part of $e^{j(\theta_1 + \theta_2)} = \cos(\theta_1 + \theta_2)$

$$\begin{aligned}
 e^{j(\theta_1 + \theta_2)} &= e^{j\theta_1} e^{j\theta_2} \\
 &= (\cos \theta_1 + j \sin \theta_1)(\cos \theta_2 + j \sin \theta_2) \\
 &= \boxed{(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2)} + j(\dots)
 \end{aligned}$$

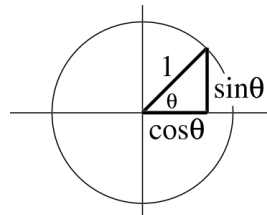
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Euler's FORMULA

- Complex Exponential**
 - Real part is cosine
 - Imaginary part is sine
 - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

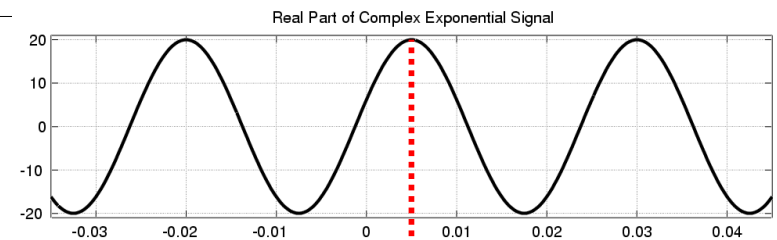
$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

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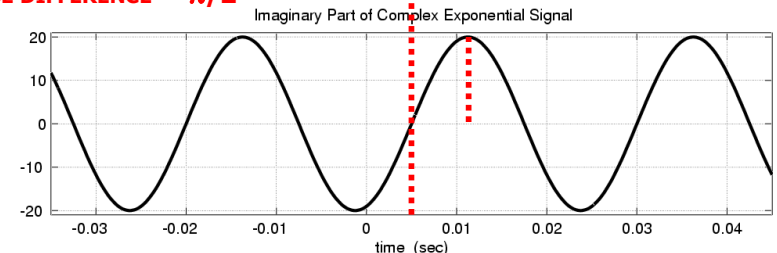
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Real & Imaginary Part Plots



PHASE DIFFERENCE = $\pi/2$



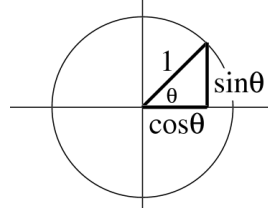
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COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

Interpret this as a **Rotating Vector**

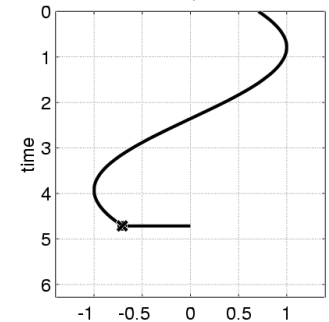
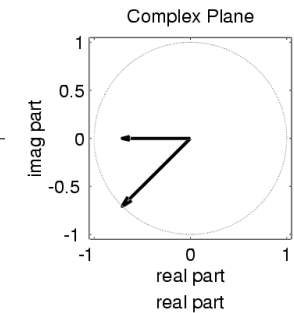
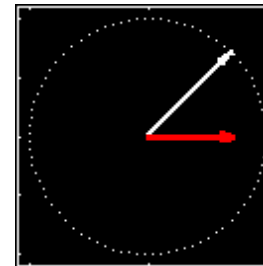
- $\theta = \omega t$
- Angle changes vs. time
- ex: $\omega = 20\pi$ rad/s
- Rotates 0.2π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

Rotating Phasor

See Demo on CD-ROM
Chapter 2



Cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A \cos(\omega t + \phi)$$

So,

$$\begin{aligned} A \cos(\omega t + \phi) &= \Re\{Ae^{j(\omega t + \phi)}\} \\ &= \Re\{Ae^{j\phi} e^{j\omega t}\} \end{aligned}$$

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A \cos(\omega t + \phi) = \Re\{Ae^{j\phi} e^{j\omega t}\}$$

Sinusoid = REAL PART of $(Ae^{j\phi})e^{j\omega t}$

$$x(t) = \Re\{Xe^{j\omega t}\} = \Re\{z(t)\}$$

Complex AMPLITUDE = X

$$z(t) = Xe^{j\omega t} \quad X = Ae^{j\phi}$$

POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3} \cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$\begin{aligned} x(t) &= \Re\left\{\sqrt{3}e^{j(77\pi t + 0.5\pi)}\right\} \\ &= \Re\left\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\right\} \end{aligned}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

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WANT to ADD SINUSOIDS

- ALL SINUSOIDS have **SAME** FREQUENCY
- HOW to GET **{Amp,Phase}** of RESULT ?

$$x_1(t) = 1.7 \cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9 \cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532 \cos(2\pi(10)t + 141.79\pi/180)$$

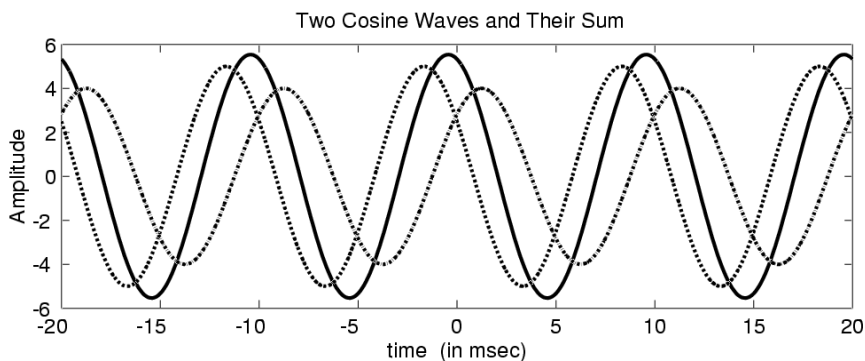
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ADD SINUSOIDS

- Sum Sinusoid has **SAME** Frequency



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PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^N A_k \cos(\omega_0 t + \phi_k)$$

$$= A \cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^N A_k e^{j\phi_k} = A e^{j\phi}$$

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Phasor Addition Proof

$$\begin{aligned}
 \sum_{k=1}^N A_k \cos(\omega_0 t + \phi_k) &= \sum_{k=1}^N \Re \{ A_k e^{j(\omega_0 t + \phi_k)} \} \\
 &= \Re \left\{ \sum_{k=1}^N A_k e^{j\phi_k} e^{j\omega_0 t} \right\} \\
 &= \Re \left\{ \left(\sum_{k=1}^N A_k e^{j\phi_k} \right) e^{j\omega_0 t} \right\} \\
 &= \Re \{ (A e^{j\phi}) e^{j\omega_0 t} \} = A \cos(\omega_0 t + \phi)
 \end{aligned}$$

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POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

$$\begin{aligned}
 x_1(t) &= \cos(77\pi t) \\
 x_2(t) &= \sqrt{3} \cos(77\pi t + 0.5\pi)
 \end{aligned}$$

- COMPLEX ADDITION:

$$1e^{j0} + \sqrt{3}e^{j0.5\pi}$$

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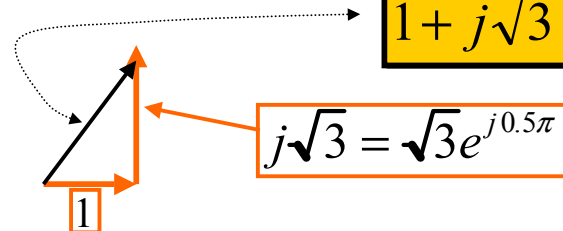
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POP QUIZ (answer)

- COMPLEX ADDITION:

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$



- CONVERT back to cosine form:

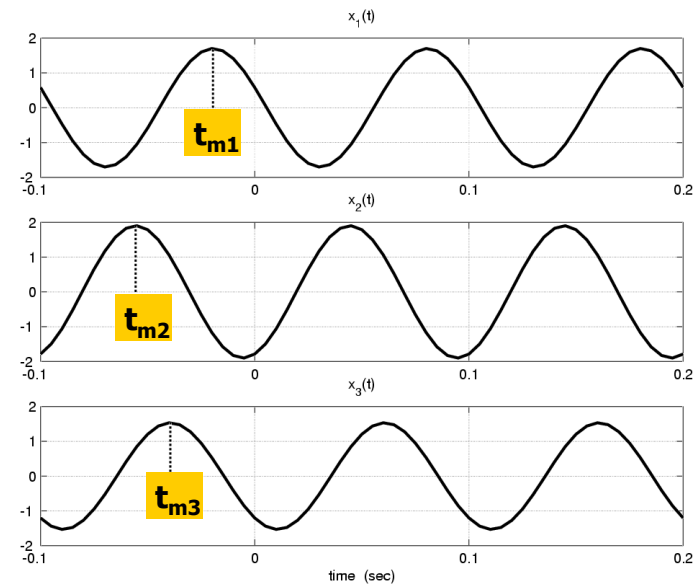
$$x_3(t) = 2 \cos(77\pi t + \frac{\pi}{3})$$

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ADD SINUSOIDS EXAMPLE



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Convert Time-Shift to Phase

- Measure **peak times**:
 - $t_{m1} = -0.0194$, $t_{m2} = -0.0556$, $t_{m3} = -0.0394$
- Convert to **phase** ($T=0.1$)
 - $\phi_1 = -\omega t_{m1} = -2\pi(t_{m1}/T) = 70\pi/180$,
 - $\phi_2 = 200\pi/180$
- Amplitudes
 - $A_1 = 1.7$, $A_2 = 1.9$, $A_3 = 1.532$

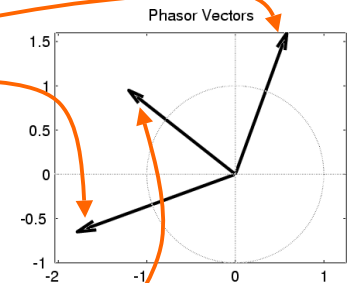
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Phasor Add: Numerical

- Convert Polar to Cartesian
 - $X_1 = 0.5814 + j1.597$
 - $X_2 = -1.785 - j0.6498$
 - sum =
 - $X_3 = -1.204 + j0.9476$
- Convert back to Polar
 - $X_3 = 1.532$ at angle $141.79\pi/180$
 - This is the sum



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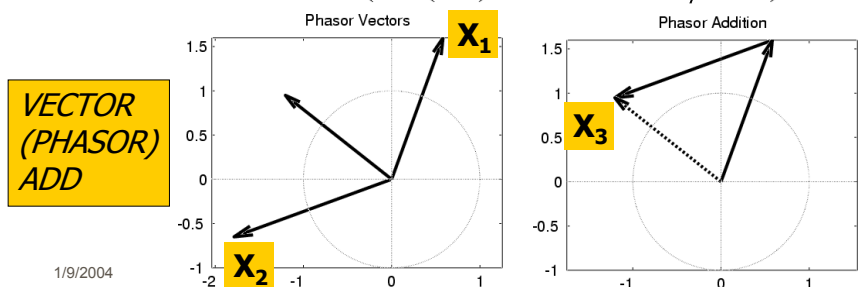
ADD SINUSOIDS

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